Exploring multiplicity fluctuations with 2-particle correlations in Au+Au collisions

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Abstract

The STAR experiment group is searching for the critical point in the phase diagram of nuclear matter. A way to do this to use different beam energies and look for fluctuations in the number of particles produced in these events. These fluctuations are most easily apparent in calculations of cumulants in event-by-event multiplicity distributions. A different, more differential, way of looking for fluctuations is with the use of the two-particle correlation function $R_2$. By integrating $R_2$ one may derive the cumulants of the multiplicity distribution and use these two different methods to investigate the same physics. These errors can be corrected by averaging the $R_2$ values over many small bins of the collision location. In this paper, we inspect how these pseudo-correlations effect the cumulant values of the multiplicity distributions.
1 Introduction

Finding the critical point of nuclear matter is of great interest to the physics community. Currently the phase transition boundary is mostly speculative, but if there is evidence that a critical point exists then this may confirm a first order phase transition. This boundary delimits the two states that nuclear matter can be in: hadronic matter and quark-gluon plasma.

The following figure (Figure 1) is a speculative QCD (Quantum Chromodynamics) phase diagram with the temperature in MeV on the y-axis and Baryon Chemical Potential ($\mu_B$), essentially density, which is monotonically related to the beam energy, on the x-axis. By using different beam energies one can effectively scan the x-axis for a location of the critical point.

![Illustration of QCD phase diagram of nuclear matter with Au+Au collisions at RHIC.][1]

Figure 1: Illustration of QCD phase diagram of nuclear matter with Au+Au collisions at RHIC. [1]
2 Critical Point and Cumulants

If the critical point exists at a certain beam energy, it will be apparent by fluctuations that differ from those of a Poisson distribution (the expectation of no fluctuation). One variable most sensitive to these fluctuations is a particle correlation variable $R_2$, described later. STAR has studied calculating cumulants of multiplicity distributions, which according to [3], should also indicate the increased fluctuations that would result from the existence of a critical point.

In the equation below, $\delta M$ is the 'deviate' and $\delta M = M - \langle M \rangle$, where $M$ is the multiplicity in single events and $\langle M \rangle$ is the average multiplicity over all events.

\[
\begin{align*}
C_1 &= \langle M \rangle \\
C_2 &= \langle (\delta M)^2 \rangle \\
C_3 &= \langle (\delta M)^3 \rangle \\
C_4 &= \langle (\delta M)^4 \rangle - 3\langle (\delta M)^2 \rangle^2
\end{align*}
\] (1)

Cumulants are statistical variables that can be used to describe the properties and shape of a distribution. The quantity $C_1$ being the mean, $C_2$ the square of the standard deviation (a.k.a. the variance), while $C_3$ and $C_4$ are related to the moments called the skewness (non-zero if the distribution is asymmetric) and kurtosis (non-zero if the distribution is non-Gaussian), respectively.

By analysing the cumulants of different multiplicities (such as that in figure 2) at varying beam energies we may be able to spot changes evident of a critical point. STAR has seen similar behavior in experiments at RHIC that require additional careful study (See figure 4).
Figure 2: Netproton multiplicity distribution of Au+Au atoms at 200GeV in STAR experiment for varying collision centralities. 0-5% being most central. [6]

Figure 3: Fluctuations from the poisson distribution indicate a critical point. Note that this is the generic theoretical expectation for the dependence of the fluctuations of C4/C2 versus the beam energy if the critical point exists and is near the beam energy near 19 GeV. [5]
Figure 4: Fluctuations in STAR experiment. A ratio of $C_4$ to $C_2$ is plotted against various beam energies. A fluctuation similar to the one in figure 3 can be seen for collision centralities of 0-5%. [3]
3 $R_2$ Correlation Variable

The variable $R_2$ describes two particle correlations in (pseudo)rapidity. The value of $R_2$ tells you how likely is it that you have another particle at some other rapidity. If $R_2 < 0$ (anti-correlated), then it is less likely than a random distribution, and if $R_2 > 0$ (correlated) then it is more likely.

Rapidity ($\eta$) represents the angle between the track’s trajectory and the beam pipe. At $\eta = 0$ the particle is traveling perpendicular to the beam pipe. Larger values of $\eta$ represent smaller angles with respect to the beam pipe.

$$\eta = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right]$$  \hspace{1cm} (2)

![Figure 5: The R correlation variable. When correlated, if $x=1$, $y$ has a high probability of being 1. When antit-correlated, if $x=1$, $y$ has a low probability of being 1.](image)

The STAR detector at RHIC has an efficiency that depends on rapidity. The STAR efficiency is a plateau at around 90% for absolute values of the rapidity less than 1, and it drops to zero for larger absolute values of the rapidity. In this STAR publication [3] a wide cut on the location of the primary collision vertex is used. However, the collision vertex can vary $\pm 30 \text{cm}$ ($\pm 50 \text{cm}$ at 7.7GeV) along the $z$-vertex. When the collision vertex moves, so does the efficiency area, causing pseudo-correlations that are easily seen in the $R_2$ variable.

So it is known from the study of $R_2$, that the use of a wide cut on the primary vertex causes pseudo-correlations caused by the finite...
reach of the acceptance in rapidity. These pseudocorrelations can be corrected in the $R_2$ variable by calculating $R_2$ in many small bins in the primary vertex location and averaging those results. The following figure (Figure 6) are distributions of the $R_2$ correlation variable utilizing z-vertex corrections.

![Figure 6](image)

Figure 6: The figure on the left is a set of random particles with no correlation that developed correlations because of the variability in the location of the primary vertex. The figure on the right shows the values of $R_2$ with (green histogram) and without (blue histogram) Z-vertex averaging.

## 4 $R_2$ and Cumulants

The $R_2$ variable can also be used to calculate the cumulants. One can integrate $R_2$ to produce the same cumulant values of the multiplicity distributions. In the following equation $\rho_1$ refers to the one particle density and $y$ refers to rapidity.

$$R_2 \Rightarrow r_2 = \frac{\int \rho_1(y_1)\rho_1(y_2)R_2(y_1,y_2)dy_1dy_2}{\int \rho_1(y_1)\rho_1(y_2)}$$

$$C_2 = \langle M \rangle + \langle M \rangle^2 r_2$$
Because the pseudocorrelations resulting from the variability of the primary vertex location effect $R_2$, this raises the question of the importance of these pseudocorrelations for the calculation of the cumulants from the multiplicity distribution. In this study we looked at events generated with zero correlations by obtaining rapidities from a random number generator. The results of the cumulants calculated from $R_2$ and the cumulants calculated from the multiplicity distribution when applying a z-vertex dependent efficiency can then be compared.

<table>
<thead>
<tr>
<th>Z-Vrtx Averaging</th>
<th>$C_1$ via Mult</th>
<th>$C_1$ via $R_2$</th>
<th>$C_2$ via Mult</th>
<th>$C_2$ via $R_2$</th>
<th>$C_3$ via Mult</th>
<th>$C_3$ via $R_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without</td>
<td>26.0972</td>
<td>26.0972</td>
<td>12.539</td>
<td>12.539</td>
<td>-0.569351</td>
<td>-0.569108</td>
</tr>
<tr>
<td>With</td>
<td>26.0972</td>
<td>26.0972</td>
<td>12.4683</td>
<td>12.4683</td>
<td>-0.554056</td>
<td>-0.554041</td>
</tr>
</tbody>
</table>

Table 1: This table represents the effect of z-vertex averaging on a fixed multiplicity of 50, including the experimental acceptance, and including the variation of the primary vertex location $\pm 50$ cm with a flat distribution.

In Table 1 the cumulant values obtained from the multiplicity distributions and from the $R_2$ values are compared with and without Zvtx averaging. When the cumulant values from z-vertex averaging agree with those without it, z-vertex averaging is not needed. However, if the cumulant values differ, then z-vertex averaging is needed.

When experiment conditions are the same the cumulant values from the multiplicity and $R_2$ are identical as they are mathematically related. However, it proves that the values of $C_2$ and $C_3$ derived from the multiplicity using wide z-vertex bins are different from those derived from $R_2$ and $R_3$ using z-vertex averaging. This means that cumulant analysis from a multiplicity should also be done with small z-vertex bin averaging as well (Not done in figures 2 and 4). However, this change is small, being 1% for $C_2$ and 2% for $C_3$.

Additionally, events generated by the UrQMD model can be used and cumulants from $R_2$ and multiplicity compared like-wise. The UrQMD model [4] is a hadronic cascade model that is widely used in this field. It is an implementation of well-known hadronic physics and does produce correlations but does not have any treatment of the physics of a critical point. However, this is more difficult and will require further experimentation and study. Figure 7 below shows how cumulant values changed when introduced to these effects.
Figure 7: The 3 columns are $C_1$, $C_2$, and $C_3$ respectively. Here the values of the cumulants are plotted against the event multiplicity using a beam energy of 7.7GeV. The top row is cumulants derived from the multiplicity, and the bottom row is cumulants derived from $R_2$ using the $z$-vertex corrections.
5 Summary

Currently the QCD phase diagram for nuclear matter is speculative. However, the STAR experiment group is searching for the critical point of nuclear matter which may confirm the existence of a first order phase transition boundary. By using different beam energies we can probe the phase diagram for fluctuations evident of a critical point. These fluctuations will be most apparent in the two-particle correlation variable $R_2$.

Current analysis uses a wide cut on the primary vertex, however this is known to cause pseudo-correlations caused by the finite reach of the acceptance in rapidity. By integrating over many small bins rather than a single large bin we can eliminate the effect of the detectors efficiency and that of the vertex position deviation, this changes the values of $R_2$ and the cumulants. The difference in the change of $R_2$ and cumulant values are rather small. Further careful study is needed to confirm if this effect is significant to the exploration of the critical point of nuclear matter.

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References


