

Momentum Fluctuations in Heavy Ion Collisions

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August 17, 2016

1 The Physics

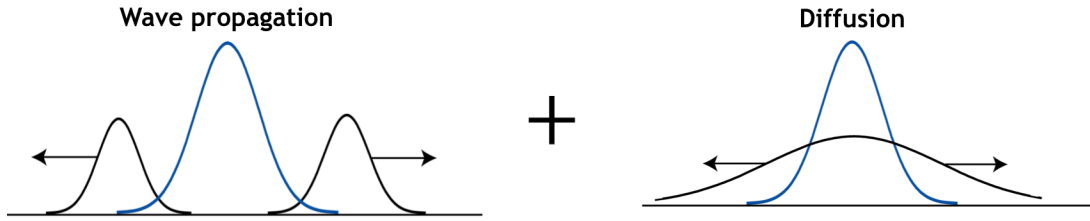
While researching the momentum fluctuations of heavy ion collisions, the focus is on the time from the collisions of the atoms, to about 1 femtosecond after. This is a very small window of time, $1 * 10^{-15}$ seconds. It is during this time just after colliding while the atoms are speeding away from each other, when a state of matter known as Quark-Gluon plasma exists. Imagine inflating a balloon, it does not have a uniform pressure throughout while expanding, there are areas that have higher pressures and areas of lower pressures. When the balloon pops, the areas of higher pressure will speed outward at a faster rate than the areas of lower pressure. This means that the momentum of the air is not uniform when the popping occurs. Now take this concept and apply it to the collision of atoms. Just after the collision, the atoms are speeding away before the constituents have time to expand outward. When they do, there is a fluctuation of the momentum over the volume of the collision area. This fluctuation is what is being focused on.

Then the collision area begins to expand, there is a fluctuation between the momentum of different areas. Due to the viscosity, the faster moving areas will transfer energy to the slower moving areas and will begin to slow down. The areas that are moving slow will increase their speed, this brings the whole system to an equilibrium. The correlation function of the momentum of the system will go to zero as time goes to infinity, however, in reality it will not. Brownian motion is the random zig-zag motion of a heavy particle suspended in a fluid and this effects the momentum correlation of the collision system. Instead of the correlation function of the momentum going to zero, it will reach a lower limit, due to this Brownian motion.

The equation for this state was derived by Dr. Gavin and Dr. Moschelli to be:

$$\left[\frac{\tau_\pi^*}{2} \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \frac{\nu^*}{\tau^2} \left(2 \frac{\partial^2}{\partial \eta_r^2} + \frac{1}{2} \frac{\partial^2}{\partial \eta_a^2} \right) \right] \Delta r_G = 0, \quad (1)$$

Where transport coefficient τ_π is a relaxation time for the shear modes. This is given by $\tau_\pi = \beta\nu$. ν is the kinematic viscosity of the system and is given by $\nu = \eta/Ts$. These are constant parameters that were previously used in Dr. Gavin's and Dr. Moschelli's paper [1]. The work I performed did not use constant parameters, but instead τ_π^* and ν^* . These are similar to the τ_π and ν above but have time-dependent denominators. $\tau_\pi^* = \tau_\pi/(1 + \kappa\tau_\pi/\tau_0)$ and $\nu^* = \nu/(1 + \kappa\tau_\pi/\tau_0)$ from equation (1). There are two parts to this equation, a wave equation and a diffusion equation. This allows a hypothesis to be formed as to how this wave will react and change through time. The wave equation side will cause the wave to propagate outwards while the diffusion equation side will cause the wave to spread and flatten through time.



With both of the behaviors at work, it can be hypothesized that the wave will begin to split and propagate while also widening and flattening. The question then can be asked, how will this wave behave through time? This is what our program does. It takes into account the various input variables and outputs graphs for the system. We are trying to match these graphs to the data collected from the STAR experiment at RHIC. You can see how the wave changes with time and the STAR data overlaid in figure 1.

2 What I did

When I first arrived, I had little to no knowledge of programming. Being that the main code is written in C++, I spent my first few days acclimating myself to the language and format. My understanding of C++ truly began to develop when I was given the unfinished, main code to look over and begin to work on. I made a few test programs with simple outputs to help understand the syntax. At this time, I was also reading through the first few chapters of Ramona Vogt's text, Heavy Ion Collisions, to introduce myself to what the physics is behind this problem we are tackling. Before long, I began to get a grasp on the main code as well as the

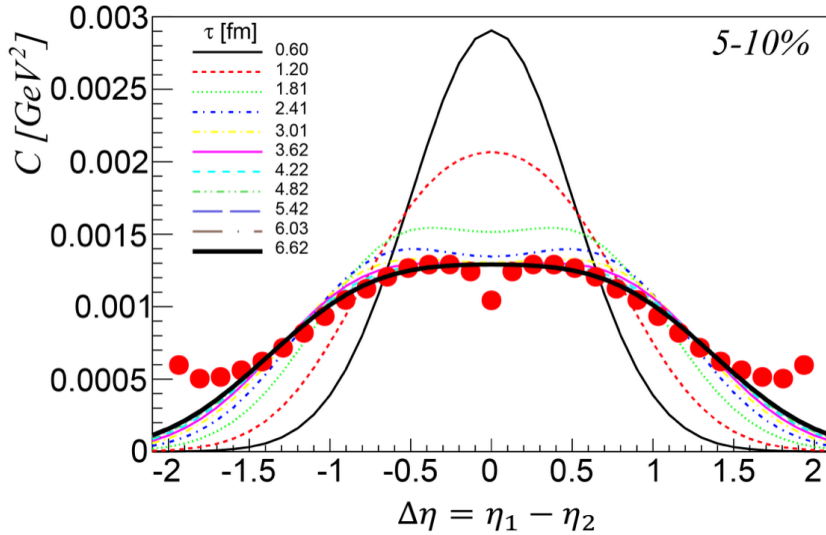


Figure 1: C vs. $\Delta\eta$

accompanying functions. I then began to choose a specific parameter to slightly change and see how it effects the output, as well as put this output into ROOT and see how the graphs are translated and adjusted while also comparing them to the data collected by the STAR experiment. By doing this parameter by parameter, I was really able to begin to view these variables not as simple numbers in an equation, but as physical conditions that really changed what our output collision information meant. This was the first time in the ten weeks of the REU that I actually felt like I was grasping the content.

It was this moment that we ran into our first real challenge. To test a different set of input parameters, the main code had to be recompiled each time a new set were input. This added about thirty seconds onto the program run time of about two minutes which did not seem problematic initially, but when testing a wide range of parameters, this is a great amount of down time. The idea of using an input text file arose, which would prevent the main code from needing to be recompiled each time. I made the input text template and began to edit the main code to read the input files from this template. It finished quite easily without any major obstacles, and now were were able to just change the values of the variable on a text file instead of having to edit the main code each time. This was used for some time to perform more runs, but this process was still very time consuming.

It was then decided that a shell script should be written to allow the program to scan through a range of variables without needing the user to change them after

each run. This is where I needed to begin to learn bash and the command line language. This was my first introduction to using a terminal, the command line interface for the UNIX system of the Mac. I really enjoyed learning this because it gave me a deeper understanding of the foundation of the Mac operating system and how to interact with the computer in a different, more native way. Using this new knowledge, I was able to streamline my interactions with the computer when coding, compiling, and operating the main code. I successfully created the script after dealing with a handful of simple syntax errors and begin to implement this new input concept. We decided on ranges to scan through the initial parameters, τ_0 , T_F , τ_{Fc} , and let it loose. Because the script would run through a range for each of the three parameters, it would take a very long time, sometimes up to two days, for the program to finish scanning through the three dimensional matrix. I would spend these down times reading more of Vogt's book and also looking through the recently published paper submitted by Dr. Gavin and Dr. Moschelli. Both of these texts continued to further my knowledge on the topic we were working with.

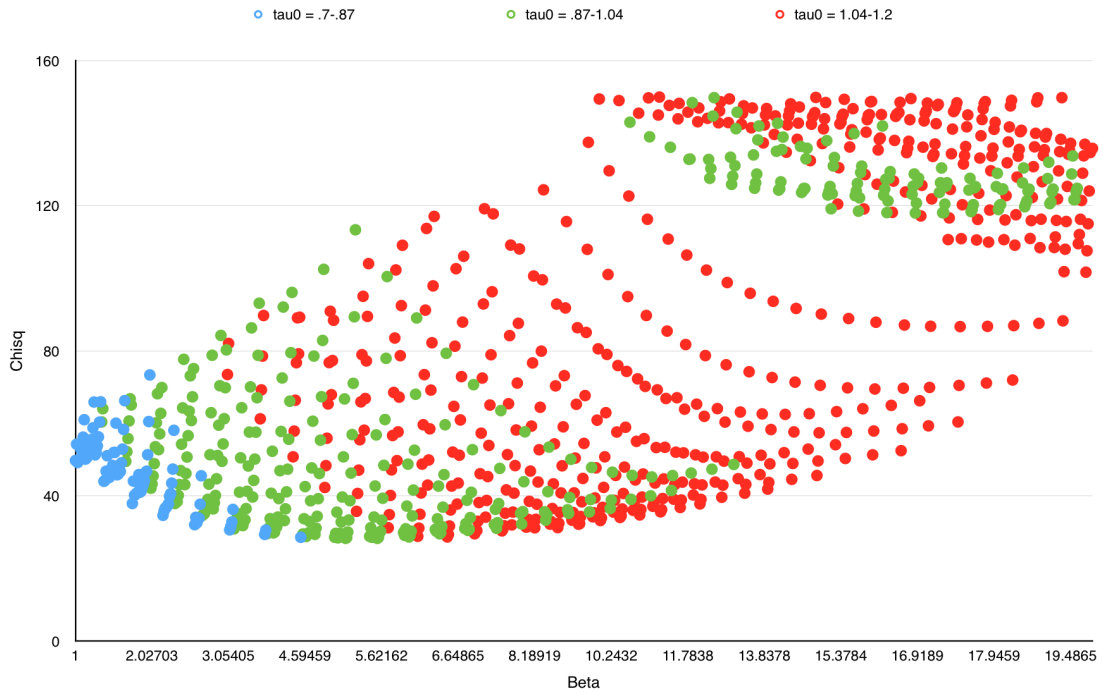
Once the script had run through a handful of ranges, the output graphs are compared to the data obtained from STAR. I sifted through all the output data and focused on the runs with a low χ^2 which correlate to the best fitting initial parameters. I programmed the script to run through the ranges for τ_0 , T_F and τ_{Fc} while β was set to 5, 10 and 14. Taking the runs with the best fitting parameters, the ones with the lowest χ^2 , the output graphs were compared between the different β s. It is very interesting to see how β has an effect on the graph shape. Up to this point, I had performed these scans with κ turned off, meaning the initial conditions were constant. Running the program for constant initial conditions, κ being off, was something that Dr. Moschelli had already spent time doing. We kept κ off so we could be sure the script and main program were working correctly. I then turned κ on, causing the conditions to change with time and this effected the output significantly. The previous scans that had been performed had to be rerun now that we had dynamic initial conditions.

After spending quite a bit of time analyzing the new scans' graphs and trying to interpret what they mean, I came across a problem that I had not noticed before. One of the variables, η Choice, had not been adjusted for κ being active. This meant that all of the scans done with κ on had to be scrapped and rerun to get correct results. Again for the third time, the scans had to be rerun; I was essentially a professional at running these scans by then. After that fiasco, I noticed a few characteristics arise while analyzing the new outputs. There seemed to be a correlation between β and τ_0 . To get the best χ^2 , lower β s seemed to favor a lower τ_0 . I also noticed that to get the best χ^2 for each of the β s, a higher τ_0 favored a lower T_F , and a lower τ_0

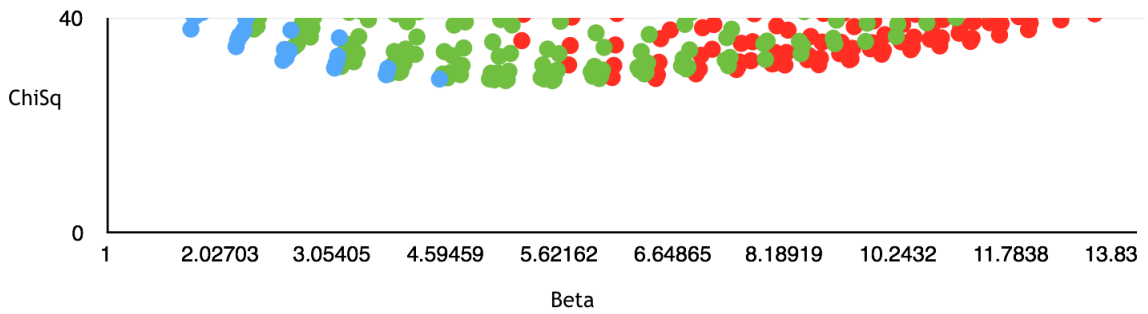
favored a higher T_F . This begins to show an interesting relationship between these three variables. By noticing these patterns, it can be hypothesized that well-fitting runs with a low β should have a low τ_0 and a high T_F . Inversely, well-fitting runs with a high β should have a high τ_0 and a low T_F . Taking this a step further, it seemed that τ_{Fc} had almost no effect on the χ^2 .

The somewhat disturbing characteristic about our results up to this point was that it seemed the data from the STAR experiment could be fit relatively well with any value of β , when paired with the right τ_0 and T_F . With the previous hypothesis in mind, it was decided that the best course of action would be to code the program to scan through β as well as the other variables. After it was written to scan through the extra dimension, β , a few short test scans were ran. These short-range scans gave rise to interesting results; results that hinted at following our hypothesis. After some analysis, I began to have the idea to command the script to perform a large run, scanning through a very wide range of β , τ_0 , T_F and τ_{Fc} . After doing some calculation, it was estimated that doing a scan of this magnitude would take approximately four weeks time. With only two weeks left in the REU, I had to figure out a way to shorten the time span this would take to run. With the help of Dr. Moschelli, a quit process was added into the code. This meant that during each individual run, if the most central collision had a χ^2 that was higher than an upper limit we set, it would quit that entire run and move onto the next set of parameters in the scan. This greatly reduced the amount of time for the scans because it would only perform the runs that had decent χ^2 s.

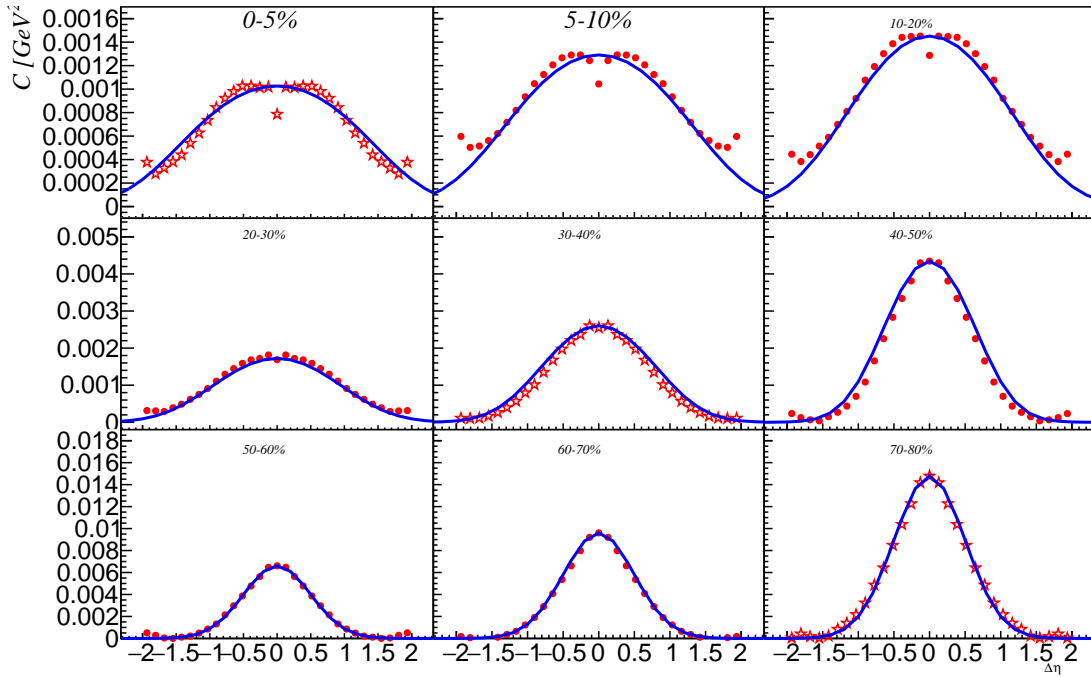
With the new process implemented, the large scan was initiated. While the run time without the quit process was estimated to be around 27 days, the scan with quit process only took about four days. After completion, there were about 4,800 runs that got decent χ^2 s out of a total of 22,800 runs in the scan. Now we finally had a large pool of runs with essentially every combination of parameters possible. This scan revealed many interesting concepts. The top thirty best fitting runs all had β s around five and τ_0 s around one. Also intriguing are the facts that the top 600 runs had a T_F of 0.15 and τ_{Fc} still continued to seemingly have no effect on the χ^2 . I created a graph of β vs. χ^2 of the runs with a χ^2 lower than 150, with the different values of τ_0 colored by range:



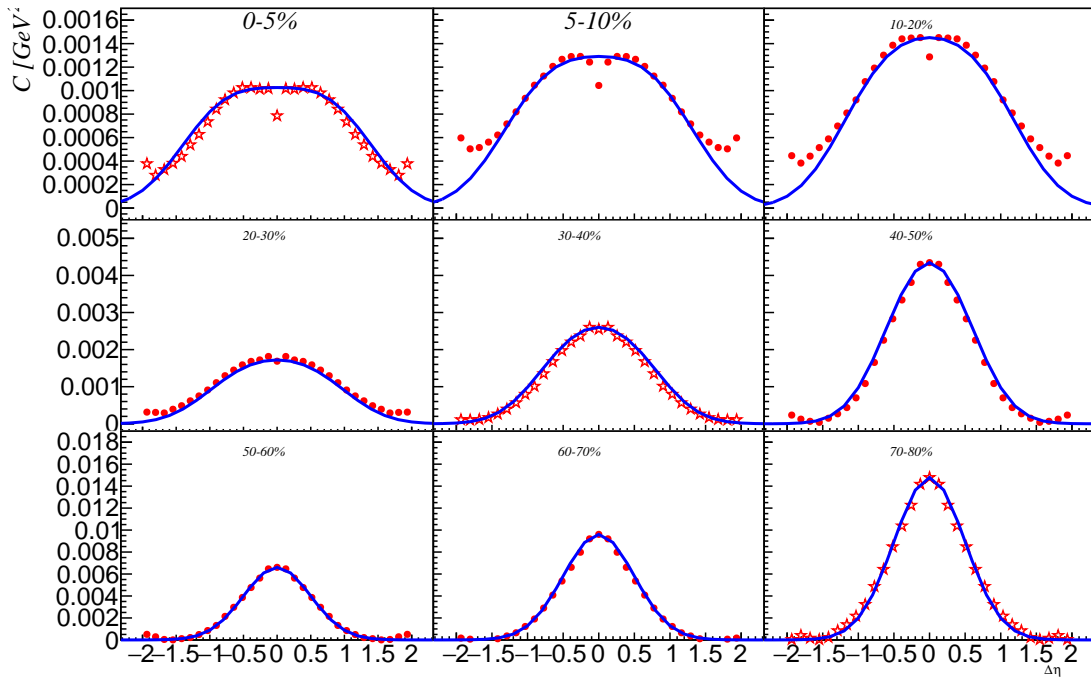
It can be seen that there seems to be a relationship between β and τ_0 . When this graph is focused on just the runs with a χ^2 lower than fifty, this relation seems more apparent:



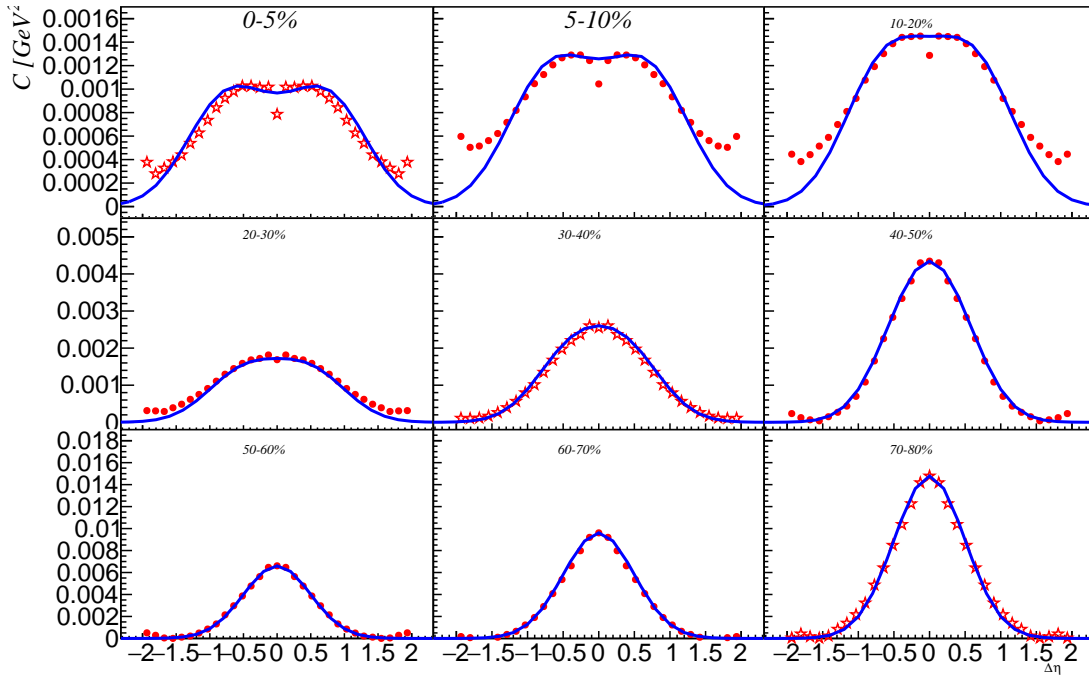
So although well-fitting parameters can be obtained from almost any value of β in the range of two through twelve, the best fitting parameters seem to be when β is approximately five, τ_0 is approximately one, and T_F is 0.15. When looking at the C vs. $\delta\eta$ graphs for all the centralities, the effects of β can be seen. When $\beta=2$:



When $\beta=5$:



When $\beta=11$:



When β is two, the graph fits the width well but does not fit the shoulders of the top and when β is eleven, the shoulders are fit well but the width is too skinny. It seems that when β is five is when the graphs fit well over the entire shape. This problem will be further studied by looking into which of these β values seem the most physically acceptable and how this holds up to other inquiries into this state of the heavy ion collision.

References

- [1] S. Gavin, G. Moschelli, and C. Zin, “Rapidity Correlation Structure in Nuclear Collisions,” [arXiv:1606.02692](https://arxiv.org/abs/1606.02692) [nucl-th].