Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

WEDNESDAY, January 3, 2018
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

Problem Number 1 (10 pts)

A parachutist jumps at an altitude of 3000 meters. If the parachute does not open, she is expected to approach a terminal speed of 30 m/sec. (g = 10 m/s^2)

(a) [4 points] Assuming that air resistance is proportional to speed, about how long does it take her to reach 90% of the terminal speed?

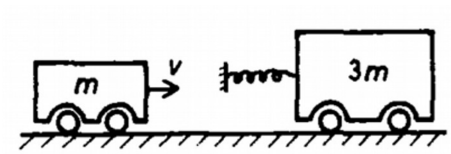
(b) [2 points] How far has she traveled in reaching 90% of the terminal speed?

After her parachute opens, her speed is slowed to 3m/sec. As she hits the ground, she flexes her knees to absorb the shock.

(c) [4 points] How far must she bend her knees in order to experience deceleration no greater than 10 g? Assume that her knees are like a spring with a resisting force proportional to displacement (mechanical energy is conserved).

Problem Number 2 (10pts)

A cart of mass *m* moves on a frictionless surface with speed *v* as it approaches a cart of mass 3*m* that is initially at rest. The spring is compressed during the head-on collision



(a) [3 points] What is the speed of the cart with 3*m* at the instant of maximum spring compression assuming conservation of energy?
(b) [3 points] What is the final velocity of the heavier cart after the collision, if the energy is conserved?
(c) [2 points] Give the final velocity of the heavier cart in the case of a completely inelastic collision (the carts move together after collision).
(d) [2 points] How much energy is dissipated in the case of a completely inelastic collision?

Problem Number 3 (10pts)

A particle of mass m is constrained to move on the parabola $z=\frac{x^2}{a}$ in the plane, a is a constant length, and there is a constant gravitational force acting in the negative z direction.

(a) [3 points] Write the Lagrangian using x as the generalized coordinate of the particle.
(b) [2 points] Write the equation of motion from the Lagrangian?
(c) [3 points] Write the equation for small oscillations about the equilibrium point.
(d) [2 points] What is the angular frequency for the small oscillations in (c)?

Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

WEDNESDAY, January 3, 2018
13:30 — 15:30

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

4. **10 points** A circular loop of radius R has a current I, and lies in the (x, y) plane with its center at the origin. At an elevation z on the $z-$ axis, it produces a field $\mathbf{B} = (0, 0, B)$ with

$$B = \frac{\mu_0 R^2 I}{(z^2 + R^2)^{3/2}}.$$

A second loop of radius δ, is also parallel to the $x - y$ plane and centered above the first loop at an elevation z. See Figure. The small loop has resistance Ω. For all answers below you can treat $\delta << R, \delta << z$.

a) (1pt.) Evaluate the magnetic flux through the second loop.

b) (3 pts.) The loop is now moved with constant velocity $v\hat{\mathbf{z}}$. Assume that $z = vt$. Evaluate the current in the loop and express it as a function of t.

c) (3pt.) Using cylindrical coordinates and one of the Maxwell equations prove that

$$B_\rho(\delta, \phi, z) \sim \frac{\delta}{2}(\frac{\partial B_z}{\partial z})_{0,0,z}$$

for the transverse field $B_\rho(\delta, \phi, z)$ acting on the small loop. The divergence operator in cylindrical coordinates is

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho B_\rho) + \frac{1}{\rho}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z}.$$

d) (3 pts.) Evaluate the force needed to keep the loop moving at constant velocity.

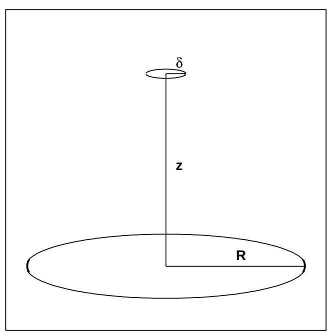


Figure 1:

5. **10 points** A solenoid is constructed with a thin wire, wrapped in a single layer without gaps around a non-magnetic cylinder of radius R and length D. The wire has resistivity ρ, and has a square cross section of area a^2. Recall that the inductance is defined as $\Phi = LI$, where Φ is the magnetic flux and I the current.

   a) (3pt.) Compute the self-inductance L of the solenoid as a function of the given variables.

   b) (3 pts.) Compute the resistance Ω of the solenoid.

   c) (4 pts.) At time $t = 0$ the solenoid is connected to a DC battery with potential V_0. Write the differential equation for the circuit and find the dependence of the current on time.

6. **10 points** A rectangular box has dimensions $L \times L \times D$. The two square sides are held at electric potentials $+V_0$ and $-V_0$ respectively, whereas the four other sides are kept at ground. Use the center of the box as the origin of your reference frame, with the z axis perpendicular to the square base.

   a) (1pt.) Describe the boundary conditions.

   b) (3 pts.) Set up the Laplace equation for the space inside the box, using separation of variables.

   c) (3 pts.) Give a series solution for the equation. Justify your choice of functions based on the boundary conditions.

   d) (2 pts.) Set up the equation to evaluate the coefficients (you will get full credit if the equation is correct, no need to solve it).

   e) (1 pt.) In the limit $L >> D$, prove that the field inside approaches the field of a capacitor.

Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART III

FRIDAY, January 5, 2018
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part III).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

7. **10 points** A particle of mass m is in the stationary state

$$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$$

where A and a are positive real constants.

(a) (2 pts.) Find A.

(b) (1 pt.) What is the energy of the state?

(c) (3 pts.) For what potential energy function does Ψ satisfy the Schroedinger equation?

(d) (4 pts.) Find the expectation values of position and momentum.

8. **10 points** The scattering of a non-relativistic electron of low energy E from a noble gas atom can be modelled in one dimension by a plane wave incident on a square well potential given by,

$$\begin{array}{lll} V(x) = 0 & \text{for} & x < -a \\ V(x) = -V_0 & \text{for} & -a < x < a \\ V(x) = 0 & \text{for} & x > a \end{array}$$

where V_0 and a are real, positive constants.

(a) (2 pts.) State analytic expressions for the time-independent wave function in each region of the potential. Assume that the wave is incident from the left, in the region $x < -a$. Define all variables that you introduce.

(b) (3 pts.) State analytic expressions for the boundary conditions.

(c) (4 pts.) A remarkable property of this system is that for certain values of the wavenumber k inside the well the transmission coefficient is exactly one. One such value is $k = \pi/(2a)$. For that value, evaluate the boundary conditions to solve for the amplitude of the reflected wave.

(d) (1 pt.) For what other values of k is the transmission coefficient one?

Note of interest: the phenomenon of enhanced electron transmission is observed and it is known as the *Ramsauer-Townsend effect.*

9. **10 points** The spin angular momentum operators for a spin-1/2 particle are given by $S_i = (\hbar/2)\sigma_i$ where $i = x, y,$ or z and the Pauli spin matrices are specified in the z-basis by,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(a) (2 pts.) Find the normalized eigenspinors of S_z and their corresponding eigenvalues.

(b) (2 pts.) Consider a spin-1/2 particle in an eigenstate of S_x with eigenvalue $+\hbar/2$. Find the normalized eigenspinor.

The spin-1/2 particle is in a region of uniform magnetic field with magnitude B and direction along $+z$. The Hamiltonian is given by $H = -\gamma B S_z$ where γ is a constant.

(c) (3 pts.) At $t = 0$ the particle is in the eigenstate of S_x with eigenvalue $+\hbar/2$. Find the normalized eigenspinor at time $t > 0$.

(d) (3 pts.) Find the expectation value of S_x versus t.

Note of interest: the time evolution of the spin angular momentum is known as *Larmor precession* and is the quantum analog of classical precession.

Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART IV

WEDNESDAY, January 5, 2018
13:30 — 15:30

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

10. **10 points**

(a) (5 pts.) State the condition for two phases to coexist at a given pressure and temperature. From this condition derive the Clausius-Clapeyron equation for the slope of the coexistence curve

$$\left.\frac{dP}{dT}\right|_{coexistence} = \frac{L}{T\Delta v},$$

where L is the latent heat and v is the specific volume.

(b) (5 pts.) At elevation of 500 m, the boiling temperature of water changes by about 1.7°C compared to that at sea level. Using this as an input, make an *estimate* (to factor of 2 accuracy) of the latent heat of evaporation for the water liquid-gas phase transition (If needed, you can use that the density of air $\rho_{air} \approx 1$ kg/m^3, and the normal atmospheric pressure $p_0 \approx 10^5$ Pa.).

11. **10 points** A neutral pion of energy $E = 2m_\pi c^2$ decays into two photons.

    (a) (5 pts.) Find the maximum and minimum energies of the photons.

    (b) (5 pts.) Find the minimum opening angle between two photons.

12. **10 points** The system of N quantum oscillators (of frequency ω) are in thermal equilibrium at temperature T.

   (a) (3 pts.) Calculate the system partition function.

   (b) (4 pts.) Find the system average energy.

   (c) (3 pts.) Find $\left\langle (\Delta E)^2 \right\rangle$, the fluctuations in the system energy.