# Multiplicity-Momentum Correlations in Relativistic Nuclear Collisions

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## 1 Intro

Over this past summer I have been working with Sean Gavin and George Moschelli on how correlations of momentum and multiplicity could indicate signs of partial thermalization of the matter produced in relativistic nuclear collisions. At the relativistic Heavy Ion Collider (RHIC) and Large Hardron Collider (LHC) large nuclei collide at relativistic speeds to produce a deconfined quark-gluon medium that might be quark-gluon plasma. Hydrodynamic models have been successful at describing the dynamics of the evolution of the medium, but these models assume local thermal equilibrium. Small systems like proton-proton and proton-nucleus show signs of hydrodynamic like evolution in high multiplicity events, but were not expected to reach thermal equilibrium. It is likely that collision events of any kind never fully equilibrate. Therefore, it is important to find an independent indicator of the level of equilibration. In this paper we discuss how multiplicity-momentum correlations could signal partial thermalization. We analyze UrQMD simulated events of Au-Au collisions at a center of mass energy 200GeV per nucleon.

## 2 Multiplicity-Momentum Correlations

The UA1 experiment shown in figure 1 indicates that there is a positive correlation between average transverse momentum per particle $\langle p_t \rangle$ and charged multiplicity $N$ in proton-proton collisions [1]. Here the transverse momentum per particle is $\langle p_t \rangle = \langle \sum p_{t_i} \rangle / \langle N \rangle$ where $\sum p_{t_i} = P_t$ is the total transverse momentum of an event, and $\langle \ ... \ \rangle$ represents an average over events.

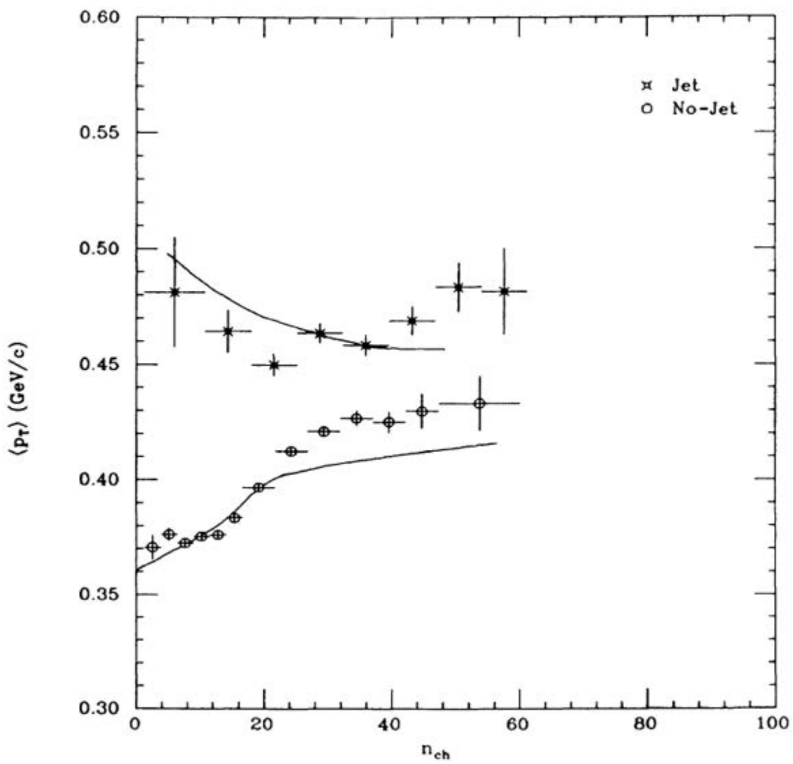


Figure 1: UA1 experimental data of average transverse momentum as a function of multiplicity [1]. top lines represent events with jets and bottom lines are events without jets. Points represent actual experiment data and solid lines are PYTHIA calculations.

As figure 1 illustrates, $\langle p_t \rangle$ indicates a positive correlation with multiplicity when there are no jets present and a negative correlation when there are jets in an event. From the data one could also argue that the introduction of jets makes no correlation between $p_t$ and $N$, but PYTHIA calculations show definitive negative correlations with jets. This is interesting because just by adding particles to the system it increases the average momentum of each particle in the no-jet case. It would be more intuitive if just $P_t$ goes up with $N$ since $P_t$ is the total transverse momentum of an event which directly depends on $N$ by $P_t = \sum_{i=1}^{N} p_{t_i}$. This behavior of an increasing $p_t$ per particle with multiplicity indicates that some other physics could be going on. Increasing $p_t$ with $N$ would be equivalent to saying that if we did a survey that compared peoples height and weight then increased the sample size, then everyone got heavier on average by including more people in the survey. This idea is happening in particle and nuclear collisions which is what we are interested in to see if some other type of physics is going on. To measure this correlation we use an observable that is a combination of the covariance of $P_t$ and $N$ and the variance of multiplicity

$$D = \frac{Cov(P_t, N) - \langle p_t \rangle Var(N)}{\langle N \rangle^2}. \tag{1}$$

Here $P_t$ is the total transverse momentum of an event and $N$ is the multiplicity. The first term on the right hand side is the covariance between $P_t$ and $N$ which measures how much the two quantities are correlated with each other and is defined by:

$$Cov(P_t, N) = \langle P_t N \rangle - \langle P_t \rangle \langle N \rangle. \tag{2}$$

From Figure 1 we see that $\langle p_t \rangle$ increases as $\langle N \rangle$ increases for the no-jet case. Because of this we expect the covariance to give a positive result since $P_t \sim N$. Like the height and weight example, where a taller person is generally heavier, the more particles produced in the event the more $P_t$ there is. However, what we are interested in focusing on is where it does not follow the typical trend to see what else could be causing the behavior. To do this in (1) we subtract off the variance of $N$ multiplied by $\langle p_t \rangle$ where

$$Var(N) = \langle N^2 \rangle - \langle N \rangle^2. \tag{3}$$

The reason we subtract off this term from the covariance is because $P_t$ directly depends on $N$ so fluctuations in $P_t$ should also depend on fluctuations in $N$. We want to take out that dependence and look at what is left over for an indication of some other physics going on in the collision. Statistics tells us that if the particle sources are independent of one another than equation (3) approaches a value of $\langle N \rangle$ and the second term in (1) approaches $\langle p_t \rangle \langle N \rangle$. This informs our expectation of how $D$ behaves. We expect that $D$ is positive and decreases by a factor of $\langle N \rangle^{-1}$ as centrality increases. In thermal equilibrium the numerator should approach zero [2]. Centrality is the measure of how much the two nuclei are overlapping with each other, the less centrality there is the less overlap the two nuclei have when colliding which leads to less nucleons interacting with one another. The more centrality a collision has, the more head-on it is and more nucleons are participating in the collision process. The trivial case for $D$ is when all the particles produced have the same $p_t$. If this happens then covariance term simplifies to the $\langle p_t \rangle$ variance term and $D$ becomes 0. If we want to write $D$ in terms of time to measure the evolution of a collision then we can write [2]

$$D = D_0 S + D_{eq}(1 - S). \quad (4)$$

Where $D_0$ is the value of $D$ in the earliest moments of the collision and $D_{eq}$ is the value of $D$ when the system is in local thermal equilibrium. Local thermal equilibrium is when different sections of the system can have different temperatures, but everything in each individual section has the same temperature. The variable $S$ is a survival probability which is the probability that a particle in the system has no interaction with the remaining system. In other words, $S = 1$ at the beginning of the collision and decreases over time until it goes to zero. In the equilibrium case, $D$ is expected to vanish [2]. Central collisions are more likely to reach equilibrium because they live longer and have more particles, combining the expectations that $D$ already trends like $\langle N \rangle^{-1}$ and $S \rightarrow 0$ with more complete equilibration, We expect $D$ to actually trend faster than $\langle N \rangle^{-1}$.

# 3 Analysis

To get an understanding of how $D$ behaves we need to test this in a collision. However, running nuclear collisions at such high energies takes a large amount of time and money for one shot. To run the millions of events and get a lot of data we can use UrQMD (Ultra-relativistic Quantum Molecular Dynamic) which allows us to simulate nuclear collisions and read the data that comes out of it. The events were given to me from W.J Llope at Wayne State University where I was running my code on their high performance computing grid. There is an assumption that come with UrQMD which is that it assumes the system is a gas of particles and resonances. We used C++ and ROOT to do the calculation of the observable and create histograms to see the results. Because UrQMD is a model that simulates the scattering between particles, there is a chance that some collisions have a long enough lifetime where partial thermalization can happen and we expect $D$ to show signs of this. All of our graphs and tables are for 20 million UrQMD events of Au-Au collisions with a center of mass energy 200GeV per nucleon. our kinematic cuts for all the graphs are the following: $0.15 < p_t < 2GeV$, $|\eta| < 0.5$, and $N$ is all charged particles. Instead of centrality as our x-axis all of our graphs use $N_{part}$ which is related to centrality but also tells how many nucleons are participating in colliding with other nucleons. In other words, the more participants there are in the collision the more centrality it has.

# 4 Results

Our results show that from figure 2, $D$ appears to be positive at the zeros in the error bars, but because of their size it does not rule out negative values for $D$. Also, $D$ does decrease with centrality but figure 4 shows that $D$ does not decrease faster than $\langle N \rangle^{-1}$ but it decreases slower than $\langle N \rangle^{-1}$. This result goes

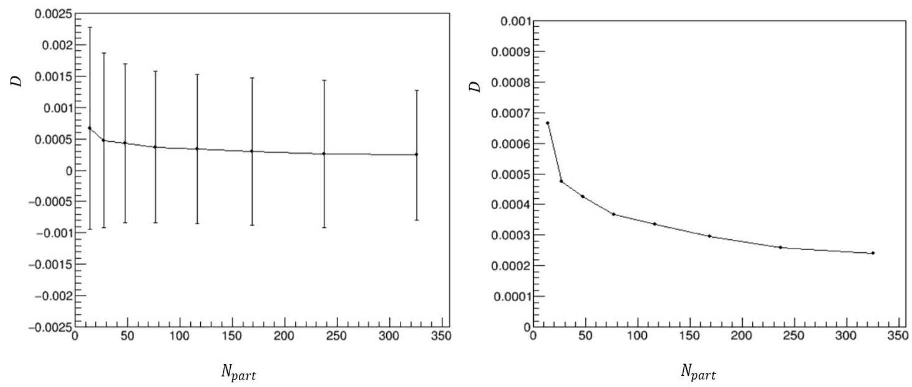


Figure 2: Observable $D$ as a function of $N_{part}$. Right figure is a zoomed in version of the left figure with no error bars for shape clarity

against our expectation for how $D$ trends with centrality. However, the value of $D$ is consistent with the zeros in the error bars, but We need better statistics to confidently determine the value of $D$. Furthermore, $D$ does show signs of partial thermalization because of its decreasing behavior with centrality.

Table 1: The values below are obtained from the zeros of the error bars in figure 2.

| $N_{part}$ | $D \cdot 10^{-4}$ | $Errors$ |
|---|---|---|
| 14.3858 | 6.65 | 0.001609 |
| 27.3827 | 4.75 | 0.001386 |
| 47.7871 | 4.24 | 0.001266 |
| 76.8777 | 3.68 | 0.001207 |
| 116.735 | 3.36 | 0.001139 |
| 169.037 | 2.96 | 0.001172 |
| 237.268 | 2.58 | 0.001168 |
| 325.477 | 2.39 | 0.001039 |

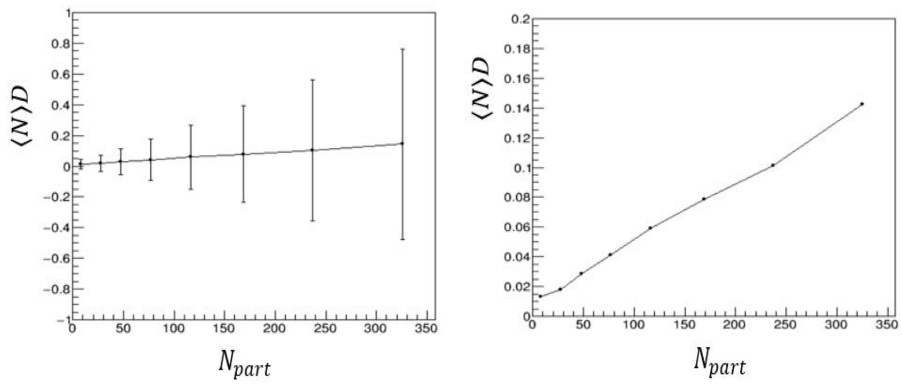


Figure 3: $ND$ as a function of $N_{part}$. Right figure is a zoomed in version of the left figure with no error bars for shape clarity

Table 2: The values below are obtained from the zeros of the error bars in figure 2.

| $N_{part}$ | $\langle N \rangle D$ | $Errors$ |
|---|---|---|
| 14.3858 | 0.01325 | 0.03209 |
| 27.3827 | 0.01789 | 0.05224 |
| 47.7871 | 0.02836 | 0.08477 |
| 76.8777 | 0.04108 | 0.1348 |
| 116.735 | 0.05907 | 0.2084 |
| 169.037 | 0.07871 | 0.3118 |
| 237.268 | 0.1014 | 0.4587 |
| 325.477 | 0.1425 | 0.6197 |

# 5 Conclusion

When we started this research, we wanted to study the observable $D$ in UrQMD to find signs of partial thermalization in nuclear collisions. We had expectations as to how $D$ would behave. Initially, we thought that $D$ would be positive and decreasing with centrality faster than $\langle N \rangle^{-1}$. However, From the data I gathered in UrQMD we can conclude that $D$ appears to be positive but our large error bars allow $D$ to have negative values. There are blast wave calculations that Gavin, Moschelli, and Zin do that assumes thermalization and typically give negative values. We are skeptical about these values which is another reason for doing this calculation with UrQMD to see if the result we get from it is consistent with the blast wave calculations. If $D$ trends faster than $\frac{1}{\langle N \rangle}$ then $\langle N \rangle D$ should be decreasing. Our data shows that $\langle N \rangle D$ is increasing with centrality which means that our $D$ trends slower than $\frac{1}{\langle N \rangle}$ and goes against our expectations. This can possibly be due to jets which we do not distinguish between events that have jets and events that do not. Events that do have jets could contribute an increase in $p_t$ as we saw in the UA1 experiment. Jets are more likely to happen in central collisions which could be why $\langle N \rangle D$ increases with centrality. Because of our large error bars we are not confident to determine what our result of $D$ in UrQMD is at this time. Better statistics and more work is needed to narrow down the value of $D$ and describe its behavior in the future.

# 6 References


[1] Sjöstrand, van Zijl, Phys.Rev. D36 (1987) 2019

[2] Gavin, Moschelli, Zin, in preperation