Model independent extraction of the axial mass parameter in CCQE anti neutrino-nucleon scattering

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October 17, 2014

Abstract

The study of neutrino oscillations depend on a consistent value for the axial mass. In order to facilitate this, a model-independent extraction of the axial mass parameter from quasielastic (anti)neutrino-nucleon scattering is needed. Most studies employ a model-dependent extraction using the dipole model of the axial form factor. This paper utilizes a model-independent description using the z-expansion of the axial form factor. Using this method on the reported mineral oil results from Fermilab's MiniBooNe experiment could lead to a consistent value for the axial mass. The first steps in analyzing the mineral oil will be to analyze the reported quasielastic antineutrino scattering data from C-12 using the z-expansion and propose a statistical model to be used to combine cross sections of carbon and hydrogen; which will be the primary focuses of this paper. The value obtained for the axial mass from the C-12 analysis, $0.85^{+0.12}_{-0.05}$ GeV differs greatly from the value used by the MiniBooNE collaboration, 1.35 GeV. We also propose a statistical model to be used to combine cross sections of carbon and hydrogen for future analysis of mineral oil data from MiniBoone.

1 Introduction

Several experiments over the past decades have shown that neutrinos can spontaneously convert from one flavor to another and back again. This phenomena is called neutrino oscillations. A solid understanding of this process could lead to a more fundamental understanding of physics beyond the Standard Model and leptonic CP violation.

In order to understand neutrino oscillations, a detailed knowledge of neutrino scattering between 0 and 10 Gev is needed. One method is to look at charged current quasielastic (CCQE) interaction of neutrinos, which allows certain parameters, like the axial mass, to be extracted which is essential for the study of neutrino oscillations.

The structure of this paper is as follows. Section 2 will discuss CCQE scattering, and the

MiniBooNE experiment whose data was used to perform this analysis. Section 3 will discuss the axial mass parameter and the axial form factor commonly used to obtain it. Section 4 will discuss the z-expansion model which was used in this analysis to obtain a model independent value of axial mass. Section 5 will present the extraction of the axial mass parameter and the results obtained. Section 6 proposes a statistical model to analyze the cross section of Mineral oil. Section 7 will discuss the results and further directions for this study.

2 Neutrino Oscillations

2.1 Charged Current Quasielastic Scattering

Charged current quasielastic scattering (CCQE) is defined as the scattering of a(n) (anti) neutrino off a nucleon such that a nucleon and a charged lepton in the same flavor as the neutrino are produced.

$$\nu_\mu + n \rightarrow \mu^- + p \tag{1}$$

$$\bar{\nu_\mu} + p \rightarrow \mu^+ + n. \tag{2}$$

In order to understand the CCQE interaction at a nucleon level, four form factors are needed, which can be obtained from the nucleon matrix element of the Standard Model weak charged current[1]

$$\begin{aligned}\langle p(p\prime)|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^{(p)}(p\prime)\{\gamma^\mu F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu \\ +F_2(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2) + \gamma^\mu\gamma_5 F_A(q^2)\}u^{(n)}(p).\end{aligned} \tag{3}$$

The first two are the vector form factors $F_1(q^2)$ and $F_2(q^2)$ which are related via isospin symmetry to the electromagnetic form factors that are measured in electron-nucleon scattering. The third from factor contribution $F_P(q^2)$ is suppressed by powers of the small lepton-nucleon mass ratio. This leaves only the final form factor $F_A(q^2)$ unconstrained. This axial-vector form factor is normalized at $q^2 = 0$ by neutron beta decay.

When working with CCQE interactions one must take into account the nucleon level interactions. To achieve this many experiments employ the Relativistic Fermi Gas model (RFG) (an in depth explanation of which can be found in [2]), and is employed in this analysis as well

2.2 MiniBooNE

MiniBooNE is a neutrino oscillation experiment at Fermi National Accelerator Lab with the primary goal to test for neutrino mass by searching for neutrino oscillations. One of the topics that the MiniBoone experiment has shed light on is charged-current (CC) scattering without pions in the final state. Most of which are charged-current quasielastic scattering (CCQE) of the muon neutrino on a bound nucleon. MiniBooNE is able to provide cross sections and differential cross sections that are normalized with a predicted neutrino flux, while also being able to extract muon energy and direction. In the energy range accessible to MiniBooNE, the axial vector form factor is the dominant contribution to the cross section and controls the Q^2 dependence. The CCQE interaction produces charged particles in the final state. These charged particles propagating through the detector oil produce optical photons via Cherenkov radiation. The identification of CCQE interactions in the detector relies solely on the detection of the Cherenkov light from the primary muon and the associated decay electron.

3 The Axial Mass Parameter

The primary focus for this research was to determine $F_A(q^2)$ in the physical region of CCQE scattering. To obtain a definition of the axial mass parameter we expand $F_A(q^2)$ at q^2=0

$$F_A(q^2)=F_A(0)[1+\frac{2}{m_A^2}q^2\ +\ldots\ ]\Rightarrow m_A\equiv\sqrt{\frac{F_A(0)}{F\prime_A(0)}}. \tag{4}$$

The world averaged value for the axial mass parameter is m_A^{dipole}=1.026 ± 0.021 GeV [3]. Recently reported values from different experiments have shown discrepancy in the value of the axial mass parameter and are larger than that of the world averaged value. These experiments include NOMAD, SciBooNE, MINOS, and MiniBooNE. These experiments are performed at lower neutrino energies using heavier nuclei as targets specifically carbon and oxygen and indicate the value of m_A to be larger than the world averaged value. Since the axial form factor is the dominant form factor in the CCQE interaction and controls the q^2 dependence, a consistent value for the axial mass parameter is needed in order to facilitate a deeper understanding of neutrino scattering at these energies, It is for this reason, we must understand the source of the discrepancy between these experiments. The most commonly used method to determine the axial mass parameter is the dipole form of the axial form factor

$$F_A^{dipole}(q^2)=\frac{F_A(0)}{[1-\frac{q^2}{(m_A^{dipole})^2}]^2}. \tag{5}$$

Since the value of the axial form factor at zero is known from neutron decay, the dipole form of the axial form factor provides a very simple method for extracting the m_A^{dipole}. However, this could be one of the primary causes of the discrepancy between recently reported values. The parameterizations within this dipole form are overly constrained and thus produce model dependent values for the axial mass. Measurements forced to fit this form factor will find different values for the axial mass parameter due to sensitivity at different energy ranges and create discrepancies.

It was for this reason that Bhattacharya, Hill, and Paz applied the method of the z expansion, which uses neutrino-nucleon CCQE scattering data from MiniBooNE, in an attempt to counteract the strong bias that the dipole form of the axial form factor introduces and distinguish nucleon-level interaction from nuclear effects. MiniBooNE has released data for the muon antineutrino double -differential CCQE scattering cross section.

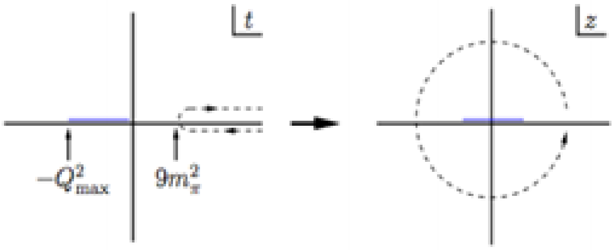

Figure 1: Confromal mapping of the cut plane onto the unit circle [1]

4 Z-Expansion

This paper pursues in the same style as the axial form factor analysis in [1]. The procedure makes model-independent constraints on the behavior of the axial form factor which makes use of the known analytic properties. The axial form factor is viewed as a function of the complex variable $t = q^2 = -Q^2$ The form factor is analytic outside of a cut at values of t beginning at the three-pion cut, t$_{cut}$=$9m_\pi^2$.

As seen in Figure 1, we can map the domain of analyticity onto the unit circle such that the physical region is mapped onto an interval

$$t(t, t_{cut}, t_0)=\frac{\sqrt{t_{cut}-t}-\sqrt{t_{cut}-t_0}}{\sqrt{t_{cut}-t}+\sqrt{t_{cut}-t_0}}, \tag{6}$$

where t_0 is a free parameter that represents the point mapping on to $z = 0$. Due to this analyticity, we can express the form factor as a power series in the new variable z,

$$F_A(q^2)=\sum_{k=0}^{\infty} a_k z(q^2)^k. \tag{7}$$

The coefficients are bounded in size to ensure convergence of the series. We use both a bound of $|a_k| \leq 5$ and a bound of $|a_k| \leq 10$. The bounding is described in detail in [4].

5 Extraction of the axial mass parameter

Using binned results for the double differential cross section, $\frac{d\sigma}{dE_u d\cos\theta}$ from MiniBooNE's antineutrino-nucleon CCQE scattering data for the proton bound within C-12, we apply the z expansion method to extract m$_A$. This analysis will use a definite nuclear model namely the Relativistic Fermi Gas Model as it is described in [1]. Using the final expression for the differential cross section of neutrino-nucleus scattering given in [1] and making changes due to the change in the incident particle type, this analysis obtained a differential cross section for antineutrino-nucleon scattering

$$\frac{d\sigma}{dE_u d\cos\theta}=\frac{G_F^2|\bar{P}_l|}{16\pi^2 m_T}\{2(E_l-|\bar{P}_l|\cos\theta_l W_1+(E_l+|\bar{P}_l|\cos\theta_l)W_2$$

$$-\frac{1}{m_T}[(E_l-|\bar{P}_l|\cos\theta_l(E_\mu+E_l)-m_l^2]W_3+\frac{m_l^2}{m_T}(E_l-|\bar{P}_l|\cos\theta_l)W_4-\frac{m_l^2}{m_T}W_5\}. \tag{8}$$

By integrating (8) over the energy-dependent $\bar{\nu}_\mu$ flux from Table XI of [5], and dividing the result by 6 to obtain the per-proton event rate, and dividing by the total flux to obtain a flux-averaged cross section, which gives us a theoretical prediction.

Numerical values used in this analysis for input parameters are the same as those used in [1]. Experimental values for the double differential cross section are obtained from (Table XIX) in [5] we also use the shape uncertainty from Table (XX) in [5]. Using this data we construct an error matrix

$$E_{ij}=(\delta\sigma_i)^2\delta_{ij}+(\delta N)^2\sigma_i\sigma_j, \tag{9}$$

where $\sigma_i=\frac{d\sigma}{dE_u d\cos\theta}\Delta E_u\Delta\cos\theta$ denotes a partial cross section, $\delta\sigma_i$ denotes the shape uncertainty, and $\delta N=0.172$ is the normalization error. Using the aforementioned error matrix a χ^2 function was constructed and minimized in order to find the best fit for m_A.

$$\chi^2=\Sigma_{ij}(\sigma_i^{expt.}-\sigma_i^{theory})E_{ij}^{-1}(\sigma_j^{expt.}-\sigma_j^{theory}). \tag{10}$$

We define error intervals by $\Delta\chi^2=1$, and employ parameters as in [1]. We define the vector form factors F_1 and F_2 by the BBA2003 parameterization [6]. The value used in [5] for the binding energy of carbon $\epsilon_b=0.030$GeV is a much higher value than that which the data favors, thus this analysis uses a value of $\epsilon_b=0.025$GeV which was extracted from electron scattering data [7]. From this data Q^2 can be reconstructed as a function of the observed muon energy, muon scattering angle, and incident antineutrino energy:

$$Q_{rec}^2=2E_\nu^{rec}E_\mu-2E_\nu^{rec}\sqrt{E_\mu^2-n_\mu^2}\cos\theta_\mu-m_\mu^2, \tag{11}$$

where E_ν^{rec} is given by

$$E_\nu^{rec}=\frac{m_N E\mu-\frac{m_\mu^2}{2}}{m_N-E_\mu+\sqrt{(E_\mu^2-m_\mu^2)}\cos\theta_\mu}. \tag{12}$$

Taking into account the effect of a cut on Q^2, This analysis investigates the sensitivity of m_A at low Q^2 data.

The results of our extraction are shown in Fig. 2 [1]and compare extractions of m_A^{dipole} in the dipole form of the axial form factor with extractions of m_A using the z expansion. The data presented here corresponds to $Q_{rec}^2\leq Q_{max}^2$ where to Q_{rec}^2 is given in [1]. For definitiveness we set $Q_{max}^2=1.0$ GeV^2 and we find that the value obtained for axial mass to be

$$m_A=0.85_{-0.05}^{+0.12}\pm 0.12\text{GeV} \ \text{(antineutrino scattering)} \tag{13}$$

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[1] At 0.1 GeV there is an error in the analysis causing the value of $|a_k|\leq 5$ for C-12 to be skewed which at the time writing this report has not been identified

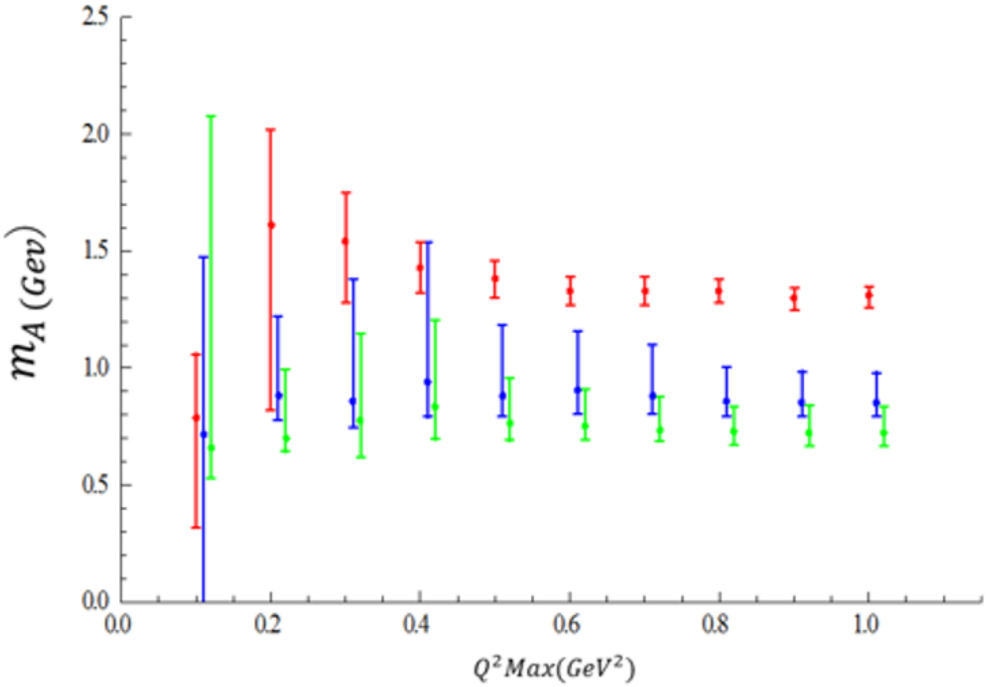

Figure 2: Extracted value of m_A versus Q^2_{max}. Dipole model results for m_A^{dipole} are shown in red; z expansion results with $|a_k| \leq 5$ are shown in blue, z expansion results with $|a_k| \leq 10$ are shown in green.

which is in agreement with the value obtained by Bhattacharya et al. in [1] for neutrino scattering

$$m_A = 0.85^{+0.22}_{-0.07} \pm 0.09\text{GeV} \ (\text{NEUTRINO SCATTERING}). \tag{14}$$

The first error is experimental, using the fit with $|a_k| \leq 5$, and the second error represents residual form factor shape uncertainty which is taken to be the maximum change of the 1σ interval when the bound is increased with $|a_k| \leq 10$. For the sake of comprehensiveness we compare values using the dipole form of the axial form factor with the same Q^2_{max} with Bhattacharya et al. in [1] . This analysis yielded $m_A^{dipole} = 1.31^{+0.04}_{-0.05}$ which is agreeable with the neutrino extraction done by Bhattacharya et al. in [1] which yielded $m_A^{dipole} = 1.31 \pm 0.05$GeV.

6 Statistical model for mineral oil analysis

Using the derived cross section for a free particle in [1] and the derived cross section for a target nucleus in [1] we were able to propose a statistical model for the mineral oil used in the MiniBooNe experiment. The Mineral oil has the chemical formula (C_nH_{2n+2} where n is $\approx$ 20). Utilizing this information and by making slight changes in the free particle and nucleus cross sections to represent hydrogen and carbon-12 we were able to create the following model

$$d\sigma = \frac{20\sigma_C + 42\sigma_H}{62}. \tag{15}$$

Where $d\sigma$ represents the newly constructed differential cross section. We obtained this model by taking into account the number of atoms present in a molecule of mineral oil, 20 for carbon and 42 for hydrogen and multiplying these factors by their respective cross sections σ_C and σ_H. We then divide this value over the total number of atoms present in the mineral oil. At the time of writing of this report we have not yet implemented this model in our analysis.

7 Summary

This analysis has presented a model independent extraction of the axial mass parameter for antineutrino-nucleon CCQE scattering on the proton with C-12. These results are in agreement with those published by Bhattacharya et al. The results of this analysis give further credence to the argument that the dipole form of the axial form factor is overly constrained and model dependent. Even though the errors from model-independent extraction are larger we obtain more a consistent value of axial mass which for further studies into the nature if CCQE interaction. This analysis has also proposed a method of combining the cross sections of carbon and hydrogen in order to understand CCQE interactions in the mineral oil used by the MiniBooNE experiment. The next step for this analysis is to implement the mineral oil data provided in [5] using the proposed statistical model, analyze the double differential cross section of the mineral oil, and extract a model-independent axial mass from it.

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