Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

WEDNESDAY, January 7, 2015
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. Your special ID number that you received from Delores Cowen;
2. The problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

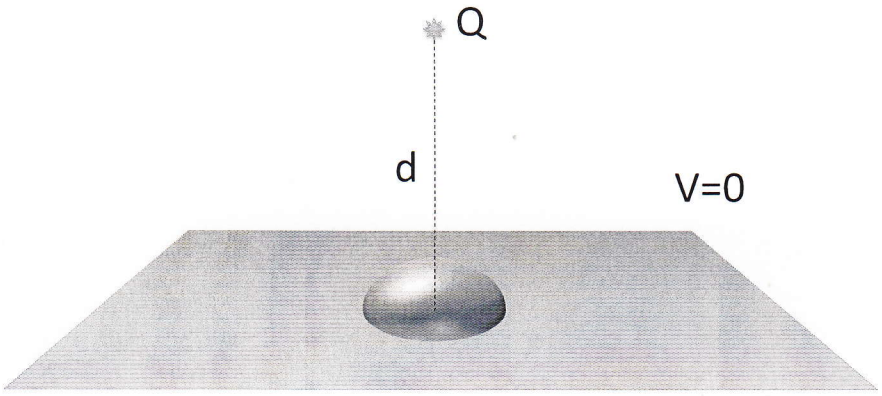
Do NOT write your name on the cover or anywhere else in the booklet!

1. (**10 points**) An infinite conducting plate has a solid hemisphere of radius R placed on it. The flat side of the hemisphere is attached to the plate via a conducting connection. This assembly is then grounded. A point charge Q is placed a distance d above the plane, directly above the center of the hemisphere.

a) (2 pts) For the region above the plane (on the side with the hemisphere), identify the boundary conditions for the potential.

b) (5 pts) Find the appropriate image charge distribution which reproduces this potential in the upper hemisphere.

c) (3 pts) Find the potential and the electric field everywhere above the plane (i.e., on the side of the plane with the hemisphere).



2. (**10 points**) A planet of mass m orbits a star of mass M. The orbit is ellyptic and the distance between the planet and the star oscillates between r_0 and r_1. The system total energy is E and its total angular momentum is L.

a) (1 pt) Write the reduced mass μ for the system .

b) (4 pts) Write an energy conservation equation for the system energy E as a function of r and its time derivative, L and μ

c) (4 pts) solve the equation for r_0 and r_1 .

d) (1 pt) Using the solution, solve for L and E as a function of the masses, the gravity constant G, and r_0, r_1.

3. (**10 points**) Consider a simple harmonic oscillato r in one dimension. The Hamiltonian H and the wavefuction ψ at time $t = 0$ are

$$H = \hbar\omega(a^+ a + 1/2) \tag{1}$$

and

$$\psi(0) = \frac{1}{\sqrt{5}}|1\rangle + \frac{2}{\sqrt{5}}|2\rangle \tag{2}$$

where $|n\rangle$ denotes the eigenfuction with energy $E_n = \hbar\omega(n + 1/2)$, and a^+ and a are the raising and lowering operators respectively.

(a) (1 pt) What is the time-dependent wavefunction $\psi(t)$?

(b) (4 pts) What is the expectation value for the energy?

(c) (5 pts) The position x can be represented with operators by $x = X_0(a^+ + a)$ where $X_0 = \sqrt{\hbar/2m\omega}$ is a constant. Derive an expression for the time-dependent expectation value for the position operator. You may find the following operator expressions helpful: $a|n\rangle = \sqrt{n}|n-1\rangle$; $a^+|n\rangle = \sqrt{n+1}|n+1\rangle$.

4. (**10 points**) A system is composed of N distinguishable atoms at rest and mutually noninteracting. Each atom has only two nondegenerate energy levels 0 and $\varepsilon > 0$. Let E be the total energy.

(a) (5pts) Assuming equilibrium condition, compute the mean energy per particle E/N as a function of positive temperature T and evaluate it in the limit of $T \to \infty$.

(b) (5pts) Compute the entropy per atom S/N as a function of T in a thermodynamic equilibrium and evaluate it in the limits of $T \to 0$ and $T \to \infty$.

5. (**10 points**) A thief gets into a spaceship and escapes at a velocity of $v = c/4$. To stop his spaceship, a policeman fires a positively charged pellet of mass m and charge q with a velocity of $v = c/2$, directly at his spaceship. An observer on the earth measures the distance between the pellet and the thief to be d at the instant the pellet leaves the gun. In an effort to stop the pellet, the thief charges a metal ring of radius R with positive charge Q, that is attached to and held behind his spaceship with its plane perpendicular to the velocities of the thief's spaceship and the pellet. Assume that the pellet was fired directly along the axis of the ring. We would like to figure out what is the minimum amount of charge that must be placed on the ring to stop the pellet from hitting the thief. Assume that the mass of the thief and spaceship $M \gg m$, and ignore any acceleration of the spaceship due to a reaction from the bullet.

a) (4 pts) This problem is most easily solved in the Rest Frame Of the Thief (RFOT). Calculate the relative velocity v_R and the distance d_R between the bullet and the thief in the RFOT.

b) (3 pts) Calculate the change in kinetic energy of the bullet as it comes to a stop.

c) (3 pts) Compute the minimum charge on the ring, required to stop the pellet.

6. A particle of mass m is confined in a potential of the form

$$U(x) = -\alpha\delta(x), \quad \alpha > 0. \tag{3}$$

(a) (5 pts) Find the particle energy and corresponding wave function

Suddenly, the parameter α changes to one half of its value $\alpha \to \alpha/2$.

(b) (5 pts) Find the probability that the particle will become free

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PART II

FRIDAY, January 9, 2015
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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1. (**10 points**) Figure 1 shows a window with double-sided glasses with air in between. The inside temperature is T_{inside} = 20°C and the outside $T_{outside}$ = -20°C. The distance between the glasses is d_a =7.5 cm and the thickness of the glasses is d_g =3.0 mm. The thermal conductivity of air is $\kappa_a = 5.6 \times 10^{-3} Cal/(m \cdot s \cdot K)$ and κ_g =0.0167 $Cal/(m \cdot s \cdot K)$ for the glass. Ignoring the convection, find the heat flux through the window.

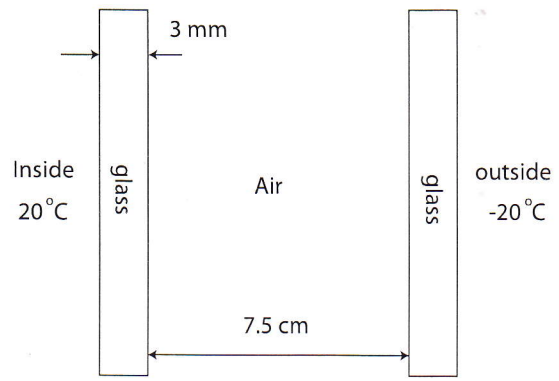

Figure 1:

2. (**10 points**) A particle of mass m is constrained to the (x, y) plane subject to the repulsive force $\mathbf{F} = k\mathbf{r}$, with $k > 0$. The initial position is (x_0, y_0).

a) (2 pts) Write the Lagrangian of the particle, and the equations of motion.

b) (2pts) Derive the equations of motion and obtain the general solution.

c) (2 pts) For this particular trajectory, $\dot{x}\dot{y} = c$, where c is a constant. Use this and the initial positions to completely solve the equations of motion.

d) (2 pts) find a relation between x_0, y_0, c, k and m.

e) (2p ts) prove that the trajectory is a hyperbola.

3. **(10 points)** Consider two spin $s = 1/2$ particles to be in a state

$$X = \frac{1}{\sqrt{5}}\chi_+^{(1)}\chi_-^{(2)} + \frac{2}{\sqrt{5}}\chi_-^{(1)}\chi_+^{(2)} \tag{1}$$

where $\chi_\pm^{(i)}$ describes i-th particle in a state with $s_z = \pm 1/2$.

(a) (2 pts) What are the possible values of the total spin j and its projection on z axis, j_z?

(b) (3 pts) Express all possible eigenstates $|j, j_z\rangle$ in terms of $\chi_\pm^{1,2}$.

(c) (3 pts) Express the spin function of the system [Eq.(1)] in terms of eigenstates $|j, j_z\rangle$

(d) (3 pts) What are the probabilities that the total spin is 0, 1/2, 1? What are the probabilities that j_z is 0, 1/2, 1?

4. (**10 points**) A wall contains n absorbing sites. Each site can absorb one molecule of surrounding gas. The wall is in thermal and chemical equilibrium with surrounding (monoatomic, ideal) gas maintained at temperature T and pressure P. The absorption energy equals to $-\varepsilon$.

   (a) (3 pts) Calculate the canonical partition function of the ideal gas.

   (b) (4 pts) Calculate the grand canonical partition function and find the chemical potential of the gas as a function of T and P.

   (c) (3 pts) Find the mean occupancy for an absorbing site as a function of external temperature and pressure.

5. (**10 points**) A disk of radius R is on a slab. At $t = 0$ a constant force F is applied to the slab (see Figure 2). Both disk and slab have mass M. The slab-support interface is frictionless, but the disk does not slip.

a) (1pt) Compute the moment of inertia of the disk.

b) (3 pts) Write equations for the forces acting on the slab and the disk

c) (3 pts) Write an equation for the torque acting on the disk.

d) (3 pts) Evaluate the linear accelerations of both the disk and the lab.

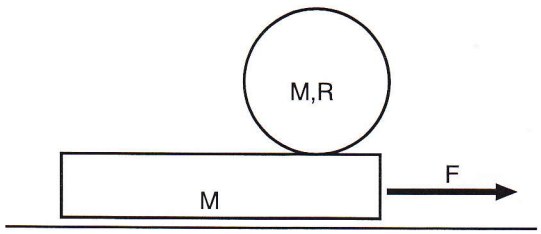

Figure 2: Figure for disk-slab problem.

6. (**10 points**) As shown in Fig. 3, a toroid with a rectangular cross section is placed within a parallel plate capacitor. We show a cross section of the toroid to indicate the rectangular shape (note: figure not to scale). The full toroid with inner radius a and outer radius b fits within the parallel plate capacitor with radius $R \gg b$. Assume that $R \gg d$, the gap between the capacitor plates and ignore fringe effects. The height of the toroid is $h \ll d$. There are N uniformly spaced turns of wire on the toroid with a total resistance R_2 (we have only shown a few turns). The coils are densely wound, $N \gg 1$. The material on which the coil is wound has a permittivity $\epsilon = \epsilon_0$ and a permeability $\mu = \mu_0$ i.e., the same as vacuum. Also assume that the space between the toroid and the capacitor is vacuum. At the start of the process, there is a charge $+Q$ on the upper plate of the capacitor and a charge $-Q$ on the lower plate. The capacitor is then discharged via an external circuit with a resistance R_1.
a) Assuming that the toroid has a negligible effect on the discharging of the capacitor, derive an expression for the time dependent charge on the capacitor plates. (2 pts).
b) Calculate the time dependent induced B field due to the change in the E field in between the capacitor plates. (3 pts).
c) Calculate the time dependent induced current in the toroid due to the induced magnetic field (5 pts).

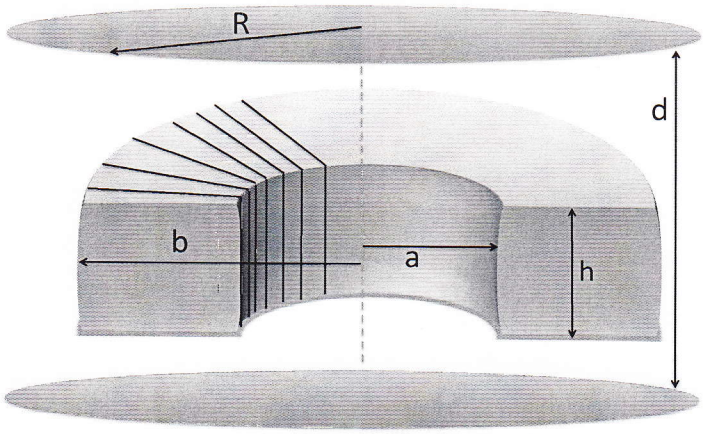

Figure 3: