Constraints on light Dark Matter from the Casimir Effect

By: Abdel Alsnayan Advisor: Alexey Petrov

August 15, 2014

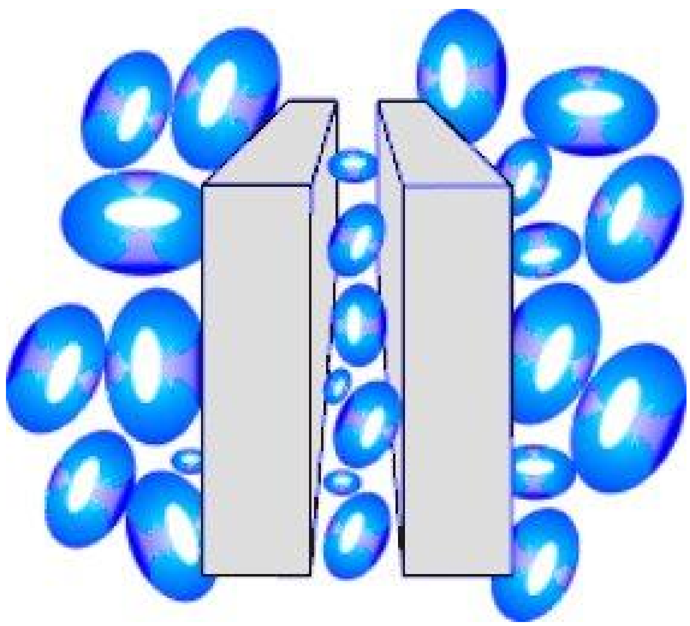

Figure 1: The Casimir Effect

1 Introduction

The search for Dark Matter has been an on going endeavor for almost a century. There have been a medley of experiments that attempt to identify Dark Matter. In this paper we attempt to look at Dark Matter from a different perceptive, one that relies on the Casimir Effect.

The Casimir Effect is an attraction between two conducting plates. This was first shown by Hendrick Casimir's in 1948. Casimir's original calculation is assumed to be a consequence of vacuum fluctuations. While, most agree that the attraction is due to vacuum fluctuations, others have argued that the Casimir Force is a result of van der waals force.

In recent events, we have allotted for a different explanation for the Casimir force. With the meticulous study of hidden sector physics we realize that light Dark Matter might play a role in the Casimir Effect. If one were to recalculate Casimir's original calculation, but with consideration to DM particles then perhaps one would get a more accurate result that agrees with the experimental side. We will consider a dark photon, a boson with mass and spin 1 that mediates the dark electromagnetic field. We will first re-derive Casimir's original results and then consider the possibility of the dark photon interacting with the photon and then calculate the Casimir force based of that interaction.

2 Casimir Effect

To measure the Casimir Effect, we need to define the vacuum when it is in ground state.Ground state can be defined as the lowest possible energy state. One way to obtain a ground state is to start off with the electromagnetic field. First we must quantize the electromagnetic field in a vacuum and then impose ground state ket notation which results in the following:

We first write down Maxwell's equations for the electromagnetic force where charge and current are set at 0.

$$\vec{\nabla} \cdot \vec{E} = 0$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

The next step is to redefine these terms as vector and scalar potentials.

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

Rewriting the electric and magnetic fields in terms of rank-2 tensor one gets the well known Lagrangian density

$$L = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{c} j_{\mu} A_{\mu}$$

Once you take the Euler-Lagrange of that Lagrangian density you get the well know Hamiltonian

$$H = \int \frac{1}{2}(E^2 + B^2) d^3 r$$

We then plug in E and B in terms of vector potentials and we are left with

$$H = \frac{1}{2} \int (-\frac{\partial A}{\partial t}^2 + (\nabla \times A)^2 d^3 r$$

Now that we have a classical formulation, we need to quantize it. In order to do that, we must promote the field vectors into operators. After doing that,we get the result below.

$$< 0|H|0 >=< 0| \sum_{k,\lambda} (\alpha_{k,\lambda}^{\hat{\dagger}} \alpha_{\hat{k},\lambda} + \frac{1}{2})|0 >$$

$$< E >= \sum_{k,\lambda} \frac{1}{2} \hbar \omega$$

Where k is the wave vector and λ is the polarization term.

Assume two conducting plates with a R^3 box and each plate is placed side by side, with length L parallel to the x and y axis. The plates are a distance a apart.

To solve for ω_n and K_n, we need to look at a one dimensional standing waves and impose the boundary conditions $\psi(a,t) = \psi(0,t) = 0$ When we have a wave equation that obeys the given boundary conditions we call them Dirichlet boundary conditions.

We plug this into the KG equation

$$\psi(x,t) = e^{-i\omega_n t} sin(k_n x)$$

We get the results

$$\omega = ck$$

where $k = \frac{k\pi}{a}$
This can be fitted for three dimensions. The wave equation becomes

$$\psi_n(x, y, z, t) = e^{-i\omega_n t} e^{-iK_x X + iK_y Y} \sin(K_n z)$$

and the wave vector becomes

$$K^2 = K_x^2 + K_y^2 + K_z^2$$

Where:

$$K_x = \frac{n_x \pi}{L}, K_y = \frac{n_x \pi}{L}, K_z = \frac{n_x \pi}{a}$$

In order to measure the vacuum energy we need to integrate over the x and y coordinates and sum over the z coordinate. The sum diverges so in order to make it converge we introduce the regulator ω^{-s} This is refereed to as the modern zeta function. We then take $\lim_{s \to 0}$ which results in a physical, measurable force.

$$< E(a) >= \frac{\hbar}{2(2\pi^2)} \int \int L^2 \sum \frac{w}{w^s} \mathrm{d}K_x \mathrm{d}K_z$$

Expanding and rearranging the equation yields:

$$< E(a, s) >= 2L^2 \int \int \frac{\hbar}{2} \frac{1}{2\pi^2} \sum \sqrt{{K_x}^2 + K_y^2 + K_z^2} \mathrm{d}K_x \mathrm{d}K_z$$

Here we make the conversion from Cartesian to polar coordinates, using $y^2 = K_x^2 + K_y^2$. Y is the resulting Jacobian.

$$< E(a, s) >= \frac{\hbar}{2\pi} L^2 \int \sum y(y^2 + \frac{{n_z}^2 \pi^2}{a^2})^{\frac{1}{2} - s} \mathrm{d}y$$

First we attack the integral with a simple u substitution.

$$u^2 = y^2 + \frac{{n_z}^2 \pi^2}{a^2}$$

$$\int_{K_z}^{\infty} u^{2-s} \mathrm{d}u$$

$$\left.\frac{u^{3-s}}{3-s}\right|_{K_z}^{\infty}$$

$$-(\frac{n_z \pi}{a})^{3-s}$$

The result is a negative answer. Integration along the x and y axis shows a physics attractive force.

After some factoring and rearranging we have:

$$< E(a,s) >= L^2 \lim_{s \to 0} -\frac{\hbar c^{1-s} \pi^{2-s}}{2a^{3-s}(3-s)} \sum_{n=1}^{\infty} \frac{1}{n^{s-3}}$$

After examining the sum we see that is has a convergent value:

$$\sum_{n=1}^{\infty} \frac{1}{n^{-3}} = \zeta(-3)$$

$$< E(a,s) >= -\frac{L^2 \hbar c \pi^2}{6a^3} \zeta(-3)$$

$$\zeta(-3) = \frac{1}{120}$$

$$\frac{< E(a,s) >}{A} = -\frac{\hbar c \pi^2}{720 a^3}$$

To find the force we simply take the $\frac{\partial E}{\partial a}$ which yields:

$$\frac{< E(a,s) >}{A} = -\frac{\hbar c \pi^2}{240 a^4}$$

As of now we have demonstrated that there is an attraction. The main question is, what causes this attraction? The derivaton we have provided assumes the Casimir Force as a result of vacuum fluctuations. As stated previously, there are different opinions among the community. We will explore more of the varying opinions next.

3 Casimir Effect and the Van der Waals Force

In Casimir's original calculations,due to the boundary condition, we assume perfectly conducting plates and this is problematic. But in nature we do not see perfectly conducting plates. So, since there isn't such a thing as perfectly conducting we must somehow incorporates α into our calculations. Each conductor has a plasma frequency and a skin depth. Both of these properties depend on α, the fine structure constant. From this we can can say that alpha plays a role in the calculations, but no reference is made by Casimir.

A calculation that is dependent on α must consider the strength of the electromagnetic field and find out when exactly α vanishes. Others have made this consideration by allowing the Casimir Effect to be a result of interaction between atoms. One particular example we can follow is that of Jaffe's. If we look at Jaffe's method, we can learn how this is possible. He looks at the Casimir Effect as a scalar, in order to simplify it. One could follow the method Jaffe constructed by looking at the following interaction Lagrangian.

$$\mathcal{L} = \frac{1}{2} g \sigma(x) \phi^2(x)$$

The boundary is considered as $g \to \infty$. He calculates the effective energy as the sum of all one-loop Feynman diagram. The calculation shows as $g \to \infty$ the dependence on the material no longer plays a role and the result in the same as Casimir.

4 Dark Photon-Photon Mixing

The Lagrangian for a non interacting electromagnetic field is

$$\mathcal{L} = -\frac{1}{4} {F_{\mu\nu}}^2$$

We can expand this Lagrangian to include the dark photon and kinetic mixing.

$$\mathcal{L} = -\frac{1}{4} {F_{\mu\nu}}^2 - \frac{1}{4} {B_{\mu\nu}}^2 - \frac{1}{2} \chi F^{\mu\nu} B_{\mu\nu}$$

Where χ is the coupling constant and

$$F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

is the electromagnetic field strength tensor and

$$B^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

is the dark electromagnetic field strength tensor.

$$\frac{1}{2}\chi F^{\mu\nu}B_{\mu\nu}$$

is the kinetic mixing term.

This Lagrangian provides us with the equation of motion for the field. If we go on to diagonalize it we end up with no mixing terms and thus we have the equation for the fields (physical eigenstates). After that, one must consider the interactions between fermions and the dark photon and this will give a new charge shift, i.e. a modified alpha. Once that is done we apply it to the Euler-Lagrange equation and we can get the Hamiltonian, i.e. the total energy of the field. We then induced ground state with $< 0|and|0 >$ and then go on to make the Casimir force calculation.

5 Summary

In conclusion, we have shown that there is definitely a Casimir force. An attraction between two plates in a vacuum that are less than a micron apart. Although the force is clear, the reason behind it is not. We have discussed the two prevailing opinions and their reasoning. Our main goal aside from discussing the reason for the force is to recalculate the Casimir effect, considering the interaction between photons and dark photons. In order to do that, we constructed a Lagrangian that can give us the energy of the field as well as the alpha between the dark photon and the photon. Similar to Jaffe, we can use an approximation that incorporates α, that is defined by the para-photon, and record a finite value for the Casimir Force.