Neutron Anti-Neutron Oscillations in Free Space and in a Nucleus

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(Dated: December 9, 2017)

Abstract

A missing piece of our understanding of how the Universe came to be is the question of why it contains more matter than anti-matter. Current model of formation of the Universe assumes that right after the Big Bang, the Universe contained equal amounts of matter and anti-matter. The fate of antimatter in the Universe's evolution is currently not established. A possible explanation of current-day matter domination lies in the process called baryogenesis. There are three conditions for generating excess matter in baryogenesis: baryon number violation, CP violation, and the evolution of the Universe out of thermodynamic equilibrium. An experimental manifestation of baryon number violating processes is neutron-anti-neutron oscillations, which we consider in this paper. We will go through the derivation of the probability of a neutron turning into an anti-neutron in free space. Then we will calculate the annihilation rate of an anti-neutron and neutron in a nucleus. The techniques developed in the project could be eventually applied to studies of neutron-anti-neutron oscillations in neutron stars.

INTRODUCTION

Current models of physics show that an event, such as the Big Bang, should have created equal parts matter and anti-matter in the universe. It appears that in today's world all antimatter has disappeared. But could this anti-matter be somewhere else in the universe? The answer is probably not. If there were large pockets of anti-matter in the universe we would be able to detect the photons released at the border of these anti-matter pockets due to annihilation. For matter to have become the dominant substance in our universe, we have to either have some sort of reactions, which makes more matter than anti-matter, and some sort of process in which matter and anti-matter spontaneously turn into each other.

In 1967, Soviet Physicist Andrei Sakharov proposed three conditions for this interaction, also known as baryogensis: baryon number violation, Charge Parity or CP violation, and the interaction must be outside of thermal equilibrium.[1]

Baryon number violation means that baryon number is not conserved in a reaction. Baryon Number is a number which is assigned to each baryon, a composite particle made up of 3 quarks, such as protons or neutrons. For matter baryons, such as protons and neutrons, this number is 1, while for anti-matter baryons, such as anti-neutrons and anti-protons this number is -1. When the Big Bang happened, the total baryon number of the universe was 0. The Standard Model of particle physics included a very complicated mechanism for baryon number violation. It is quite possible that models beyond the Standard Model have a simpler method for baryon number violation, such as presence of baryon-number violating interactions, which could be tested experimentally

CP denotes combined action of charge conjugation and parity symmetry. Charge conjugation transforms a particle into an anti-particle, for example a neutron to an anti-neutron. Parity means that after that particle is transformed it will become it's corresponding anti-particle. Violations of C or CP symmetries imply that interactions among particles differ from the interactions among anti-particles.

The third condition, the out-of-thermal equilibrium evolution of the Universe, assures that the reactions that created excess of baryons over anti-baryons are not reversed. This can happen according to the principle of detailed balance as in equilibrium each process must be equilibrated by its reverse process. While all of these conditions are important, this project is attempting to explain the first condition, baryon number violation. To do this, we can either look at protons or neutrons, since they both have a baryon number of 1. Since protons carry electrical charge, transformation of protons into anti-protons leads to non-conservation of electrical charge, which is forbidden. Neutrons are electrically neutral, so we only have to worry about baryon number violation. There is some speculation that we can recreate these neutron anti-neutron transitions by looking at neutrons within a particle accelerator such as the LHC. These particles are hard to detect, simply because even in the right conditions, baryogenesis is rare. Another strategy for looking at this is looking at system with many neutrons such as a neutron star. Neutrons stars are stars made mostly of neutrons with masses that are at least 1.4 times larger than the mass of the sun. They are held together by neutron degeneracy pressure The goal of this project is to eventually derive formulas for the densities of neutrons and anti-neutrons in neutron stars so that we can form a theoretical ground work to investigate baryon number violation. To work up to that goal, this paper will start out by looking at the probability of neutrons and anti-neutron oscillations in free space. This will create a simple framework for us to look at neutron anti-neutron oscillation in more complex cases. Then we can find the annihilation factor for neutron anti-neutron oscillations. This will allow us to check our probability calculations because they should add up to the annihilation factor. Then we will look at the probability of neutron anti-neutron oscillations in matter using the framework we set up in the first calculation. Once we get that probability we can apply the potential of a neutron star, and calculate those densities.

NETURON ANTI-NEUTRON OSCILLATIONS IN FREE SPACE

In this section, we will derive the formula for a neutron turning into an anti-neutron. We will begin by looking at the hamiltonian matrix. From there we will calculate the eigenvalues of the matrix which represent the energy states of the neutron and the anti-neutron in free space. From here we can write a basis equation for a neutron in terms of the neutron anti-neutron basis. Then we can calculate the probability that a neutron, will turn into an anti-neutron. We will follow the derivation in the paper "Neutron-Antineutron Oscillations: Theoretical Status and Experimental Prospects" [2] and then adding on to their calculation. First we need an Hamiltonian Matrix. We'll need a transition matrix elements which are

$$\langle n| H_{eff} |\bar{n}\rangle = \langle \bar{n}| H_{eff} |n\rangle = \delta m. \tag{1}$$

Where H_{eff} is some effective hamiltonian for the neutron and the anti-neutron. Calculating the diagonal matrix elements we get

$$\langle n| H_{eff} |n\rangle = M_{11}, \langle \bar{n}| H_{eff} |\bar{n}\rangle = M_{22}. \tag{2}$$

M_{11} and M_{22} both have an imaginary competent which comes from the effective hamiltonian.

$$M_{jj} = \frac{-i\lambda}{2}, j = 1, 2. \tag{3}$$

The imaginary part is there to represent the decay of the neutron in free space. Within free space neutrons are unstable and will decay after some time. The reason neutrons decay is because of its quarks. A neutron has three quarks; two down quarks, and an up quark . A proton has two up quarks and one down quark. The down quark has a slightly larger mass than an up quark. Subatomic particles want to be at the smallest mass possible, so a neutron is likely to decay in a proton via the weak interaction, by changing one of its down quarks into an up quark giving it a smaller mass. It is both unnecessary, and a hassle to keep track of the imaginary part in most of these calculations. We will revisit this when we calculate the probability of a neutron turning into an anti-neutron. Since we have our matrix elements, we can now put together our matrix:

$$M = \begin{bmatrix} M_{11} & \delta m \\ \delta m & M_{22} \end{bmatrix}. \tag{4}$$

Diagonalizing this matrix gives us the eigenstates

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix} \tag{5}$$

where

$$\tan 2\theta = \frac{2\delta m}{\Delta M}. \tag{6}$$

The energy eigenvalues are

$$E_{1,2} = \frac{1}{2}[M_{11} + M_{22} \pm \sqrt{\Delta M^2 + 4(\delta m)^2}]. \tag{7}$$

where

$$\Delta M = M_{11} - M_{22}.$$

Using this information we can solve for $|n_1\rangle$ and $|n_2\rangle$. Then we solve for $|n\rangle$ and $|\bar{n}\rangle$ in terms of $|n_1\rangle$ and $|n_2\rangle$ giving us

$$|n\rangle = \cos\theta\,|n_1\rangle - \sin\theta\,|n_2\rangle \tag{8}$$

and

$$|\bar{n}\rangle = \sin\theta\,|n_1\rangle + \cos\theta\,|n_2\rangle\,. \tag{9}$$

We can figure out the time dependance of the $|n\rangle$ equation by solving the time dependent Schrodinger equation giving us

$$|n(t)\rangle = \cos\theta e^{\frac{-iE_1 t}{2}}\,|n_1\rangle - \sin\theta e^{\frac{-iE_2 t}{2}}\,|n_2\rangle \tag{10}$$

We're only focusing on the $|n(t)\rangle$ because our particle will be starting as a neutron. To solve for the probability, we have to square the modulus of the function representing the anti-neutron, times the function representing the time dependence of the neutron. What is inside the modulus is called the probability amplitude and looks like $\langle\bar{n}|n(t)\rangle$. We solved for $ketn(t)$ up above, so we can calculate that probability.

$$|\,\langle\bar{n}|n(t)\rangle\,|^2 = |-\sin\theta\cos\theta e^{\frac{-iE_1 t}{2}} + \cos\theta\sin\theta e^{\frac{-iE_2 t}{2}}|^2 \tag{11}$$

By separating the exponents and using some trigonometric identities we get the answer they get in the paper.

$$|\,\langle\bar{n}|n(t)\rangle\,|^2 = \sin^2 2\theta \sin^2\frac{\Delta E t}{2} e^{-\lambda t} = P_{\bar{n}} \tag{12}$$

with $e^{-\lambda t}$ being the decay rate of a neutron in free space.

Now we can do our work which was to check the calculation done in the paper. For the probability of a neutron staying a neutron, we get,

$$|\,\langle n|n(t)\rangle\,|^2 = (1 - \sin^2 2\theta \sin^2\frac{\Delta E t}{2})e^{-\lambda t} = P_n \tag{13}$$

When can add our probability to the probability they get in the paper giving us

$$P_n + P_{\bar{n}} = e^{-\lambda t}. \tag{14}$$

This is what we expected because even though it seems like the probability of our particle turning into a anti-neutron or staying a neutron should be one. As stated earlier, neutrons in free space decay. The solution we got is the decay rate. We can also see why neutron-anti-neutron transitions are called oscillations The cosines and sines in the probability equations show that neutrons oscillate into anti-neutrons. Therefore we can refer to neutron anti-neutron transitions as neutron anti-neutron oscillations.

NEUTRON ANTI-NEUTRON OSCILLATIONS WITHIN A NUCLEUS

This process is similar in matter, but the main difference is that neutrons and anti-neutrons now have different potentials. This is because within a nucleus, the masses of the neutrons change due to binding energy as well as interaction with other parts of the nucleus. This makes it more complicated because we have to deal with various imaginary parts that canceled out before. Again we will be following a calculation from the paper used above [2]. The imaginary parts of the potential are there because when a neutron turns into an anti-neutron, it will quickly annihilate another neutron in the nucleus. We will be using a similar matrix to M in the last section, but instead of both hamiltonians for both the neutron and the anti neutron having the same imaginary part, only the anti-neutron has an imaginary part. In a nucleus, the diagonal matrix elements are

$$\langle n| H_{eff} |n\rangle = m_{n,eff} = m_n + V_{nr} \tag{15}$$

and

$$\langle \bar{n}| H_{eff} |\bar{n}\rangle = m_{n,eff} = m_n + V_{\bar{n}r} - iV_{\bar{n}I} \tag{16}$$

where r and I stand for imaginary and real. These potentials come from looking at the Schrodinger equation in circular coordinates. The transitional matrix elements are the same meaning

$$\langle n| H_{n,eff} |\bar{n}\rangle = \langle \bar{n}| H_{eff} |n\rangle = \delta m. \tag{17}$$

This ends up creating the matrix,

$$M_a = \begin{pmatrix} m_{n,eff} & \delta m \\ \delta m & m_{\bar{n},eff} \end{pmatrix}. \tag{18}$$

The energy eigenvalues of this matrix are

$$E_{1,2} = \frac{1}{2} m_{n,eff} + m_{\bar{n},eff} \pm \sqrt{(m_{n,eff} - m_{\bar{n},eff})^2 + 4(\delta m)^2}. \tag{19}$$

Using the rotational matrix from the last section, we can find the relationship between the various values in M_a which ends up being

$$\tan 2\theta = \frac{2(\delta m)}{m_{n,eff} - m_{\bar{n},eff}}. \tag{20}$$

Next we need find the annihilation rate of a neutron turning into an anti-neturon. This is represented by Γ. We can get this from the eigenvalues calculated earlier.

$$E_1 = m_{n,eff} + \frac{i}{2}\Gamma \tag{21}$$

To get our E_1 to look like that, we used some algebraic manipulation and taylor series, giving us,

$$E_1 = m_{n,eff} + \frac{(\delta m)^2 (m_{n,eff} - m_{\bar{n},eff})}{(m_{n,eff} - m_{\bar{n},eff})^2} \tag{22}$$

We can plug in for $m_{n,eff}$ and $m_{\bar{n},eff}$

$$E_1 = m_n + V_n - i\frac{(\delta m)^2}{V_n - V_{\bar{n}r} + iV_{\bar{n}I}} \tag{23}$$

$$E_1 = m_n + V_n - i\frac{(\delta m)^2 + V_{\bar{n}i}}{V_n - V_{\bar{n}r} + iV_{\bar{n}I}} + \frac{V_n - V_{\bar{n}}}{(V_n - V_{\bar{n}r})^2 + (V_{\bar{n}I})^2} \tag{24}$$

Because V_n, $V_{\bar{n}r}$, and$V_{\bar{n}I}$ are all on the same order of magnitude so $V_n - V_{\bar{n}r}$ should be extraordinarily small or close to zero this gives us

$$E_1 = m_n + V_n - i\frac{(\delta m)^2 + V_{\bar{n}i}}{V_n - V_{\bar{n}r} + iV_{\bar{n}I}} \tag{25}$$

From this can plug this in to the earlier equation to get an annihilation rate of

$$\Gamma = \frac{(\delta m)^2 + V_{\bar{n}i}}{V_n - V_{\bar{n}r} + iV_{\bar{n}I}}. \tag{26}$$

CONCLUSION AND FUTURE WORK

The next step is to calculate the probability of a neutron turning into an anti-neutron inside a nucleus. We can do this in a way that similar to the probability calculations in the free space, The main difference between these two calculations is that we have to switch the eigenvalues to those to do with the neutrons in a nucleus. Once we do this, we can then apply the potentials of a neutron star to this equation and then calculate the densities.

[1] A. D. Sakharov, Soviet Physics Uspekhi **34**, 392 (1991).

[2] D. G. Phillips, II *et al.*, Phys. Rept. **612**, 1 (2016).