Hydrodynamic Flow in Pb-Pb and p-Pb collisions
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1 Introduction

Over the Summer, I have been studying various aspects of high-energy particle collisions and working alongside faculty mentor Sean Gavin. More specifically, my research was centered on the effects of Radial Flow and Jets in Pb-Pb and p-Pb collisions. We used comparisons between our fluctuation observables defined below and values from the Independent Source Model to distinguish how particle correlations contribute to the data.

Fluctuation observables R, D, and $<\delta p_{t1}\delta p_{t2}>$, defined in (5), (7), and (11), have been used to study the extent to which nuclear collisions produce a thermal system manifesting hydrodynamic flow [1]. Previous REU students M. Catanzaro and D. Taylor have worked on the development of D and the comparison to other observables [2, 3]. In this paper, we focus on the comparison between proton-nucleus and nucleus-nucleus collisions. Nucleus-nucleus collisions produce many hundreds of particles which have long been expected to produce a thermal system manifesting hydrodynamic flow. In contrast, proton-nucleus collisions produce fewer particles. The multiplicities of Pb-Pb and p-Pb at LHC energy in the rapidity range -0.8 < y < 0.8 are shown in Figure 1 as a function of the number of participants.

Recent measurements of the "ridge" and related azimuthally anisotropic flow coefficients v_n, which are described in [4], at the LHC raise the possibility that flow is also present in the smaller proton-nucleus system. See [5], [6], and references therein for recent results on flow in p-Pb. We ask whether fluctuation observables R, D, and $<\delta p_{t1}\delta p_{t2}>$ can shed additional light on this question. We use the event generator AMPT [7] to simulate p-Pb and Pb-Pb collisions at the LHC. AMPT is known to describe flow in Pb-Pb, but has very little flow in p-Pb. Our analysis of R, D, and $<\delta p_{t1}\delta p_{t2}>$ support that conclusion.

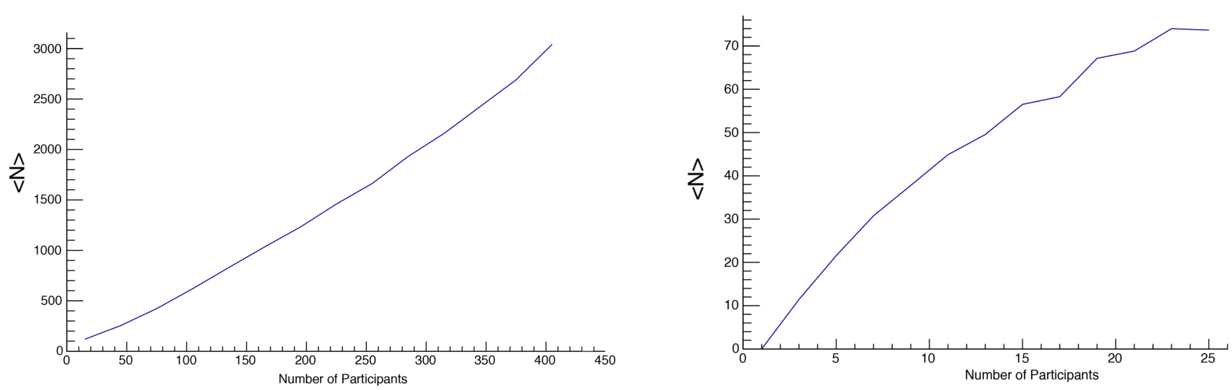


Figure 1: Multiplicity as a function of number of participants for Pb-Pb (left) and p-Pb (right).

Jet production and jet quenching also contribute to the ridge and v_n. These effect also alter R, D, and $< \delta p_{t1} \delta p_{t2} >$ [2, 3]. Moreover, measurements of the different fluctuation observables can potentially help us distinguish the effects of flow and jets. This was illustrated using the jet-quenching code HIJING [3]. One can further study this effect by changing the strength of jet quenching in AMPT. In addition, one can define a class of jet free events in AMPT.

1-1 Symbols

Throughout this paper I will be using symbols which represent various properties of particles and events. For clarity, I will define them upfront:

$< ... >$ = Event average
N = Number of charged particles in event
N_{part} = Number of participants in event
P_t = Total transverse momentum in event
p_t = Transverse momentum of a particle

1-2 Jets

Jets are produced when quarks or gluons scatter head-on, creating high transverse momentum quarks or gluons. Due to confinement of quarks, as the scattered particles separate further it becomes more energetically favorable for them to form into hadrons in a process known as fragmentation. Since a high N will make jet production more probable, the existence of jets increases the correlation between P_t and N.

1-3 Hydrodynamic Flow

In Heavy Ion Collisions, pressure gradients cause particles to move away from the beam axis. Central Collisions create symmetric pressure gradients which send particles radially outwards with uniform transverse momenta. Peripheral Collisions create asymmetric pressure gradients which also send particles radially outwards but with varying transverse momenta. The most apparent contribution due to Flow is an increase in P_t.

1-4 AMPT and ROOT

AMPT is a Monte Carlo transport model used for Heavy Ion Collisions. It uses HIJING for generating the initial conditions, ZPC for modeling the partonic scatterings, and ART model for treating hadronic scatterings.

ROOT is an object-oriented framework for large scale data analysis developed by CERN. I used ROOT as a quick way to store and read data from AMPT and to create histograms for data analysis.

I wrote a shell script containing a loop which changes the random number seed for AMPT, runs events, compresses the data file into a ROOT Tree file, and renames the file chronologically. I used this method to generate 5000 events of both Pb-Pb and p-Pb collisions.

2 Data Analysis

When analyzing data from a large number of heavy-ion collisions, statistical calculations such as event-by-event fluctuations and correlations become increasingly useful. The main focus for my data analysis was on dynamic fluctuation variables, specifically, R, D, and $<\delta p_{t1} \delta p_{t2}>$. These variables have the general formula (measured fluctuations – statistical fluctuations). When a system has local thermal equilibrium, fluctuations follow poisson statistics in which the variance is equal to the average $(<X^2> - <X>^2 = <X>)$, which makes the observables equal to 0. We take advantage of this principle by using it to determine how far the system is from equilibrium.

2-1 Number of Participants

When creating histograms for observables such as R, D, and $<\delta p_{t1} \delta p_{t2}>$, binning by different event data will give different results. One useful bin, which is the one that I used, is Number of Participants because it is inversely proportional to impact parameter, making it easy to see which events are more central and which are more peripheral.

2-2 R

Correlation measurements have proved useful in studying heavy-ion collisions. From here, I will be following the work of Mike Catanzaro and Daniel Taylor [2, 3] by focusing on these studies which use two-body correlation functions:

$$r(p_1, p_2) = \varrho_2(p_1, p_2) - \varrho_1(p_1)\varrho_1(p_2) \tag{1}$$

where $\varrho_2(p_1, p_2) = dN/dy_1 d^2 p_{t1} dy_2 d^2 p_{t2}$ and $\varrho_1(p) = dN/dy d^2 p_t$ are the densities of particle pairs and single particle densities, respectively. To make these particle densities statistically useful, we can use them to calculate multiplicity:

$$<N> = \int \varrho_1(p) dp \tag{2}$$

A convenient way to study these correlations is through fluctuation studies, which measure quantities such as multiplicity or transverse momentum and their variation over an event ensemble. For example, multiplicity variance is calculated by $<N^2> - <N>^2$, which is related to (1) because $<N(N-1)>$ represents the average number of particle pairs:

$$\int \varrho_2(p_1, p_2) dp_1 dp_2 \; = \; <N(N-1)> \; = \; <N^2> \; - \; <N>, \tag{3}$$

and if no correlation exists between particles,

$$\int \varrho_2(p_1, p_2) dp_1 dp_2 \; = \int \varrho_1(p_1)\varrho_1\,(p_2) dp_1 dp_2 \; = \; <N>^2, \tag{4}$$

which leads to

$$R := \; \frac{<N^2> - <N>^2 - <N>}{<N>^2} \; = \frac{1}{<N>^2} \int r(p_{1,}\, p_2) dp_1 dp_2 \tag{5}$$

In equilibrium, the multiplicity variance is equal to its average. The observable R measures how far a system is from equilibrium by calculating how far the multiplicity variance is from its average.

2-3 D

In equilibrium, p_t and $<N>$ are uncorrelated and therefore:

$$<NP_t> \; - \; <N><P_t> \; = \; <p_t>(<N^2> \; - \; <N>^2) \tag{6}$$

The variable D was created by utilizing this concept, subtracting one term from the other, as a way to show how from a system is from equilibrium:

$$D \; = \; \frac{1}{<N>^2}\left[(<NP_t> \; - \; <N><P_t>) \; - \; <p_t>(<N^2> \; - \; <N>^2)\right] \tag{7}$$

Since the presence of jets increases correlation between N and P_t, it also causes the value of D to increase. However, flow tends to cause the value of D to decrease, since it increases $<p_t>$ without increasing correlation between $<N>$ and p_t.

Relating D to pair densities, we can write

$$<N>^2 D \; = \; \int (p_{t1} - <p_t>) r(p_1, p_2) dp_1 dp_2 \tag{8}$$

Expanding this using (1) leads to

$$<N>^2 D \; = \; \int (p_{t1} - \; <p_t>)[\varrho_2(p_1, p_2) - \varrho_1(p_1)\varrho_1(p_2)] dp_1 dp_2 \tag{9}$$

$$= \int \; [\, p_{t1}\varrho_2(p_1, p_2) - p_{t1}\varrho_1(p_1)\varrho_1(p_2)$$

$$- <p_t> (\varrho_2(p_1, p_2) - \varrho_1(p_1)\varrho_1(p_2)] dp_1 dp_2$$

$$= \; \int \; [\, p_{t1}\varrho_2(p_1, p_2) - p_{t1}\varrho_1(p_1)\varrho_1(p_2) - <p_t> r(p_1, p_2)] dp_1 dp_2$$

Statistical information can now be extracted from this equation by using (2), (3), (5) and recognizing that $<p_t><N> \ = \ <P_t>$ and $\int p_{t1}\varrho_1(p_1)\varrho_1(p_2) \ = \ \int p_{t1}\varrho_1(p_1)dp_1 \ \int \varrho_1(p_2)dp_2$ is $<P_t><N>$ by definition, which leads back to (7):

$$
\begin{aligned}
<N>^2 D \ &= \ <P_t(N-1)> \ - \ <P_t><N> \ - \ <p_t> R <N>^2 \\
&= \ <NP_t> \ - \ <P_t> \ - \ <P_t><N> \ - \ <p_t>(<N^2>-<N>^2-<N>) \\
&= \ <NP_t> \ - \ <p_t><N> \ - \ <P_t><N> \ - \ <p_t>(<N^2>-<N>^2-<N>) \\
&= \ <NP_t> \ - \ <P_t><N> \ - \ <p_t>(<N^2>-<N>^2)
\end{aligned}
$$

2-4 $<\delta p_{t1}\delta p_{t2}>$

Following the general formula for dynamic fluctuation variables, another observable can be formulated from the variances $\sigma^2_{P_t} \ = \ <(P_t - N<p_t>)^2>$ and $\sigma^2_{P_t,stat} \ = \ <N>(<p_t^2>-<p_t>^2)$

$$
<\delta p_{t1}\delta p_{t2}> \ = \ \frac{\sigma^2_{P_t} - \sigma^2_{P_t,stat}}{<N(N-1)>} \tag{10}
$$

Using methods similar to those used for the derivation of D, $<\delta p_{t1}\delta p_{t2}>$ can be formulated in the following way:

$$
<\delta p_{t1}\delta p_{t2}> \ = \ \frac{<\sum\limits_{i\neq j}(p_{ti} - <p_t>)(p_{tj} - <p_t>)>}{<N(N-1)>} \tag{11}
$$

The presence of jets cause $<\delta p_{t1}\delta p_{t2}>$ to increase while flow causes it to decrease.

3 Independent Source Model

The Independent Source Model uses values from p-p collisions and N_{part} to generate values that are comparable to simulated values. Since its only parameter is N_{part}, the model assumes no contribution from particle correlations. Comparing these simplified values of R, D, and $<\delta p_{t1}\delta p_{t2}>$ with the simulated values allows us to differentiate between contributions due to flow and jets. If we let M represent the number of independent sources in an event, then $\varrho_1 = \widehat{\varrho_1}M$ and $<N>$ can be rewritten as

$$
<\int \varrho_1 p_1(p)dp> \ = \ <\int \widehat{\varrho_1}(p)Mdp> \ = \ \left(\int \widehat{\varrho_1}(p)dp\right)<M> \tag{12}
$$

If we let $\mu := \int \widehat{\varrho_1}(p)dp$, then $<N> \ = \mu <M>$ and $\varrho_2 = \widehat{\varrho_2}M + \widehat{\varrho_1}\widehat{\varrho_1}M(M-1)$. Additionally, we can write

$$< P_t > \;=\; \left(\int \widehat{\varrho_1}(p) p_t dp \right) < M > \tag{13}$$

Using these formulas, we can now write R in terms of independent sources:

$$R = \frac{1}{<(\int \varrho_1)^2>} < \int dp_1 dp_2 (\varrho_2 - \varrho_1 \varrho_1) >$$

$$= \frac{1}{<(\int \varrho_1)^2>} < \int d^3p_1 d^3p_2 \left[(\widehat{\varrho}_2 M + \widehat{\varrho_1}\widehat{\varrho_1} M(M-1)) - \widehat{\varrho_1}\widehat{\varrho_1} M^2 \right] >$$

$$= \frac{1}{<(\int \varrho_1)^2>} < \int d^3p_1 d^3p_2 \widehat{\varrho}_2 M + \int d^3p_1 d^3p_2 \widehat{\varrho_1}\widehat{\varrho_1} M(M-1) - \int d^3p_1 d^3p_2 \widehat{\varrho_1}\widehat{\varrho_1} M^2 >$$

$$= \frac{1}{(\widehat{\varrho}_1 M)^2} \left[a_o < M > \; + \mu^2 < M(M-1) > \; - \mu^2 < M >^2 \right]$$

where $a_o := \int d^3p_1 d^3p_2 \widehat{\varrho}_2$ .This leads to another way writing R:

$$R = \left[\frac{a_o - \mu^2}{\mu^2} \right] \frac{1}{<M>} + \frac{<M^2> - <M>^2}{<M>^2} \tag{14}$$

An independent source model for D can be calculated from (7) in a similar way by letting $b_o := \int \widehat{\varrho}_2 p_{t1}$. These calculations lead to

$$D = \frac{b_o - a_o p_{to}}{\mu^2 <M>} \tag{15}$$

When we use (14) and (15) for nuclear collisions, M and N_{part} become equivalent. Keeping in mind that N_{part} for p-p collisions is always two, (14) and (15) reduce to

$$R_{AA} = \frac{2R_{pp}}{N_{part}} + \frac{<N_{part}^2> - <N_{part}>^2}{<N_{part}>^2} \tag{16}$$

and

$$D_{AA} = \frac{2D_{pp}}{N_{part}} \tag{17}$$

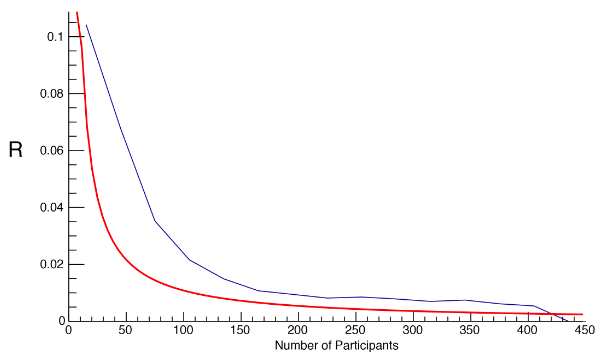
While both (16) and (17) are dependent on N_{part} , only R is dependent on volume fluctuations, which is represented by the second term in (16).

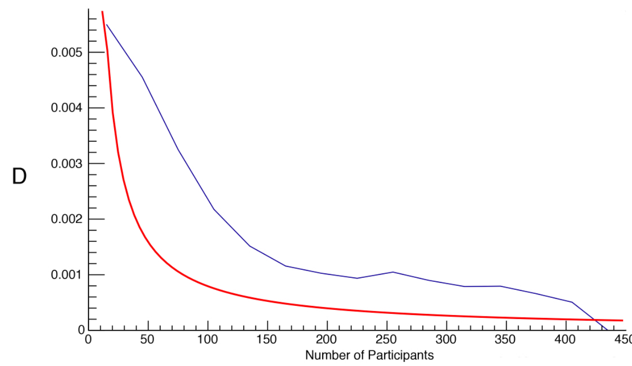
The derivation for $< \delta p_{t1} \delta p_{t2} >_{AA}$ can be found in [5]:

$$< \delta p_{t1} \delta p_{t2} >_{AA} = \frac{2 < \delta p_{t1} \delta p_{t2} >_{pp}}{N_{part}} \frac{1 + R_{pp}}{1 + R_{AA}} \tag{18}$$

3-1 Pb-Pb

Figure 2 shows graphs of observables from 5000 events of Pb-Pb. Simulated R, D, and $<\delta p_{t1} \delta p_{t2}>$ all exceed the independent source model. v_n data suggests large contributions due to hydrodynamic flow in Pb-Pb [5, 6], which is consistent with our analysis.





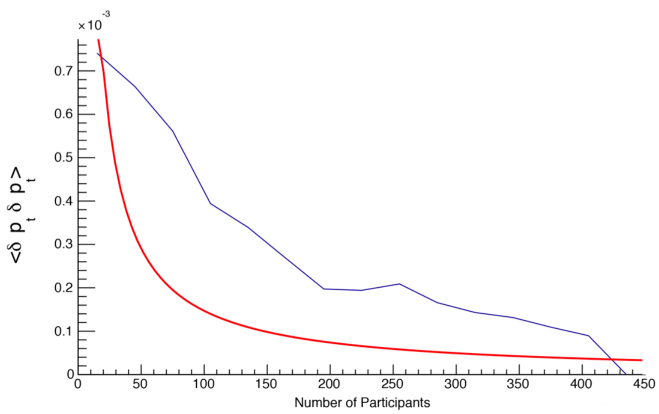
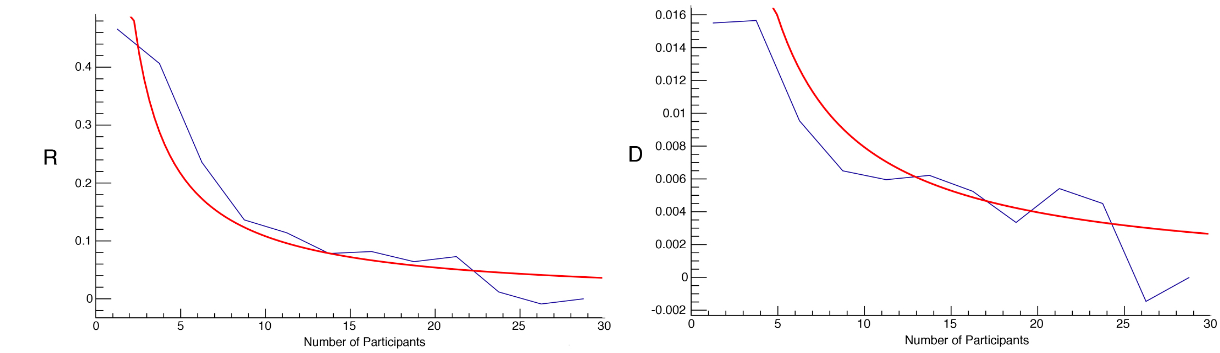


Figure 2: Graphs created from 5000 events of AMPT Pb-Pb. Blue lines represent simulated values, red lines represent independent source model values.

3-2 p-Pb

Figure 3 shows graphs of observables from 5000 events of p-Pb. The observables in p-Pb seem to be consistent with the independent source model, with the exception of $<\delta p_{t1} \delta p_{t2}>$ which is especially sensitive to statistics. Since v_n data from [5, 6] seem to disagree with this, we believe that AMPT p-Pb has insufficient hydrodynamic flow to produce significant correlations, but we need more events to be sure.



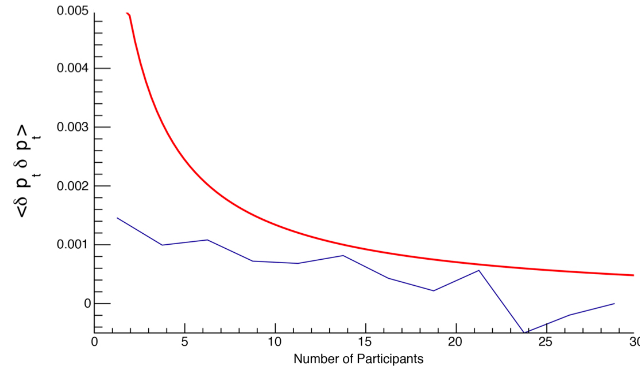


Figure 3: Graphs created from 5000 events of AMPT p-Pb. Blue lines represent simulated values, red lines represent independent source model values.

4 Looking Back and Ahead

I have spent the summer studying various aspects of heavy-ion collisions. I was introduced to statistical methods of solving problems in Physics and given a means by which to do so. Although the number of events generated during my research experience was too low, we have high hopes for the future. Thankfully, I will be continuing my research with Sean Gavin on hydrodynamic flow. I plan to gather more events from AMPT in the immediate future for more accurate results.

5 Acknowledgements

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6 References

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