Untangling nuclear and form factor effects in neutrino-nucleon scattering

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Abstract

When studying neutrino oscillations an understanding of charged current quasielastic (CCQE) neutrino-nucleon scattering is imperative. This interaction depends on a nuclear model as well as knowledge of form factors. We started by familiarizing ourselves with the Relativistic Fermi Gas (RFG) nuclear model which is the standard for analyzing CCQE scattering. We then verified calculations concerning the scattering cross section from the RFG nuclear model. An alternative to this is the Correlated Fermi Gas (CFG) nuclear model. Our analysis using this new model yielded additional functions as the result of four possible transitions between the momenta of the initial and final nucleons, as opposed to the single transition from the RFG model. These new functions are given as integrals. We were able to present analytical solutions to these integrals which will simplify their implementation in a computer code. In the future we hope to use these functions to model the CCQE cross section using the CFG model.

1 Introduction

Experiments of the past few decades have shown evidence of a quantum mechanical phenomenon known as neutrino oscillations. In this process, a neutrino with specific flavor (electron, muon, or tau) can later be measured to have a different flavor. The observation of neutrino oscillations implies that neutrinos have non-zero mass contrary to the Standard Model of particle physics. The 2015 Nobel Prize in physics was awarded to Takaaki Kajita and Arthur B. McDonald for their discovery of neutrino oscillations. Understanding more about neutrino oscillations could provide insight into physics beyond the Standard Model.

When analyzing scattering CCQE cross sections form factors and a nuclear model are both necessary. One must sufficiently describe both the nuclear-level interaction and the nucleon-level interaction in order to generate a consistent result. While the effects of modifying the nuclear level analysis to include z expansion have been done previously ([1] and [2]), we focus on the implications of modifying the nuclear model.

This paper is structured as follows. Section 2 will discuss CCQE scattering. Section 3 will discuss nuclear models which can be used to analyze neutrino scattering, in particular how the Correlated Fermi Gas Model can be used. Section 4 will discuss the results of using the CFG model to analyze the scattering cross section, and the further directions of the project.

2 Neutrino Oscillations

2.1 Charged Current Quasielastic Scattering

Charged current quasielastic scattering (CCQE) is defined as the scattering of a(n) (anti) neutrino off a nucleon such that a nucleon and a charged lepton of the same flavor as the neutrino are produced.

$$\nu_\mu + n \rightarrow \mu^- + p \qquad (1)$$
$$\nu_\mu^- + p \rightarrow \mu^+ + n \qquad (2)$$

The (anti) neutrino-nucleus scattering has two levels: the interaction of the (anti) neutrino and the nucleus, and the (anti) neutrino and a nucleon. Both of these interactions must be understood in order to produce consistent results. While working with CCQE interactions one must take into account the nuclear level interactions. In order to achieve this many experiments employ the Relativistic Fermi Gas model (RFG) (discussed in section 3, and even more in depth in [3]).

In order to understand the CCQE interaction at a nucleon level, four form factors are used, which can be obtained from the nucleon matrix element of the Standard Model weak charged current [3]. The first two are the vector form factors $F_1\,(q^2)$ and $F_2\,(q^2)$ which are related via isospin symmetry to the electromagnetic form factors that are measured in electron-proton scattering. The next from factor contribution is $F_P\,(q^2)$ which is suppressed by powers of the small lepton-nucleon mass ratio. This leaves only $F_A(q^2)$ unconstrained, but one can use z expansion to the axial-vector form factor to represent it as a Taylor series in z. The use of z expansion introduces more parameters in the axial-vector form factor allowing more freedom in fits. Further details of this method can be found in [1] and [2].

3 Nuclear Models

3.1 Relativistic Fermi Gas Model

The Relativistic Fermi Gas model (RFG) discussed in [3] has been widely used to model scattering experiments. Here we view the nucleus cross section as a statistical average of the free nucleon cross section. The incoming (anti) neutrino interacts with a nucleon determined by some momentum distribution $n_i(\mathbf{p})$, and the final state nucleon phase space is limited by a factor of $[1-n_f(\mathbf{p}')]$ enforcing Fermi statistics. In this case $\mathbf{p}' = \mathbf{p} + \mathbf{q}$ where $\mathbf{q}$ is the momentum transfer. The formula for this cross section as given in [1] is,

$$\sigma_{bound} = n_i(\mathbf{p}) \otimes \sigma_{free} \otimes n_f(\mathbf{p}+\mathbf{q}) \tag{3}$$

In the RFG model we assume the maximum allowed momentum of the initial nucleon is p_F and the minimum allowed momentum of the final nucleon is also p_F. As given in [3] the initial and final state distributions for the RFG model are,

$$n_i(\mathbf{p}) = \begin{cases} 1 & \mathbf{p} < p_F \\ 0 & \mathbf{p} > p_F \end{cases} \tag{4}$$

$$(1 - n_f(\mathbf{p}+\mathbf{q})) = \begin{cases} 0 & \mathbf{p}+\mathbf{q} < p_F \\ 1 & \mathbf{p}+\mathbf{q} > p_F \end{cases} \tag{5}$$

A plot of the initial state distribution is below in Figure 1. In which the Fermi momentum is the cutoff point signaling the transition of momentum between nucleons.

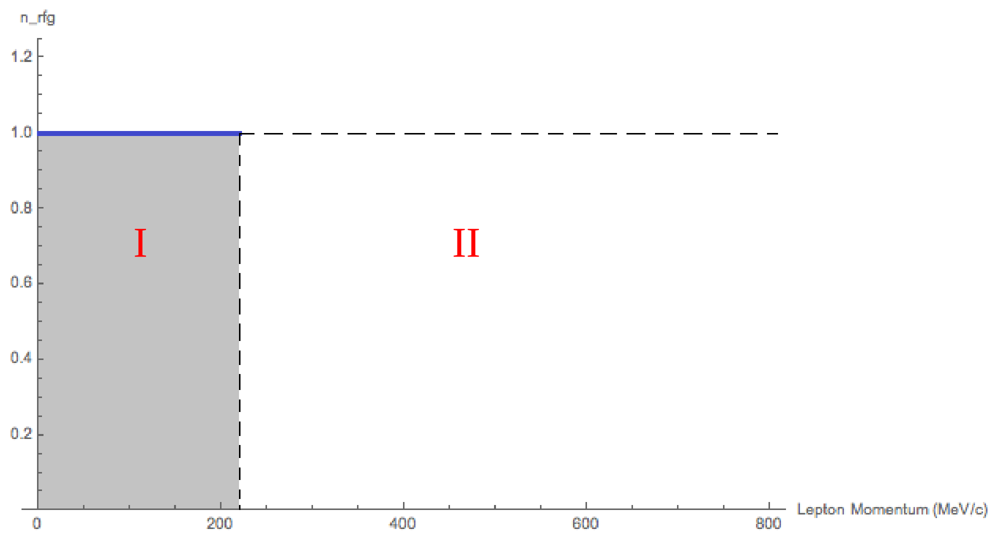

Figure 1: Plot of the initial state distribution for CCQE scattering from the RFG model in [3] using p_F = 220 MeV/c [5]. The first region corresponds to the momentum of the initial nucleon, while the latter region relates to the final state momentum.

The cross section in (3) is multi layered, and can be further expressed in terms of a hadronic tensor and leptonic tensor. Of which the hadronic tensor can be further broken down in terms of W_i functions, which are in turn products of a_i functions and H functions [3].

$$\sigma_{bound} = \frac{G_F^2}{4}\frac{1}{4|k \cdot p_T|}\int \frac{d^3\mathbf{k}'}{(2\pi)^3 2E_{k'}} L^{\mu\nu}W_{\mu\nu} \tag{6}$$

$$W_{\mu\nu} = -g_{\mu\nu}W_1 + \frac{p_\mu^T p_\nu^T}{m_T^2}W_2 - i\frac{\varepsilon_{\mu\nu\rho\sigma}}{2m_T^2}p_T^\rho q^\sigma W_3 + \frac{q_\mu q_\nu}{m_T^2}W_4 + \frac{\left(p_\mu^T q_\nu + q_\mu p_\nu^T\right)}{2m_T^2}W_5 \tag{7}$$

The momentum **k** is the momentum of the neutrino and is related to the momentum transfer **q** by the formula **k′** = **k** - **q** from [1].

These H functions are related to form factors, while the a_i functions are related to the nuclear model as in [3] and can be calculated from the momentum distributions in (4) and (5).

We can proceed to perform these calculations relating to the scattering cross section in terms of a_1, a_2, a_3, …, a_7 as in [1] and [3]. An example calculation for the a_1 function is provided below.

$$a_1 = \int d^3\mathbf{p}\, f(\mathbf{p},\mathbf{q}) \tag{8}$$

$$f(\mathbf{p},\mathbf{q}) = \frac{m_T V}{4\pi^2} n_i(\mathbf{p})\left[1 - n_f(\mathbf{p}+\mathbf{q})\right]\frac{\delta\left(\varepsilon_{\mathbf{p}} - \varepsilon'_{\mathbf{p+q}} + q^0\right)}{\varepsilon_{\mathbf{p}}\varepsilon'_{\mathbf{p+q}}} \tag{9}$$

Here an arbitrary volume V was chosen such that the final state nucleon momentum distribution can be effectively limited by the aforementioned factor of $n_f(\mathbf{p}')$.

$$a_1^{RFG} = \int d^3\mathbf{p}\,\frac{m_T V}{4\pi^2}\,n_i(\mathbf{p})\left[1-n_f(\mathbf{p}+\mathbf{q})\right]\frac{\delta\left(\varepsilon_{\mathbf{p}}-\varepsilon'_{\mathbf{p+q}}+q^0\right)}{\varepsilon_{\mathbf{p}}\varepsilon'_{\mathbf{p+q}}} \tag{10}$$

$$= \frac{m_T V}{2\pi|\mathbf{q}|}\int d|\mathbf{p}|\,d\cos(\theta)\frac{|\mathbf{p}|}{\left(E_{\mathbf{p}}-\varepsilon_b\right)}\delta\left(\cos(\theta)-\frac{\left(\omega_{eff}^2-|\mathbf{q}|^2+2\omega_{eff}E_{\mathbf{p}}\right)}{2|\mathbf{p}||\mathbf{q}|}\right)$$

$$= \frac{m_T V}{2\pi|\mathbf{q}|}\int dE_{\mathbf{p}}\frac{E_{\mathbf{p}}}{\left(E_{\mathbf{p}}-\varepsilon_b\right)} \equiv b_0^{RFG}$$

In this case the normalization V is fixed by requiring A/2 neutrons below the Fermi surface, such that

$$\frac{A}{2} = 2V\int_o^{p_F}\frac{V}{A_0}\frac{d^3\mathbf{p}}{(2\pi)^3}n_i(\mathbf{p}) \rightarrow V = \frac{3\pi^2 A}{2\mathbf{p}_F^3} \tag{11}$$

It should be noted that there is only a single possible non-zero combination of the momentum distributions (4) and (5) in the case of the RFG model. This combination is the result of a momentum transfer between the initial and final state nucleons where the initial nucleon has momentum less than the Fermi momentum, and the final nucleon's momentum is greater than the Fermi momentum. Additionally upon calculating each a_i function it is possible to express the results in terms of three b_i functions and various constants shown in the appendix of [1].

3.2 Correlated Fermi Gas Model

When working with CCQE interactions the Relativistic Fermi Gas model (RFG) is commonly used to account for the nucleon level interactions. Yet experiments show that tensor force induced short-range correlations (SRC) between proton-neutron pairs shift nucleons to high-momentum in symmetric nuclear matter (SNM) [4]. In order to account for this momentum shift we use the Correlated Fermi Gas model (CFG) when analyzing CCQE cross sections from neutrino nucleon scattering.

As given by [4], the initial state distribution from the CFG model is,

$$\tilde{n}_i(\mathbf{p}) \equiv n_{SNM}^{SRC}(\mathbf{p}) = \begin{cases} A_0 & \mathbf{p} < p_F \\ C_\infty/\mathbf{p}^4 & p_F < \mathbf{p} < \lambda p_F^0 \\ 0 & \mathbf{p} > \lambda p_F^0 \end{cases} \tag{12}$$

Where $p_F^0 = p_F = 220$ MeV/c [5] and $\lambda = 2.75 \pm 0.25$ [4]. C_∞ is the phenomenological height factor as given in [4], and A_0 is a constant from the normalization.

While an initial state momentum distribution is given in [4] we had to rework the distribution to keep our results consistent. We had to undo the normalization done in [4] because the authors normalized the momentum distribution to the number of nucleons as shown below in (13).

$$\frac{4\pi}{(2\pi)^3}\int_0^{\lambda \mathbf{p}_F^0} 2n_{SNM}^{SRC}(\mathbf{p})\mathbf{p}^2 d\mathbf{p} \equiv 1 \tag{13}$$

In order to represent this difference we in distributions we labeled the momentum distribution from [4] with $\tilde{n}_i(\boldsymbol{p})$ as shown in (12). This distribution differs from our initial momentum distribution due to a difference in normalizations because we normalized to A/2 as in [1], while [4] normalized to 1. This difference corresponds to the product of an arbitrary volume like that in (11) from the RFG model and our normalized momentum distribution given by $Vn_i(\mathbf{p})$. This gives us the relation $\tilde{n}(\mathbf{p}) = Vn_i(\mathbf{p})$, where once again our normalized momentum is rescaled such that $0 \leq n_i(\mathbf{p}) \leq 1$.

For the cases when $\lambda \neq 1$, which are the cases that result in the tail region we are interested in, we must divide each term in (12) by a factor of A_0 from [4]. The resulting renormalized initial and final state distributions can be calculated. After a change of variables ($C_\infty = C_0 * p_F$) and the renormalization from A_0 the resulting distribution(s) can be of the same form as those in [1] and [3]. They are included below accompanied by a plot of the initial state distributions in Figure 2.

$$n_i(\mathbf{p}) = \begin{cases} 1 & \mathbf{p} < p_F \\ \dfrac{C_0 p_F}{\mathbf{p}^4 A_0} & p_F < \mathbf{p} < \lambda p_F \\ 0 & \mathbf{p} > \lambda p_F \end{cases} \tag{14}$$

$$1 - n_f(\mathbf{p}+\mathbf{q}) = \begin{cases} 0 & \mathbf{p}+\mathbf{q} < p_F \\ 1 - \dfrac{C_0 p_F}{(\mathbf{p}+\mathbf{q})^4 A_0} & p_F < \mathbf{p}+\mathbf{q} < \lambda p_F \\ 1 & \mathbf{p}+\mathbf{q} > \lambda p_F \end{cases} \tag{15}$$

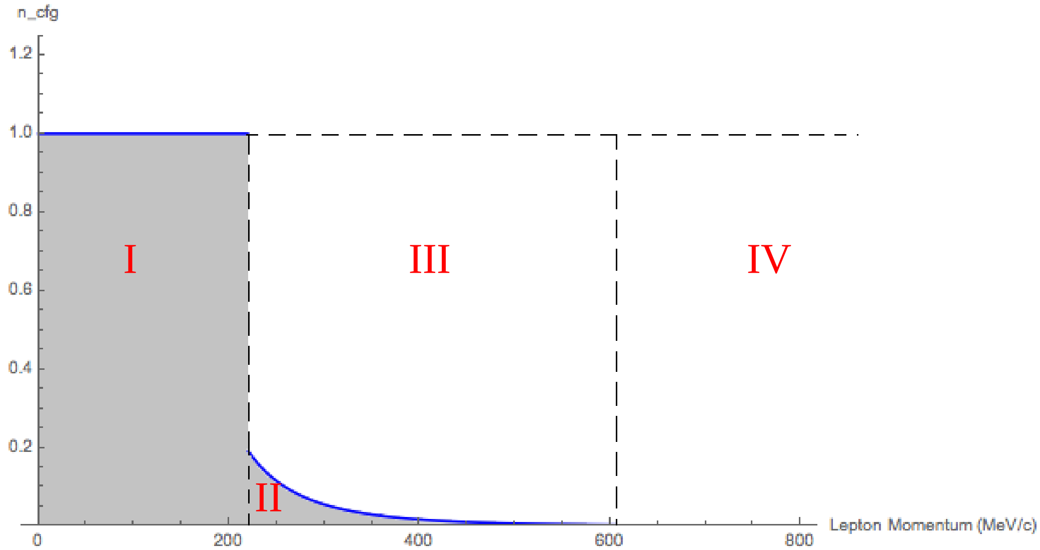

Figure 2: Plot of the initial state distribution for CCQE scattering using average values for C_0 and λ as given by [4], and p_F value given by [5]. The first two regions correspond to the momentum of the initial nucleon, while the latter two regions relate to the final state momentum.

Now notice what happens in the case where λ=1. When this is true the arbitrary volume V from the RFG model is equivalent to the A_0 in (12) [4]. In fact, in the limit λ=1 the tail-region of the momentum distribution disappears altogether, generating a distribution like that of the RFG model. For this case the constant V can be calculated in the limit that λ=1.

$$\tilde{n}(\mathbf{p}) = Vn_i(\mathbf{p})_{\lambda\to1} = \begin{cases} A_0 & \mathbf{p}<p_F \\ C_\infty/\mathbf{p}^4 & p_F<\mathbf{p}<\lambda p_F^0 \\ 0 & \mathbf{p}>\lambda p_F^0 \end{cases} = \begin{cases} A_0 & \mathbf{p}<p_F \\ 0 & \mathbf{p}>p_F^0 \end{cases} \rightarrow$$
$$\frac{\tilde{n}(\mathbf{p})}{A_0} = \frac{Vn(\mathbf{p})_{\lambda\to1}}{A_0} = \begin{cases} 1 & \mathbf{p}<p_F \\ 0 & \mathbf{p}>p_F^0 \end{cases} \tag{16}$$

$$1 = 2V\int_o^{p_F} \frac{V}{A_0}\frac{d^3\mathbf{p}}{(2\pi)^3} n_i(\mathbf{p}) \quad \rightarrow \quad V = A_0\big|_{\lambda=1} = \frac{3\pi^2}{p_F^3} \tag{17}$$

During scattering experiments momentum is transferred, and there are multiple transitions that can result. Of the nine possible combinations of the values for the initial state and final state distributions from (14) and (15), only four are non-zero, which correspond to the four transitions previously mentioned. Depending on the momentum transfer as well as the momentum of the initial nucleon, transitions can occur from region I to regions III and IV, and from region II to regions III and IV.

Using equations (14) and (15) and substituting them into,

$$f(\mathbf{p},\mathbf{q}) = \frac{m_T V}{4\pi^2} n_i(\mathbf{p})\left[1 - n_f(\mathbf{p}+\mathbf{q})\right] \frac{\delta\left(\varepsilon_{\mathbf{p}} - \varepsilon'_{\mathbf{p+q}} + q^0\right)}{\varepsilon_{\mathbf{p}} \varepsilon'_{\mathbf{p+q}}} \tag{18}$$

from [1], we can once again proceed to perform the necessary calculations relating to the scattering cross section in terms of a_1, a_2, a_3, …, a_7 as in [1] and [3]. However because of the introduction of the CFG model, there are now four different functions f(**p**,**q**) (one for each of the now possible transitions) that will each yield a different a_i function.

3.2.1 CFG Transition I → IV

Of the four f(**p**,**q**) functions, one of them is identical to the function given in [1] and [3] for the RFG model that is used to solve for the a_i functions. This case pertains to the momentum transition from region I to region IV where both corresponding values from equations (14) and (15) above are 1. Because this is equivalent to the results from using the RFG model to analyze the interaction the resulting a_i for this particular f(**p**,**q**) is the same as shown in [1] and [3] and as shown above.

$$a_1 = \int d^3\mathbf{p}\, f(\mathbf{p},\mathbf{q}) \tag{19}$$

$$a_1^{CFG}(I \to IV) = \int d^3\mathbf{p} \frac{m_T V}{4\pi^2} n_i(\mathbf{p})\left[1 - n_f(\mathbf{p}+\mathbf{q})\right] \frac{\delta\left(\varepsilon_{\mathbf{p}} - \varepsilon'_{\mathbf{p+q}} + q^0\right)}{\varepsilon_{\mathbf{p}} \varepsilon'_{\mathbf{p+q}}} \tag{20}$$

$$= \frac{m_T V}{2\pi|\mathbf{q}|} \int d|\mathbf{p}| d\cos(\theta) \frac{|\mathbf{p}|}{\left(E_{\mathbf{p}} - \varepsilon_b\right)} \delta\left(\cos(\theta) - \frac{\left(\omega_{eff}^2 - |\mathbf{q}|^2 + 2\omega_{eff} E_{\mathbf{p}}\right)}{2|\mathbf{p}||\mathbf{q}|}\right)$$

$$a_1^{CFG}(I \to IV) = a_1^{RFG} = \frac{m_T V}{2\pi|\mathbf{q}|} \int dE_p \frac{E_{\mathbf{p}}}{\left(E_{\mathbf{p}} - \varepsilon_b\right)} \tag{21}$$

In these cases where $n_i(\mathbf{p}) = 1$ and $(1 - n_f(\mathbf{p}+\mathbf{q})) = 1$, the resulting a_i integrals will all be equivalent to their RFG counterparts. Put simply, the transition from region I to region IV is equivalent to the transition modeled with the RFG model in that they both generate the same a_i functions, however the limits of integration for these CFG integrals have yet to be confirmed.

3.2.1 CFG Transition II → IV

By substituting the corresponding values from equations (14) and (15) into equation (18) The three other transitions (I to III, II to III, and II to IV) each result in a different function f(**p**,**q**). The a_i integrals relating to each of these transitions can be calculated, but as shown below become increasingly more tedious. The method used to solve for the a_1 integrals related to these three transitions is provided below, and could be used to calculate the other a_i functions in the future.

First start with an a_i function as seen in [1] and [3], here we start with a_1 as in equation (19):

$$a_1 = \int d^3\mathbf{p}\, f(\mathbf{p},\mathbf{q}) \tag{22}$$

Substitute the appropriate values for the initial and final state momentum distributions for the desired transition into equation (18), and then substitute that new f(**p**,**q**) function into your chosen a_i equation.

$$a_1^{CFG}(II \to IV) = \int d^3\mathbf{p}\, \frac{m_T V}{4\pi^2}\left(\frac{C_0 p_F}{(\mathbf{p})^4 A_0}\right)\frac{\delta\left(\varepsilon_{\mathbf{p}} - \varepsilon'_{\mathbf{p+q}} + q^0\right)}{\varepsilon_{\mathbf{p}}\varepsilon'_{\mathbf{p+q}}} \tag{23}$$

Next pull out constants from the integrand and notice that the integral has no dependence on phi, so a factor of 2π can also be pulled out.

$$a_1^{CFG}(II \to IV) = \frac{m_T V C_0 p_F}{2\pi A_0}\int d|\mathbf{p}|\, d\theta \sin(\theta)\left(\frac{|\mathbf{p}|^2}{(\mathbf{p})^4}\right)\frac{\delta\left(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p+q}} + q^0\right)}{\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p+q}}} \tag{24}$$

The next step is to modify the delta function so that it is a function of cos(θ), which can be done given (25), (26), and (27) below from the appendix of [1].

$$\varepsilon_{\mathbf{p}} = E_{\mathbf{p}} - \varepsilon_b, \quad \varepsilon_{\mathbf{p+q}} = E_{\mathbf{p+q}}, \quad \omega_{eff} = \omega - \varepsilon_b \tag{25}$$

$$\delta(g(\mathbf{p})) = \frac{1}{|g'(\mathbf{p}_i)|}\delta(\mathbf{p} - \mathbf{p}_i) \tag{26}$$

$$\delta\left(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p+q}} + q^0\right) = \delta\left(E_{\mathbf{p}} - E_{\mathbf{p+q}} + \omega_{eff}\right) = \frac{2E_{\mathbf{p+q}}}{2|\mathbf{p}||\mathbf{q}|}\delta\left(\cos(\theta) - \frac{1}{2|\mathbf{p}||\mathbf{q}|}\left(\omega_{eff}^2 - |\mathbf{q}|^2 + 2\omega_{eff}E_{\mathbf{p}}\right)\right) \tag{27}$$

Plugging this back into (24) yields

$$a_1^{CFG}(II \to IV) = \frac{m_T C_0 V p_F}{2\pi A_0}\int d|\mathbf{p}|\, d\cos(\theta)\frac{|\mathbf{p}|^2}{(E_{\mathbf{p}} - \varepsilon_b)E_{\mathbf{p+q}}}\frac{1}{\mathbf{p}^4}\frac{2E_{\mathbf{p+q}}}{2|\mathbf{p}||\mathbf{q}|}\delta\left(\cos(\theta) - \frac{1}{2|\mathbf{p}||\mathbf{q}|}\left(\omega_{eff}^2 - |\mathbf{q}|^2 + 2\omega_{eff}E_{\mathbf{p}}\right)\right) \tag{28}$$

Which can be further simplified due to the delta function and by pulling more constants out of the integrand to give:

$$a_1^{CFG}(II \to IV) = \frac{m_T C_0 V p_F}{2\pi A_0 |\mathbf{q}|}\int d|\mathbf{p}|\frac{|\mathbf{p}|}{(E_{\mathbf{p}} - \varepsilon_b)}\frac{1}{\mathbf{p}^4} \tag{29}$$

In combination with $\boldsymbol{p}^2 = E_{\boldsymbol{p}}^2 - m_N^2$, this integral can now finally be expressed in terms of an integration over the energy of the initial nucleon. It should also be noted at this point that when dealing with transitions to region III there will be a factor of $(\mathbf{p}+\mathbf{q})^4$ in the denominator of the integrand. These terms can also be simplified using the equation from energy momentum relations as well as the delta function found in (27). The process used to perform this simplification is included below.

$$\begin{aligned}\left((\mathbf{p}+\mathbf{q})^2\right)^2 &= \left((E_{\mathbf{p}}+\omega_{eff}-m_N)(E_{\mathbf{p}}+\omega_{eff}+m_N)\right)^2 = \left(E_{\mathbf{p}}{}^2+2E_{\mathbf{p}}\omega_{eff}+\omega_{eff}{}^2-m_N{}^2\right)^2 \\ &= \left(2E_{\mathbf{p}}\omega_{eff}+\omega_{eff}{}^2+\mathbf{p}^2\right)^2 = \left(\mathbf{p}^2+\mathbf{q}^2+2|\mathbf{p}||\mathbf{q}|\cos(\theta)\right)^2\end{aligned} \tag{30}$$

$$2|\mathbf{p}||\mathbf{q}|\cos(\theta) = \omega_{eff}{}^2 - |\mathbf{q}|^2 + 2E_{\mathbf{p}}\omega_{eff} \rightarrow \cos(\theta) = \frac{1}{2|\mathbf{p}||\mathbf{q}|}\left(\omega_{eff}{}^2 - |\mathbf{q}|^2 + 2E_{\mathbf{p}}\omega_{eff}\right) \tag{31}$$

From here we can see that this gives us our equation from our delta function which allows us to simplify our expressions.

Continuing with our calculation for the transition from region II to region IV we obtain:

$$a_1^{CFG}(II \rightarrow IV) = \frac{m_T C_0 V p_F}{2\pi A_0 |\mathbf{q}|}\int dE_{\mathbf{p}} \frac{E_{\mathbf{p}}}{(E_{\mathbf{p}}-\varepsilon_b)(E_{\mathbf{p}}^2-m_N^2)^2} \tag{32}$$

In order to evaluate this integral the method of partial fractions can now be used to expand the integrand into multiple terms that can be easily integrated.

$$\begin{aligned}&\int dE_{\mathbf{p}} \frac{E_{\mathbf{p}}}{(E_{\mathbf{p}}-\varepsilon_b)(E_{\mathbf{p}}^2-m_N^2)^2} = \\ &= \int dE_{\mathbf{p}} \left(\begin{aligned} &\frac{\varepsilon_b}{(E_{\mathbf{p}}-\varepsilon_b)(\varepsilon_b+m_N)^2(\varepsilon_b-m_N)^2} + \frac{-1}{4m_N(E_{\mathbf{p}}-m_N)(\varepsilon_b-m_N)^2} + \frac{1}{4m_N(E_{\mathbf{p}}+m_N)(\varepsilon_b+m_N)^2} \\ &+ \frac{-1}{4m_N(E_{\mathbf{p}}-m_N)^2(\varepsilon_b-m_N)} + \frac{1}{4m_N(E_{\mathbf{p}}+m_N)^2(\varepsilon_b+m_N)} \end{aligned} \right)\end{aligned} \tag{33}$$

Upon integrating each of these terms individually the final equation for the a_1 transition between regions II and IV is:

$$a_1^{CFG}(II \to IV) = \frac{-m_T C_0 V p_F}{2\pi A_0 |\mathbf{q}|}\left[-\frac{\varepsilon_b Log[E_\mathbf{p}-\varepsilon_b]}{(m_N-\varepsilon_b)^2(m_N+\varepsilon_b)^2} + \frac{Log[E_\mathbf{p}-m_N]}{4m_N(m_N-\varepsilon_b)^2} - \frac{Log[E_\mathbf{p}+m_N]}{4m_N(m_N+\varepsilon_b)^2} + \frac{1}{4m_N(E_\mathbf{p}-m_N)(m_N-\varepsilon_b)} + \frac{1}{4m_N(E_\mathbf{p}+m_N)(m_N+\varepsilon_b)} \right]\Bigg|_{E_{low}}^{E_{high}} \tag{34}$$

As we calculated the remaining integrals for the a_1 functions their respective partial fraction integrands became less and less trivial and involved more and more terms. However the method shown above was used to solve for each of these a_1 functions, and could be used to solve the remaining a_i functions from [1] and [3], for the CFG model.

4 Summary

After looking at the RFG nuclear model, we conducted an analysis using the CFG nuclear model for CCQE scattering. The four possible transitions from the CFG model each have unique functions, a_i, used to represent them similar to the RFG model. The equations relating to a_1 were calculated, and the method used to perform these calculations was shown. The future steps for this project are to determine the expressions for the remaining integrals for each a_i function, as well as confirm the limits of integration for said integrals. Furthermore it has not yet been determined if these expressions can be simplified for easier implementation (such as the b_i equations from [1] and [3]). Finally an analysis involving the use of the CFG model and z expansion could be performed to determine a value of the axial radius.

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References

[1] B. Bhattacharya, R. J. Hill and G. Paz, Phys. Rev. D 84, 073006 (2011) [arXiv:1108.0423 [hep-ph]].

[2] R. J. Hill and G. Paz, Phys. Rev. D 82, 113005 (2010) [arXiv:1008.4619 [hep-ph]].

[3] R. A. Smith and E. J. Moniz, Nucl. Phys. B 43, 605 (1972) [Erratum-ibid. B 101, 547 (1975)].

[4] O. Hen, B. A. Li, W. J. Guo, L. B Weinstein & E. Piasetzky, PR C 91, 025803 (2015) [arXiv:1408.0772 [nucl-ex]].

[5] E. J. Moniz, I. Sick, R. R. Whitney, J. R. Ficenec, R. D. Kephart, & W. P. Trower, Phys. Rev. Lett. 26, 445-448 (1971).