PAN SEMINAR, WAYNE STATE UNIVERSITY

FEBRUARY 5 2021

Constraining the Effective Shear Viscosity of Hot Hadron Matter

RAINER J FRIES

TEXAS A&M UNIVERSITY

Zhidong Yang and RJF, arXiv:1807.03410
Zhidong Yang and RJF, arXiv:2007.11777

Outline

- Shear viscosity. What is it in nuclear matter?
- Our idea to get to shear viscosity in hadron matter; viscous blastwave
- Data and Bayesian Analysis
- Results
- Uncertainty analysis
- Summary

Shear Viscosity

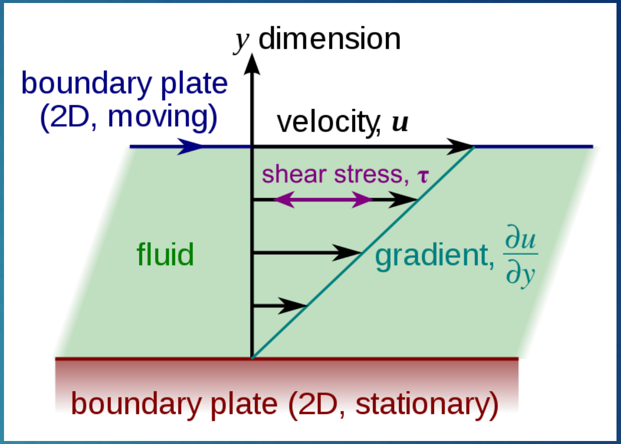
- Shear Viscosity η (Navier-Stokes):

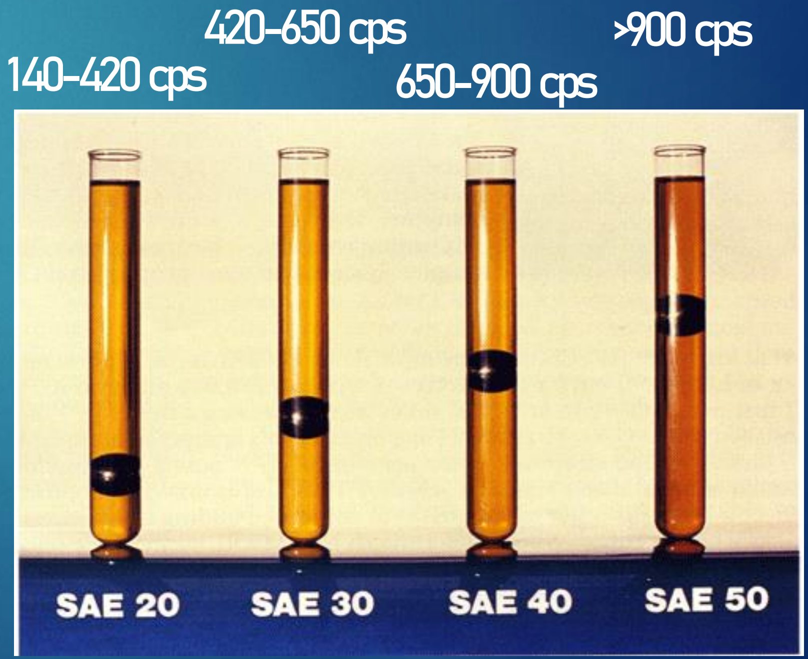
$$\frac{F}{A} = -\eta \frac{\partial u_x}{\partial y}$$

- In relativistic fluid dynamics it appears as the dimensionless transport coefficient $\frac{\eta}{s}$ (s = entropy density)

- Measures the ability of momentum transfer

$$\frac{\eta}{s} \sim T\lambda\bar{v}$$





http://www.vp-scientific.com/Viscosity_Tables.htm

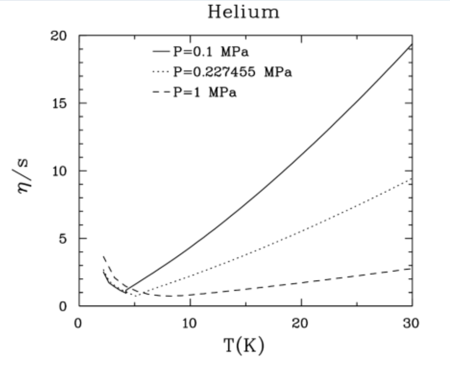
Shear Viscosity

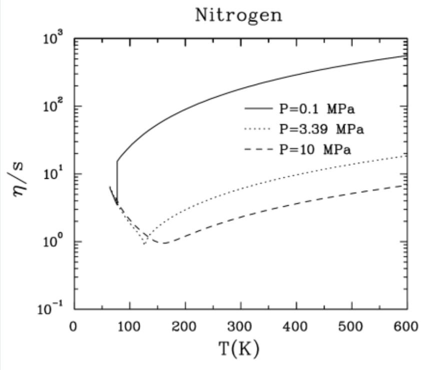
- Conjectured lower quantum bound $\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$

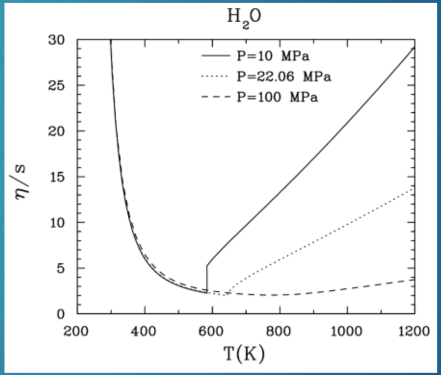
P. Kovtun, D.T. Son, A.O. Starinets, Phys. Rev. Lett. 94, 111601 (2005)

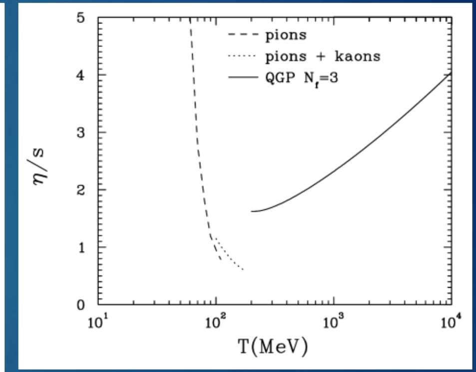
- Conjectured minimum of $\frac{\eta}{s}$ at phase transitions

L.P. Csernai, J.I. Kapusta, L.D. McLerran, Phys. Rev. Lett. 97, 152303 (2006)







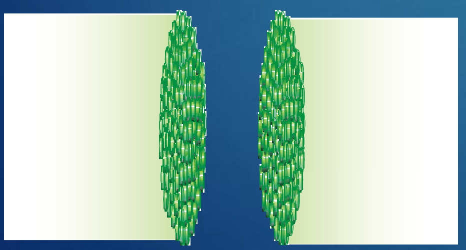


- How does hot nuclear matter fit into the picture?

High Energy Nuclear Collision

1. Initial condition: nuclear wave functions
2. Initial interaction: strong gluon fields → glasma.
3. Approach to kinetic and chemical equilibrium after ~ 0.2 – 1.0 fm/c; QGP phase with initial temperatures up to ~400-600 MeV (RHIC/LHC)
   - Transverse expansion and cooling of the fireball (~hydrodynamic behavior)
4. Hadronization around T_c (~ 160 MeV), subsequent hot hadron matter phase
   - HRG may fall out of chemical equilibrium at chemical freeze-out.
5. Decoupling of hadrons (kinetic freeze-out) and free streaming of hadrons to detectors.

1



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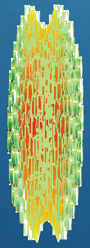

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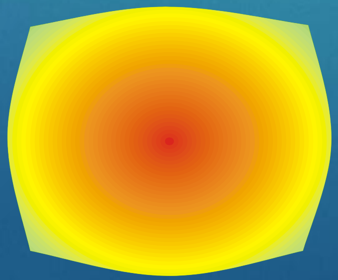

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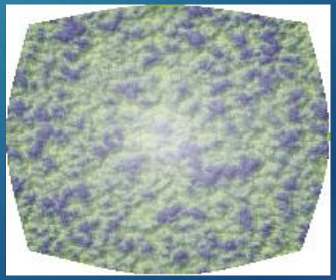

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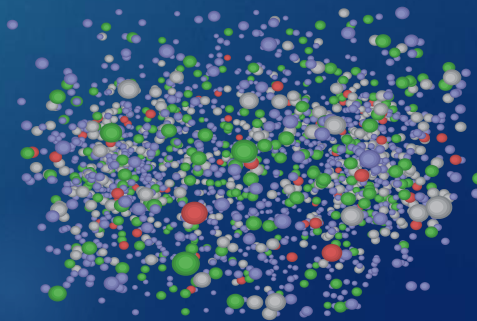
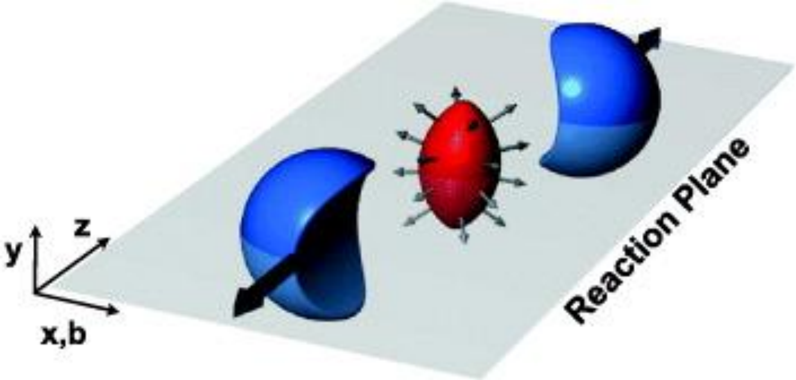
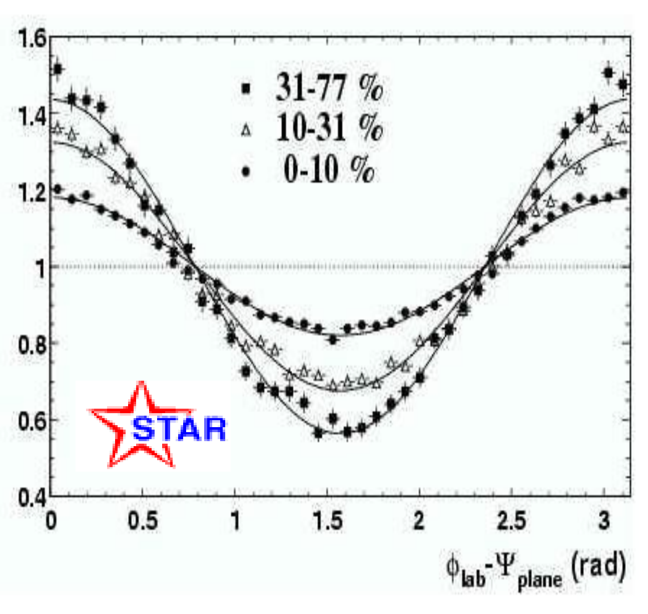


High Energy Nuclear Collisions

- Quark gluon plasma and hot hadron matter are produced in A+A collisions at RHIC and LHC.
- Flow anisotropies (elliptic flow v_2) is easily measured.
- Expansion and cooling phase close to equilibrium: use relativisitv fluid dynamic with η as an extractable parameter; fit parameters to measured data.

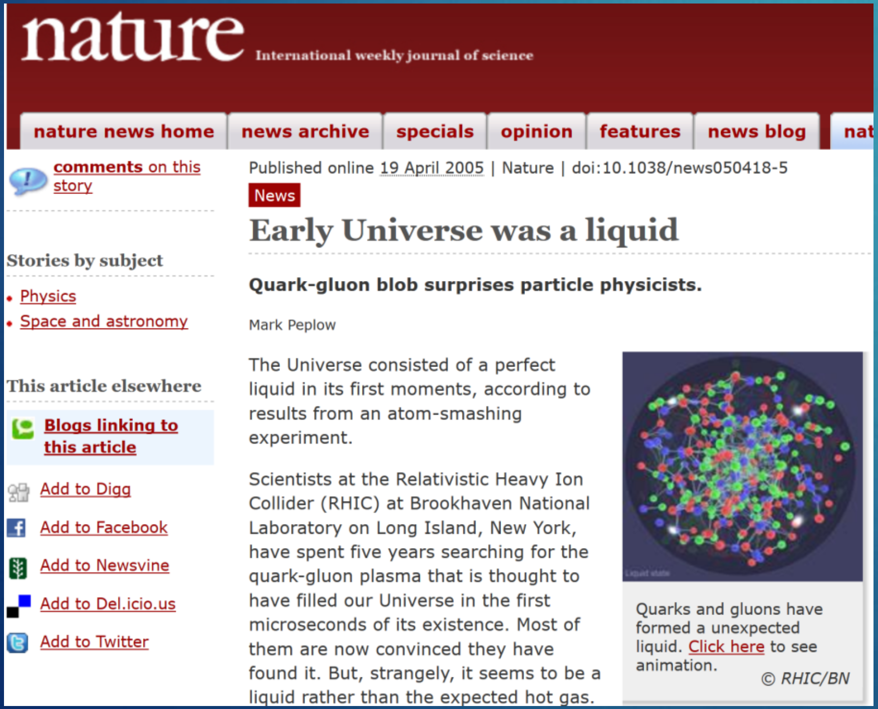


$v_2 =$ second order Fourier coefficient of the particle spectrum transverse to the beam direction.



Current Status

- Heavy ion collisions at RHIC/LHC: perfect liquid paradigm for QGP around T_c.
- Extractions from data using fluid dynamics: $\frac{\eta}{s} \sim (1 \ldots 2)\frac{1}{4\pi}$ @ T_c
- Consistent with lattice QCD and NLO pQCD calculations.



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Published online 19 April 2005 | Nature | doi:10.1038/news050418-5

News

Early Universe was a liquid

Quark-gluon blob surprises particle physicists.

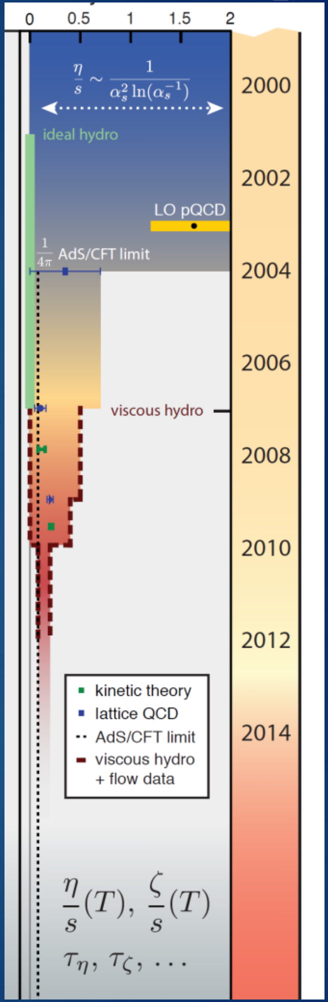
Mark Peplow

The Universe consisted of a perfect liquid in its first moments, according to results from an atom-smashing experiment.

Scientists at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory on Long Island, New York, have spent five years searching for the quark-gluon plasma that is thought to have filled our Universe in the first microseconds of its existence. Most of them are now convinced they have found it. But, strangely, it seems to be a liquid rather than the expected hot gas.

Quarks and gluons have formed a unexpected liquid. Click here to see animation.

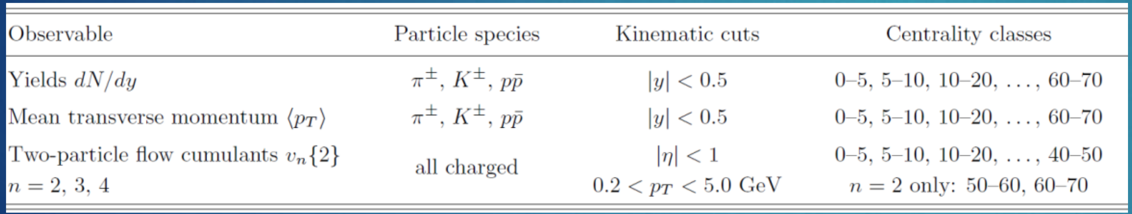
© RHIC/BN



C. Gale, S. Jeon, B. Schenke,
Int. J. Mod. Phys. A 28, 1340011 (2013)

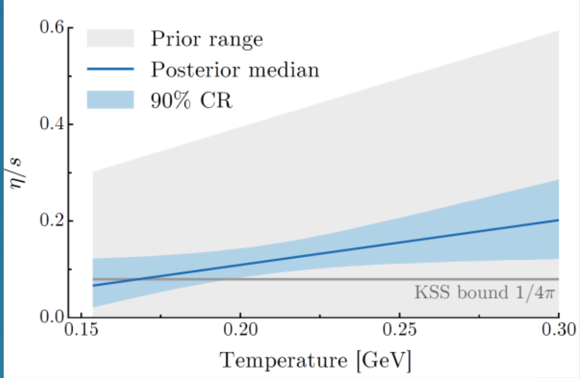
QGP Side: Bayesian Analysis

- State of the art: viscous fluid dynamics plus hadronic afterburner (URQMD/SMASH)
- Paradigm: switch to transport in the hadronic phase
  - Helps with deviations from chemical equilibrium, realistic freeze-out.
- 2+1D hydro + URQMD with 2.76 TeV Pb+Pb data; only p_T-integrated observables



Observable	Particle species	Kinematic cuts	Centrality classes
Yields dN/dy	$\pi^\pm$, $K^\pm$, $p\bar{p}$	$\lvert y\rvert < 0.5$	0–5, 5–10, 10–20, . . . , 60–70
Mean transverse momentum $\langle p_T \rangle$	$\pi^\pm$, $K^\pm$, $p\bar{p}$	$\lvert y\rvert < 0.5$	0–5, 5–10, 10–20, . . . , 60–70
Two-particle flow cumulants $v_n\{2\}$ $n = 2, 3, 4$	all charged	$\lvert\eta\rvert < 1$ $0.2 < p_T < 5.0$ GeV	0–5, 5–10, 10–20, . . . , 40–50 $n = 2$ only: 50–60, 60–70

J. E. Bernhard et al., Phys. Rev. C 94, 024907 (2016)



- 2+1D hydro + SMASH (JETSCAPE)

D. Everett et al., arXiv:2010.03928, 2011:01430

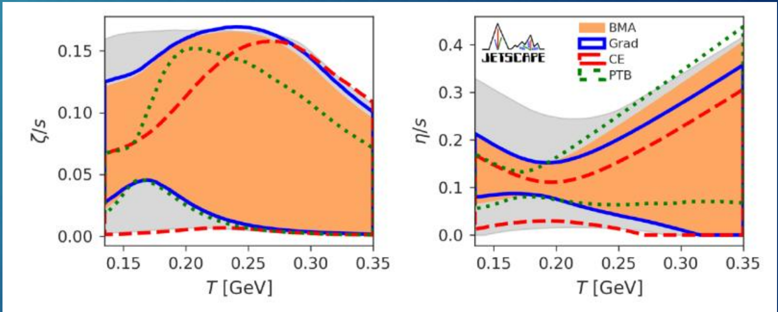
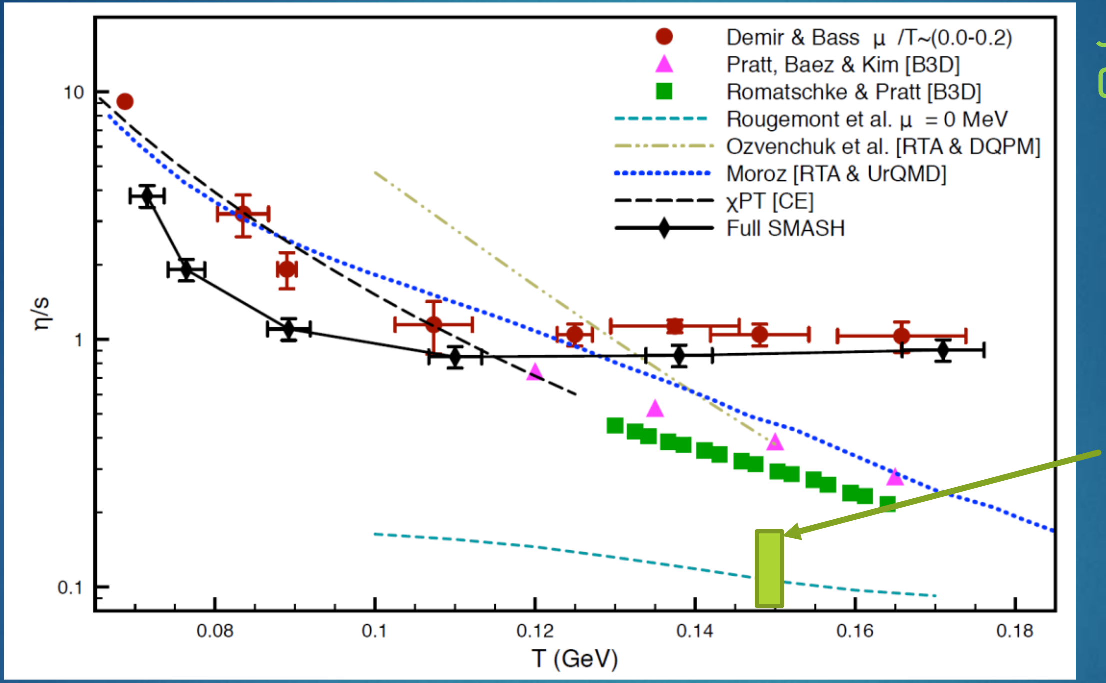


FIG. 1. The 90% credible intervals for the prior (gray), the posteriors of the Grad (blue), Chapman-Enskog (red) and Pratt-Torrieri-Bernhard (green) models, and their Bayesian model average (orange) for the specific bulk (left) and shear (right) viscosities of QGP.

Hot Hadron Matter

- Various hadronic transport calculations available



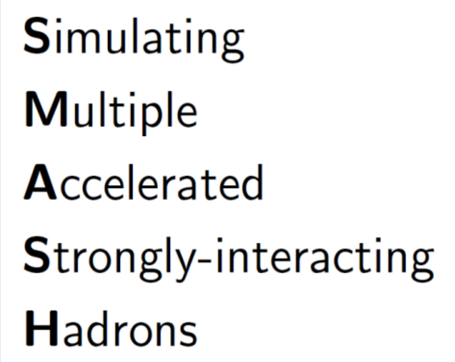
J. B. Rose et al., Phys. Rev. C 97, 055204 (2018)

QGP @ Tc
Extracted from fluid dynamics

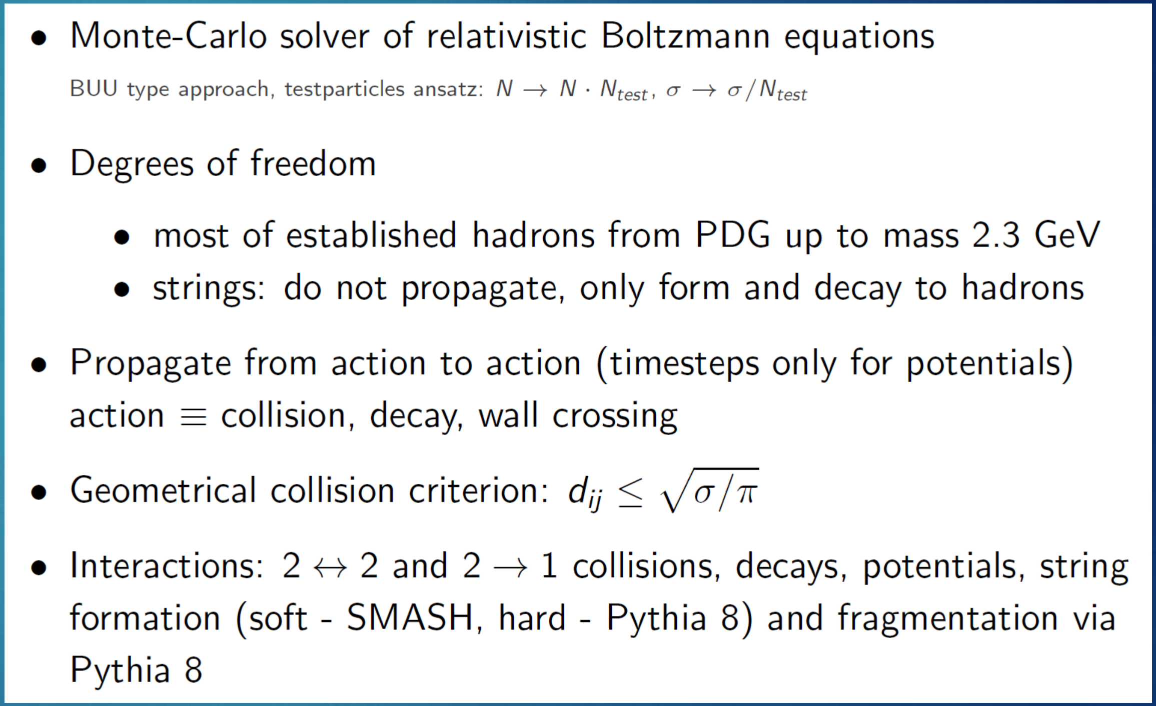
- Results vary by an order of magnitude
- The most popular models predict very large η/s. Just below T_c.

Hadronic Transport: SMASH

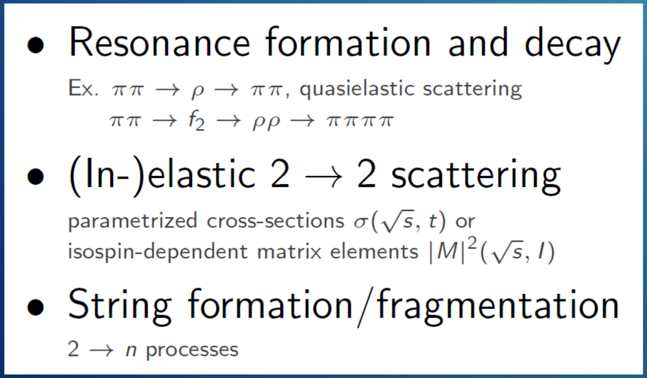
- The successor to URQMD J. Weil et al., Phys. Rev. C 94, 054905 (2016)
- In their own words (adapted from slides of D. Oliinychenko):



Simulating
Multiple
Accelerated
Strongly-interacting
Hadrons



- Monte-Carlo solver of relativistic Boltzmann equations
  BUU type approach, testparticles ansatz: $N \to N \cdot N_{test}$, $\sigma \to \sigma / N_{test}$
- Degrees of freedom
  - most of established hadrons from PDG up to mass 2.3 GeV
  - strings: do not propagate, only form and decay to hadrons
- Propagate from action to action (timesteps only for potentials)
  action $\equiv$ collision, decay, wall crossing
- Geometrical collision criterion: $d_{ij} \leq \sqrt{\sigma/\pi}$
- Interactions: $2 \leftrightarrow 2$ and $2 \to 1$ collisions, decays, potentials, string formation (soft - SMASH, hard - Pythia 8) and fragmentation via Pythia 8



- Resonance formation and decay
  Ex. $\pi\pi \to \rho \to \pi\pi$, quasielastic scattering
  $\pi\pi \to f_2 \to \rho\rho \to \pi\pi\pi\pi$
- (In-)elastic $2 \to 2$ scattering
  parametrized cross-sections $\sigma(\sqrt{s}, t)$ or
  isospin-dependent matrix elements $|M|^2(\sqrt{s}, I)$
- String formation/fragmentation
  $2 \to n$ processes

Hadronic Transport: SMASH

- Viscosity from a Kubo formula

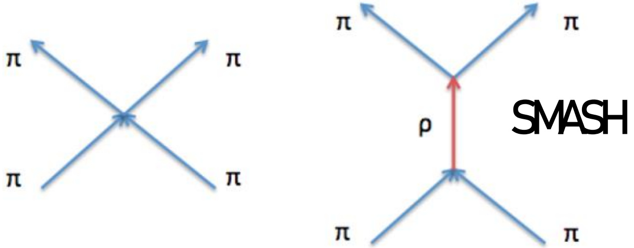
$$\eta = \frac{V}{T}\int_0^\infty dt\ C^{xy}(t)$$

$$C^{xy}(t) = \langle T^{xy}(0)T^{xy}(t)\rangle_{eq}$$

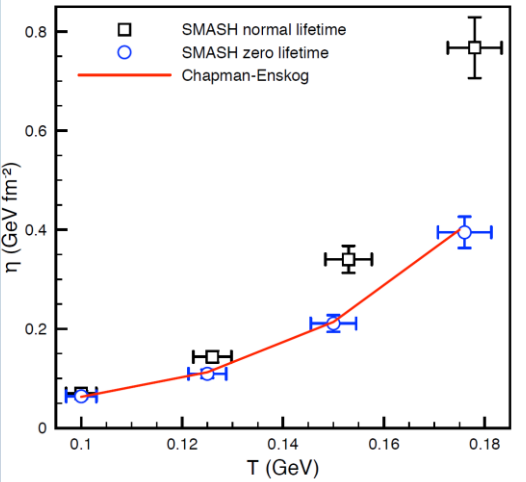
$$T^{\mu\nu} = \frac{1}{V}\sum_{i=1}^{N_{part}} \frac{p_i^\mu p_i^\nu}{p_i^0}$$

- Their result: $\eta/s \sim 1$ at T_c.
- Their explanation for large η/s:
  - Almost all relevant interactions in SMASH through resonances
  - Finite resonance life times = delay in momentum transport

Transport with parameterized 2→2 cross sections



- They convincingly demonstrate the effect in a simple $\pi - \rho$ system:

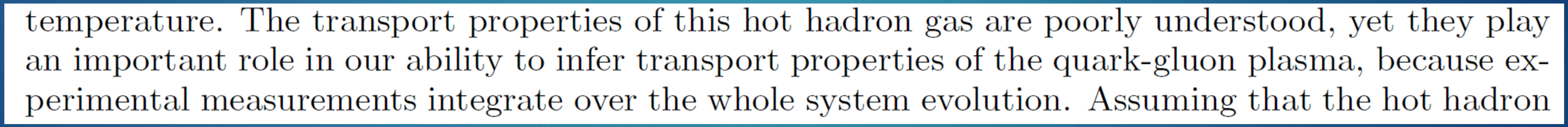


Current Status?

- The current situation leaves unanswered questions.

- Do we go from water to honey at T_c?
  - It ought to make a difference in observables somehow (or are we less sensitive to viscosity than we thought?)
- Hadronic transport improves certain aspects but introduces new uncertainties!

P. Romatschke and S. Pratt, 1409.0010:

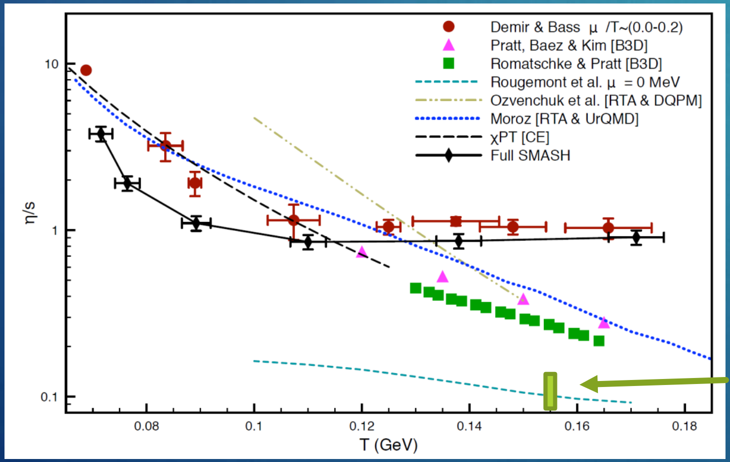


temperature. The transport properties of this hot hadron gas are poorly understood, yet they play an important role in our ability to infer transport properties of the quark-gluon plasma, because experimental measurements integrate over the whole system evolution. Assuming that the hot hadron

- What if hadronic transport gets viscosity wrong, at least around T_c? Do we underestimate η/s in QGP?

Our Idea

- What is η/s in a hot hadron gas?
- Take an agnostic approach. What if we didn't understand the microphysics at all?
- Can it be extracted from data, independent of fluid dynamics or transport?

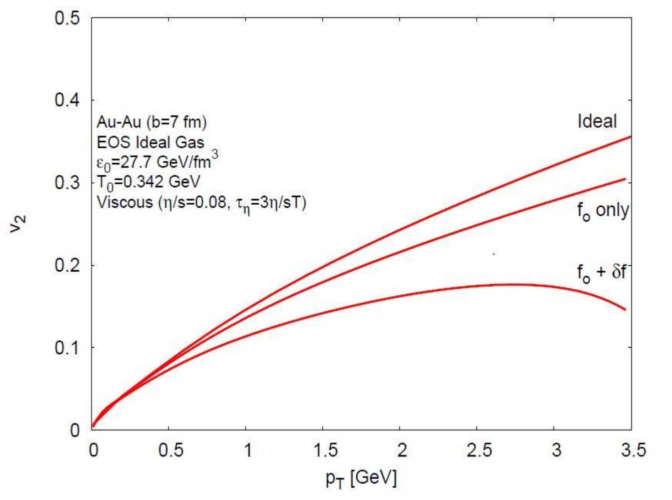


J. B. Rose et al., Phys. Rev. C 97, 055204 (2018)

Extracted from fluid dynamics

Strategy: Back To Basics

- Direct effect of shear stress on particle distributions at freeze-out.
- Can be captured by a blastwave with viscous corrections in Navier-Stokes approximation.



D. Teaney, arXiv:0905.2433

Fluid Dynamics

- Calculates flow field and f.o. hypersurface
- Uncertainties from initial conditions, equation of state
- Instantaneous Cooper-Frye freeze-out
- η/s affects f.o. + flow field and entire dynamics
- Sensitive to η/s integrated over space-time history of fireball

Blastwave

- Fits f.o. flow fields and hypersuface
- Uncertainties from simple ansatz for hypersurface and flow field
- Instantaneous Cooper-Frye freeze-out
- η/s affects freeze-out only
- Sensitive to η/s at one temperature (hadron gas!)

Complementary approaches!

Viscous Blastwave

- Start from the Retiere Lisa (RL) blastwave
- Instantaneous freeze-out on hypersurface Σ:

F. Retiere and M. Annan Lisa, Phys. Rev. C 70, 044907(2004)

$$E\frac{d^3N}{d^3p} = \frac{g}{(2\pi)^3}\int_\Sigma f(r,p)p^\mu \, d\Sigma_\mu$$

D. Teaney,
Phys. Rev. C68, 034913 (2003)

- Particle distribution $f(r,p) = f_0 + \delta f$
  - Equilibrium distribution f_0, correction $\delta f \ll f_0$
- Shear correction in Navier-Stokes approximation (here $\lambda = 2$)

A. Jaiswal and V. Koch,
arXiv:1508.05878

$$\delta f = \frac{\eta}{s}\frac{\Gamma(6)}{\Gamma(4+\lambda)}\left(\frac{E}{T}\right)^{2-\lambda}\frac{p^\mu p^\nu}{T^3}\sigma_{\mu\nu} f_0$$

K. Dusling, G.D. Moore, D. Teaney,
Phys. Rev. C 81, 034907 (2010)

M. Damodarain et al.,
arXiv:1707.00793

- Gradients of flow field $\sigma^{\mu\nu} = \frac{1}{2}(\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{1}{3}\Delta^{\mu\nu}\nabla_\kappa u^\kappa$

Viscous Blastwave

F. Retiere and M. Annan Lisa, Phys. Rev. C 70, 044907 (2004)

- Hypersurface:
  - $\tau = \sqrt{t^2 - z^2} = \text{const.}$
  - Elliptic shape in the transverse plane (axes R_x, R_y)
- Flowfield: boost invariant (Bjorken flow)
- Transverse velocity parameterization

$$v_T = (\alpha_0 + \alpha_2 \cos 2\phi_b)\rho^n$$

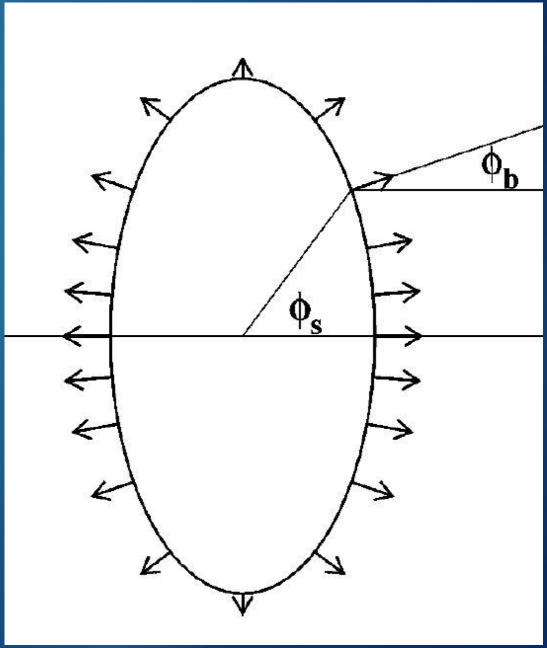
Average surface radial speed

Elliptical deformation

Tilted flow angle

Reduced radius

  - Flow tilt $\tan\phi_b = \left(\frac{R_x}{R_y}\right)^2 \tan\phi_s$



Viscous Blastwave

- Calculate $\sigma^{\mu\nu} = \frac{1}{2}(\nabla^{\mu}u^{\nu} + \nabla^{\nu}u^{\mu}) - \frac{1}{3}\Delta^{\mu\nu}\nabla_{\kappa}u^{\kappa}$ from given flow field parameterization.
- Spatial derivatives are tedious but straight forward

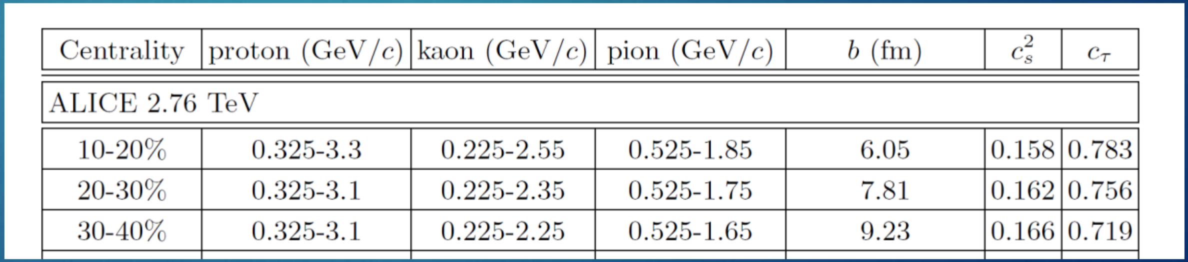
$$\partial_2 u^1 = \frac{\partial u^1}{\partial y} = n \sinh\eta_T \cosh^2\eta_T \frac{\sin\phi}{\rho R_y}\cos\phi_b - \sinh\eta_T \frac{\tan^2\phi_b}{(1+\tan^2\phi_b)^{3/2}} \frac{1}{\rho R_y \sin\phi}$$

etc…

- Time derivatives: solve fluid dyamics equations of motion
    - Ideal case is sufficient for Navier-Stokes approximation.
    - $De = -(e+p)\nabla_{\kappa}u^{\kappa}$, $Du^{\mu} = \frac{\nabla^{\mu}p}{e+p}$
    - E.g. $(1 - c_s^2 \tanh^2\eta_T)\partial_\tau \cosh\eta_T = c_s^2 \tanh^2\eta_T(\partial_1 u^1 + \partial_2 u^2 + \frac{\cosh\eta_T}{\tau}) - \frac{u^1\partial_1 u^0}{u^0} - \frac{u^2\partial_2 u^0}{u^0}$
    - η_T=transverse rapidity, c_s^2= speed of sound squared

Data Selection and Analysis

- Simultaneous fit to spectra and elliptic flow v_2 for stable hadrons (p,K,π).
- ALICE 2.76 TeV data (LHC) and PHENIX 200 GeV data (RHIC); several centrality bins except very central and very peripheral bins.
- Bayesian analysis for parameter set $\mathscr{P}$. https://madai-public.cs.unc.edu
- Experimental errors are input into statistical analysis
  - Statistical + systematic errors summed in quadrature
- Choice of fit range: "not too low, not too high"
  - "Regular" fit range



Centrality	proton (GeV/c)	kaon (GeV/c)	pion (GeV/c)	b (fm)	c_s^2	c_τ
ALICE 2.76 TeV						
10-20%	0.325-3.3	0.225-2.55	0.525-1.85	6.05	0.158	0.783
20-30%	0.325-3.1	0.225-2.35	0.525-1.75	7.81	0.162	0.756
30-40%	0.325-3.1	0.225-2.25	0.525-1.65	9.23	0.166	0.719

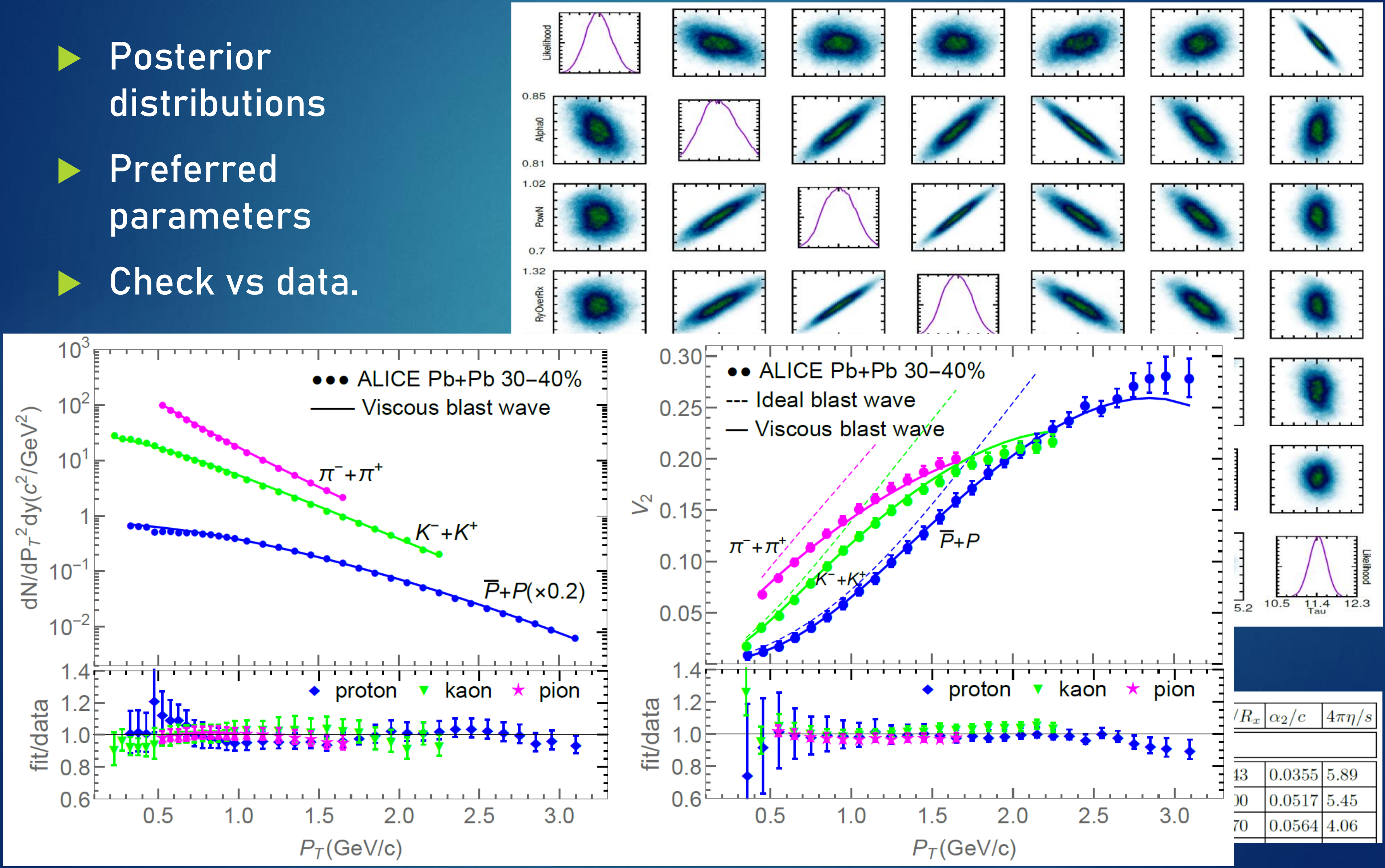
  - Vary for uncertainty analysis

Data Selection and Analysis

- Set $\lambda = 2$ for simplicity.
- Parameter set for statistical analysis: $\mathscr{P}$=$(\tau, T, \alpha_0, n, \alpha_2, R_y/R_x, \eta/s)$
- Missing from this list: several parameters that exhibit high correlations. Seek other guidance e.g. from theoretical considerations (c_s^2, chemical potentials, R_x, …),
- Perform uncertainty analysis for these “external” parameters

A Look at ALICE 30-40%

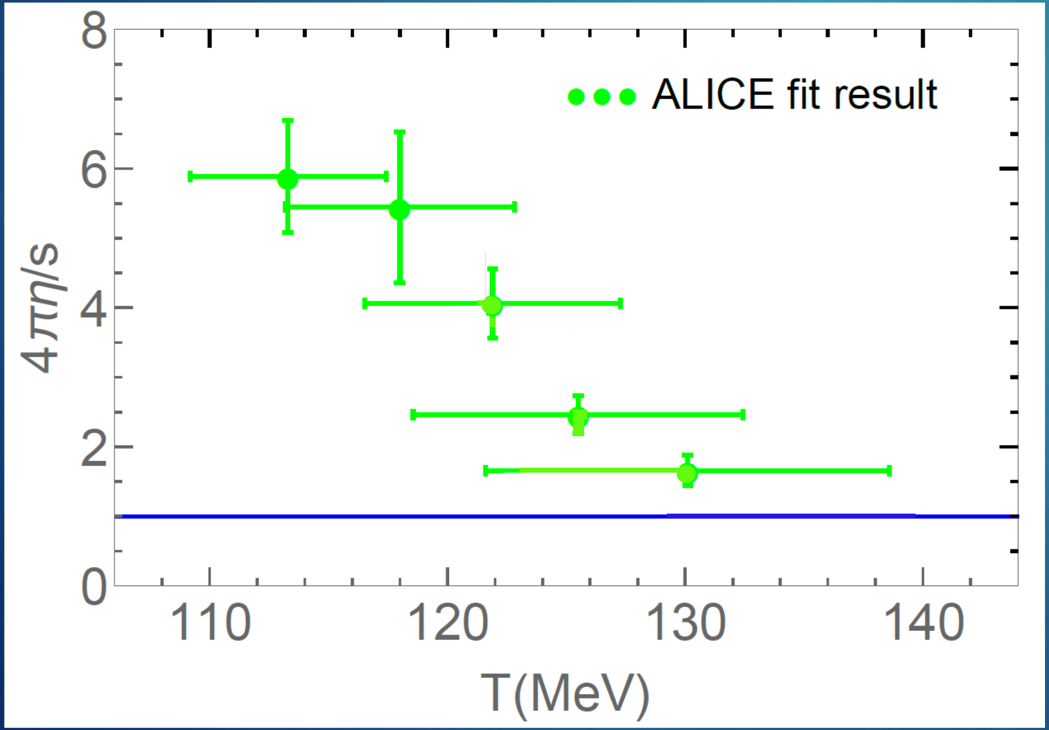
- Posterior distributions
- Preferred parameters
- Check vs data.



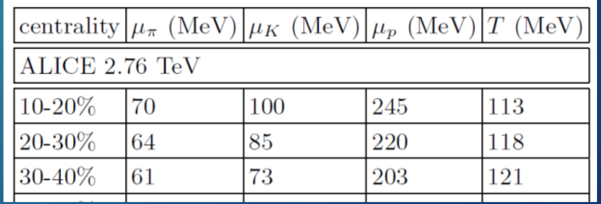
/R_x	α_2/c	$4\pi\eta/s$
43	0.0355	5.89
00	0.0517	5.45
70	0.0564	4.06

LHC: Full Picture

- Parameter changing with centrality qualitatively consistent with expectations.
- Can extract shear viscosity vs temperature curve.
- Extracted temperatures cover range ~110…130 MeV.
- η/s drops quickly toward higher temperatures



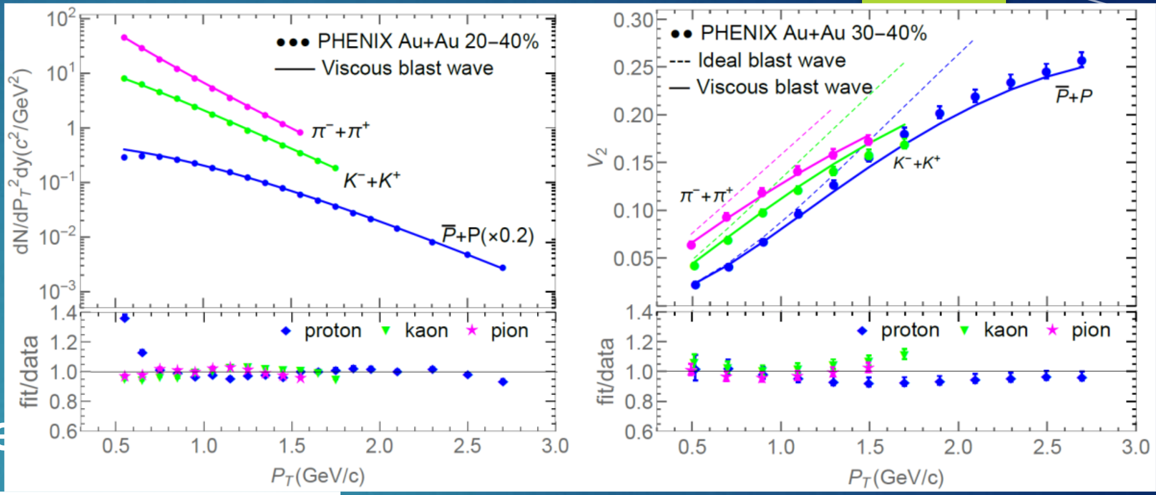
- Keep in mind: chemical potentials for stable particles can be sizeable

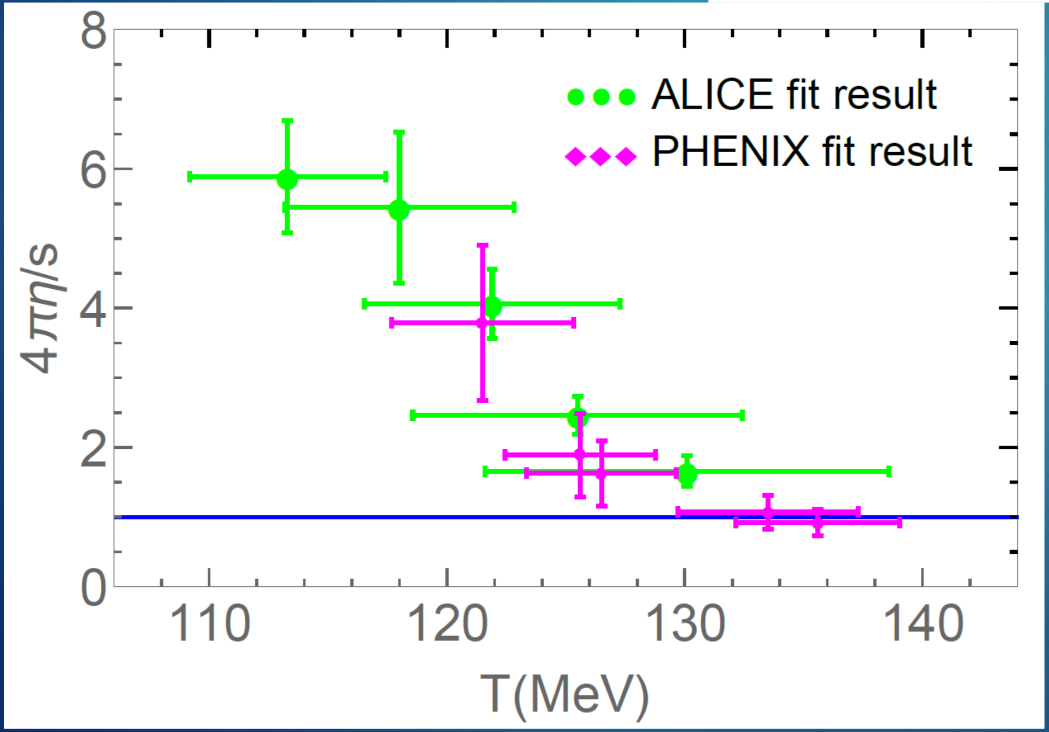


centrality	μ_π (MeV)	μ_K (MeV)	μ_p (MeV)	T (MeV)
ALICE 2.76 TeV				
10-20%	70	100	245	113
20-30%	64	85	220	118
30-40%	61	73	203	121

Adding RHIC

- Consistent with LHC extracted points within uncertainties!
- Slightly higher temperatures accessible





- This is our “raw” result.

Uncertainty Analysis

- Classify uncertainties in 4 categories:

(I) Fundamental uncertainties in the approach; shared with fluid dynamics!

  - Instantaneous freeze-out?
  - True shape of δf, beyond Navier-Stokes?

(II) Uncertainties from using blastwave vs fluid dynamics

  - Simple ansatz for f.o. hypersurface and flow field
  - Missing bulk stress effect on δf, missing resonances and decay

(III) Systematic uncertainties due to choices made in the analysis

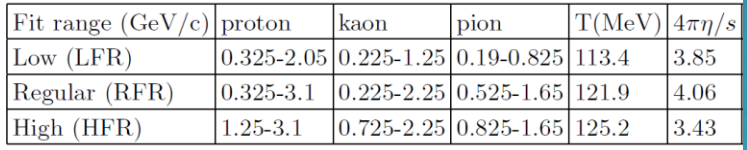
  - Choice of fit ranges
  - Choices for modeling external parameters

(IV) Uncertainties from the statistical analysis

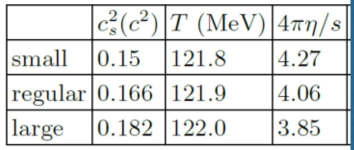
  - From experimental errors, GP emulator etc.

Uncertainty Analysis

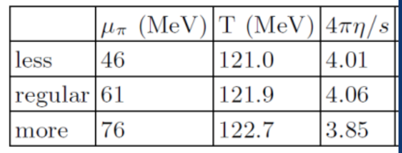
- Type (IV): Uncertainties taken from the MADAI statistical analysis.
- Type (III): Various uncertainties estimated by systematic variation of fit ranges and external parameters. Examples:



Fit range (GeV/c)	proton	kaon	pion	T(MeV)	$4\pi\eta/s$
Low (LFR)	0.325-2.05	0.225-1.25	0.19-0.825	113.4	3.85
Regular (RFR)	0.325-3.1	0.225-2.25	0.525-1.65	121.9	4.06
High (HFR)	1.25-3.1	0.725-2.25	0.825-1.65	125.2	3.43

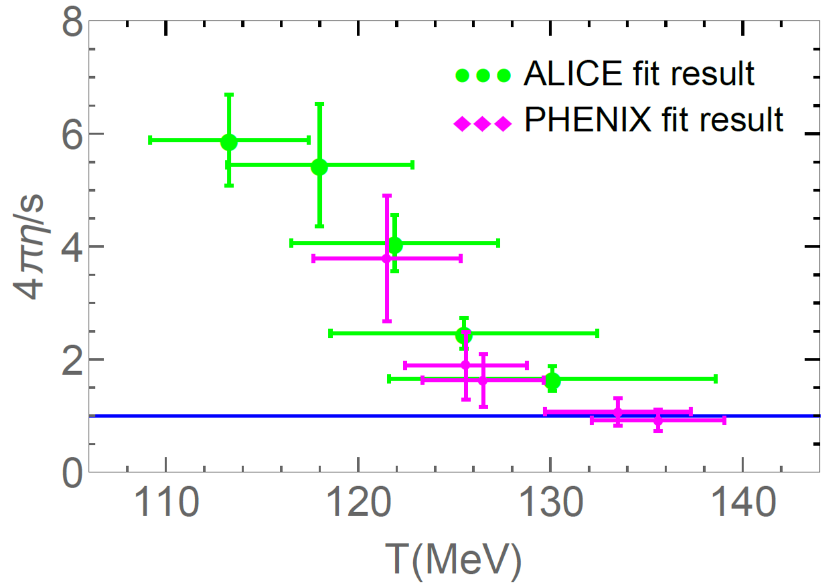


	$c_s^2(c^2)$	T (MeV)	$4\pi\eta/s$
small	0.15	121.8	4.27
regular	0.166	121.9	4.06
large	0.182	122.0	3.85



	μ_π (MeV)	T (MeV)	$4\pi\eta/s$
less	46	121.0	4.01
regular	61	121.9	4.06
more	76	122.7	3.85

- Combined uncertainties of type (III) and (IV) are shown in η/s vs T plot:
- What about type (II) uncertainties?



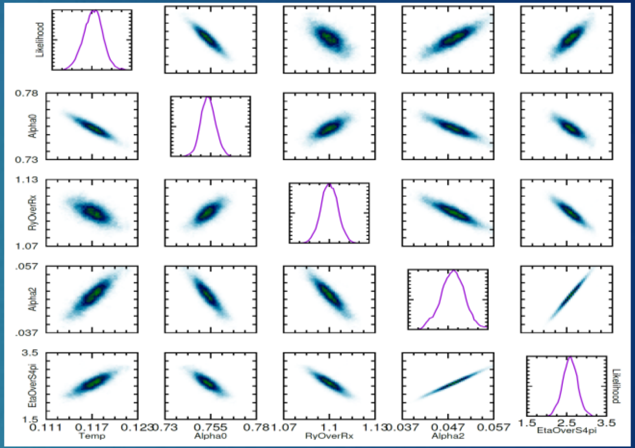
Uncertainty Analysis: Fluid Dynamics

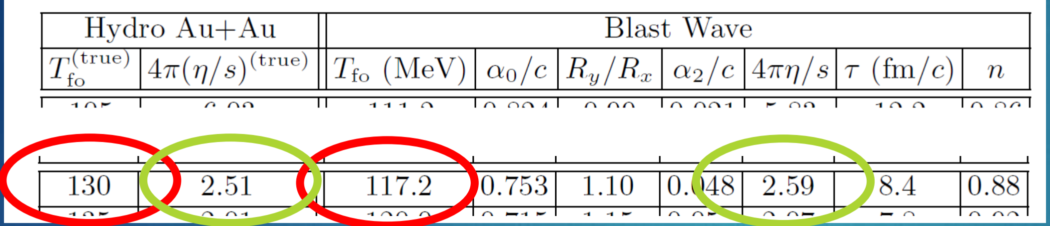
- Type (II) uncertainties: How well do the assumptions in the blastwave approximate fluid dynamics?
- Key differences:
  - Shape of hypersuface
  - Shape of flow field
  - Absence of resonance decays
  - Absence of bulk viscosity
  - Navier Stokes approximation vs 2nd order fluid dynamics

- To estimate systematic bias: viscous blastwave analysis of spectra and elliptic flow generated from fluid dynamics with known freeze-out temperature and η/s.
- Use smooth Au+Au/Pb+Pb events in MUSIC, with bulk viscosity and resonance decays.

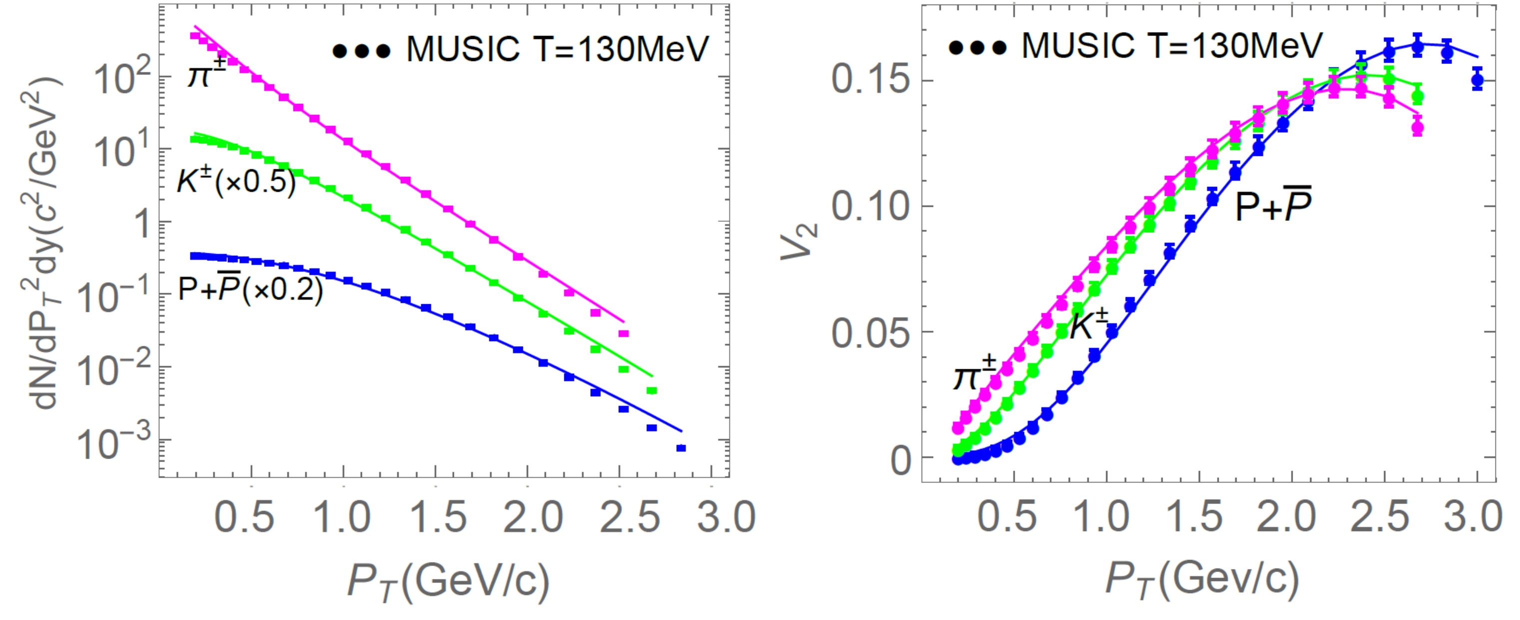
Fluid Dynamics Comparison

- Example: Au+Au, $T_{\rm fo} = 130$ MeV, $\eta/s = 2.5/4\pi$.
- Choose error bars for the MUSIC results roughly consistent with errors at RHIC.





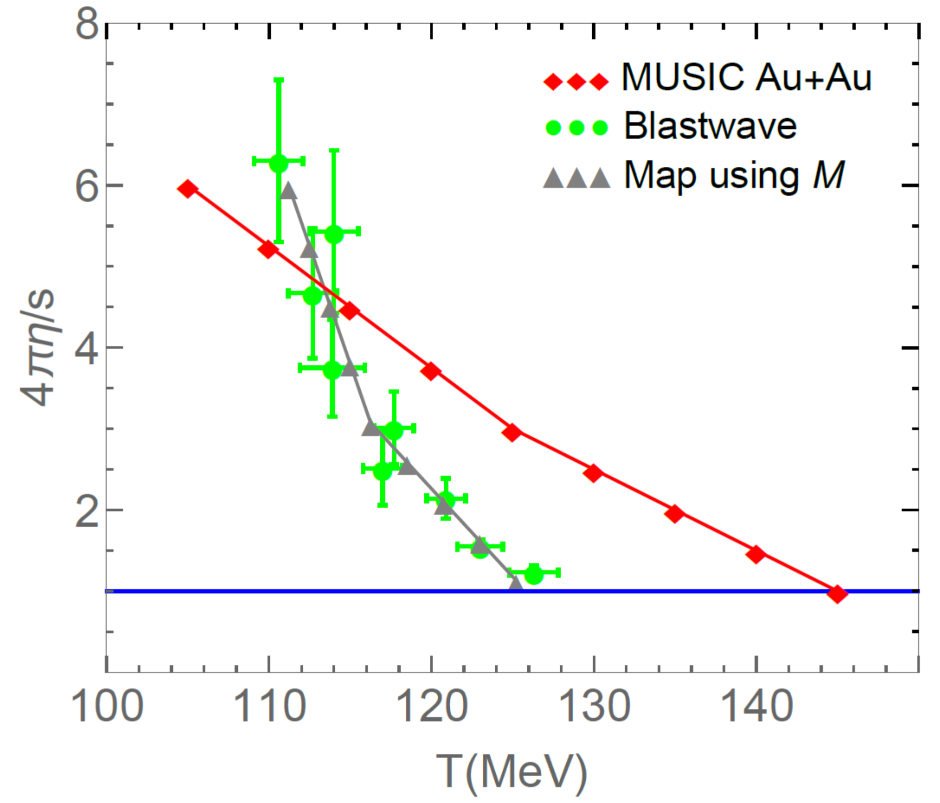
Hydro Au+Au		Blast Wave						
$T_{\rm fo}^{\rm (true)}$	$4\pi(\eta/s)^{\rm (true)}$	$T_{\rm fo}$ (MeV)	α_0/c	R_y/R_x	α_2/c	$4\pi\eta/s$	τ (fm/c)	n
130	2.51	117.2	0.753	1.10	0.048	2.59	8.4	0.88



Fluid Dynamics Comparison

- Bias in temperature extraction: smaller apparent (fitted) temperature compared to the true temperatured
  - Small effect at lower temperature, significant at higher temperature.
- Apparent η/s roughly consistent with true η/s within error bars
- Assess type (III) and (IV) uncertainties for this analysis.
- To eliminate bias: define a linear map between true and apparent values

$$\begin{pmatrix} T^{(\mathrm{extr})} \\ (\eta/s)^{(\mathrm{extr})} \end{pmatrix} = M \begin{pmatrix} T^{(\mathrm{true})} \\ (\eta/s)^{(\mathrm{true})} \end{pmatrix}$$



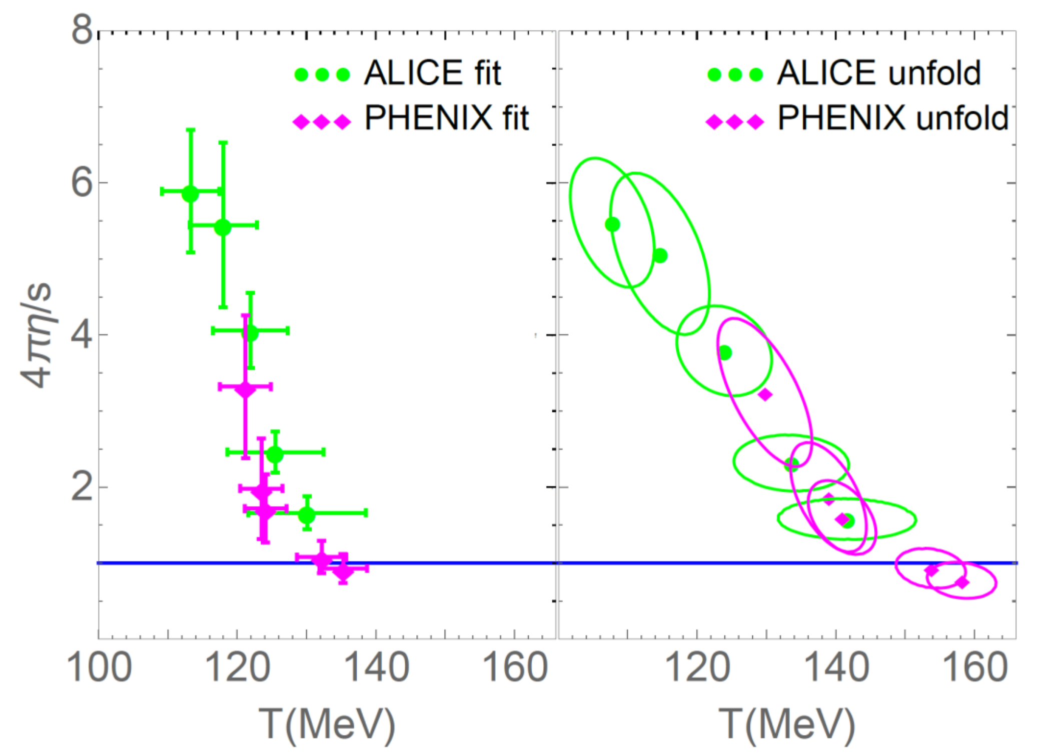
Partially Correcting Bias

- Use the inverse map to (approximately) remove bias of type (III) from the analysis of data.

$$\begin{pmatrix} T^{(\mathrm{corr})} \\ (\eta/s)^{(\mathrm{corr})} \end{pmatrix} = M^{-1} \begin{pmatrix} T^{(\mathrm{extr})} \\ (\eta/s)^{(\mathrm{extr})} \end{pmatrix}$$

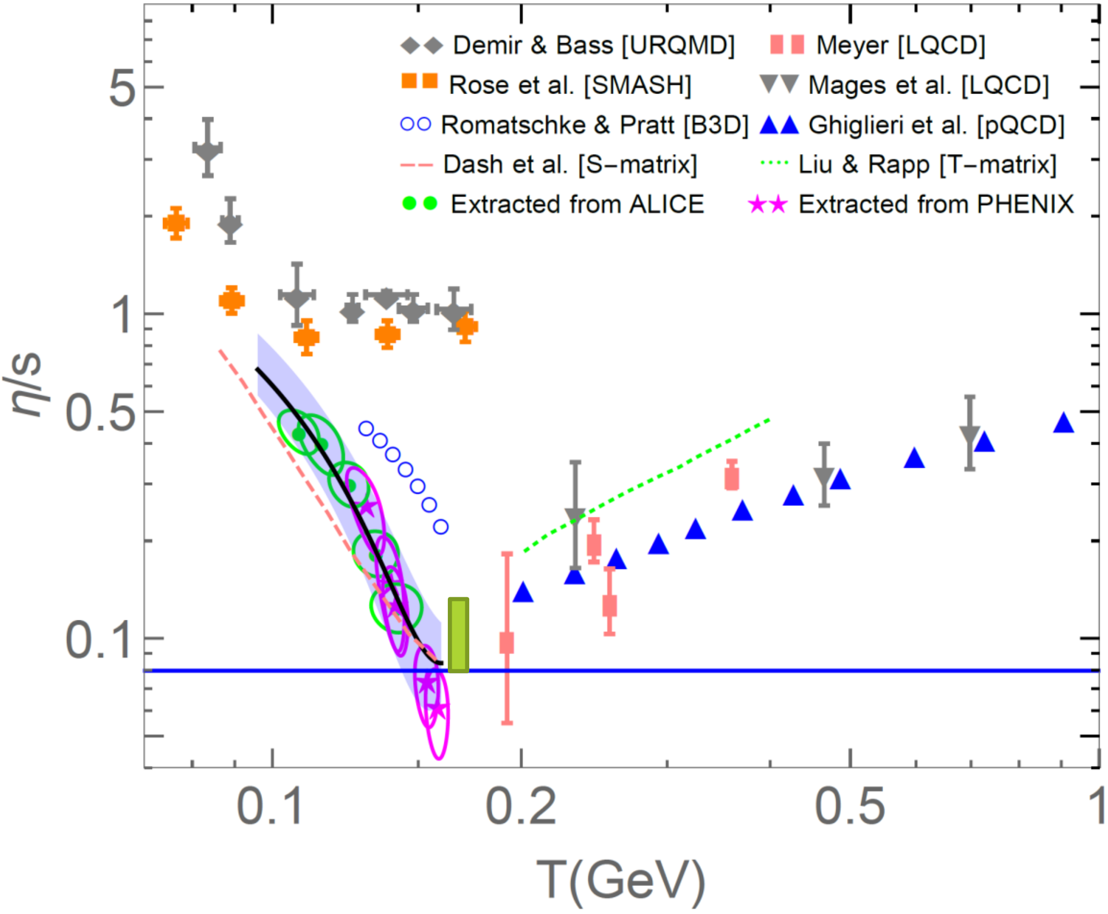
- Add uncertainties in the definition of the map M.

- Corrected results show a less steep decline of η/s with temperature.



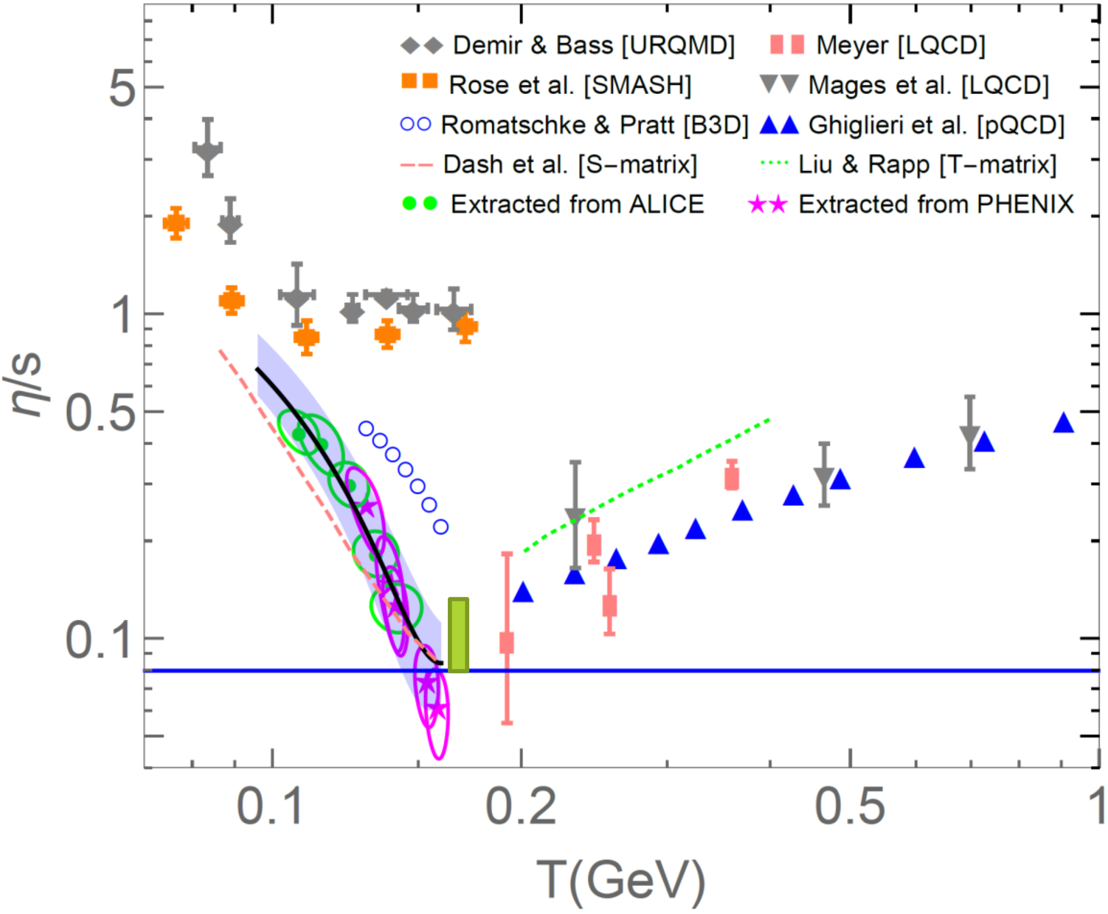
Summary I: Shear Viscosity

- Putting everything together: we find η/s falling steeply between T~110 and 150 MeV.
- Together with Bayesian fits and calculations in QGP, and the new result from Dash et al. in hadron matter this hints at a *broad* minimum of η/s around T_c, extending well into the hadronic phase.
- Caveat: our points are at finite chemical potentials
- What about type (I) uncertainties?



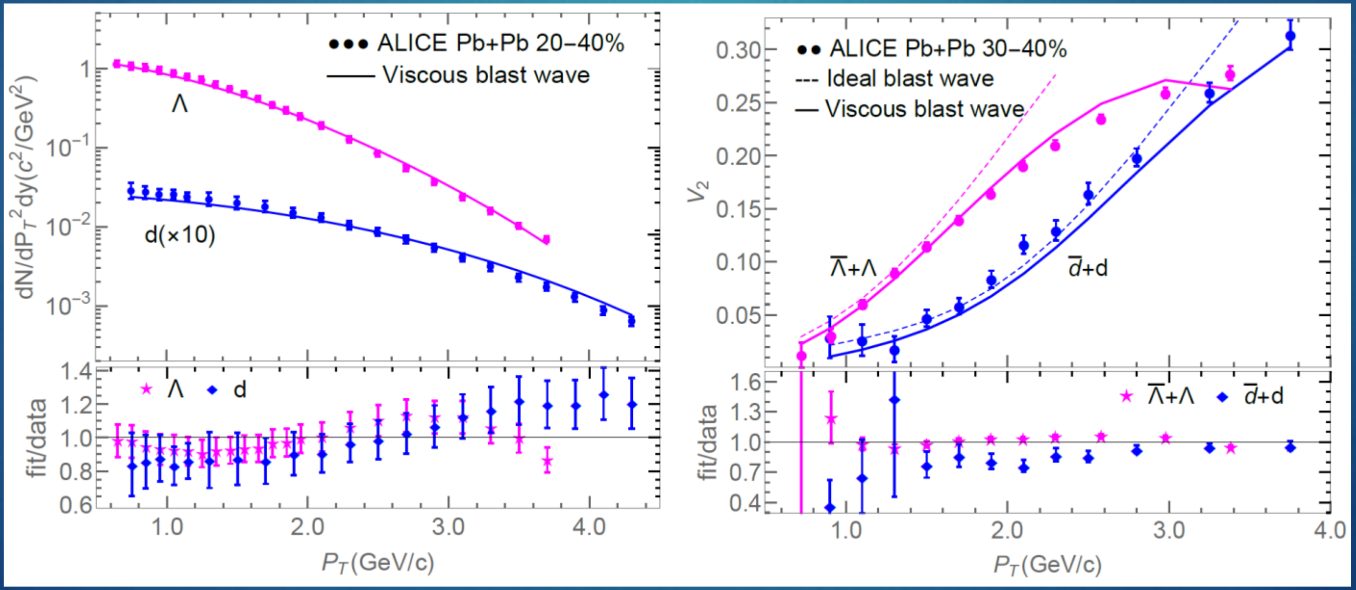
Summary II: Uncertainties

- Type (I) uncertainites difficult to assess.
- Probably one would need a comparison of hydro and transport at freeze-out.
- This remains as a big caveat for our results.



Summary III: Blastwave

- The viscous blastwave in Navier-Stokes can be a nifty tool for quick analyses of spectra, elliptic flow, etc.
- Fits are of good quality and extend to fairly large P_T.
- Example: “Predictions” for Lambda and deuteron data from ALICE using preferred fit parameters extracted from stable hadrons.



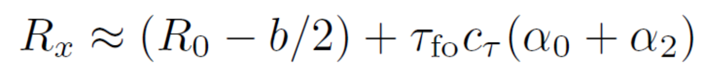
Outlook

- Several ways to improve the analysis:
  - λ-parameter in δf.
  - v_4 (Need new parameterization of hypersurface and flow field. More parameters!)
  - Include bulk viscous effects
  - Include resonance decays?

- More important: need to converge theory calculations of transport coefficients in hadron matter!

Backup

- Eliminate one geometric parameter from τ, R_x, R_y:



$$R_x \approx (R_0 - b/2) + \tau_{\mathrm{fo}} c_\tau (\alpha_0 + \alpha_2)$$

  - c_τ = time averaged acceleration on the boundary. Can be estimated to be between 0.6 and 0.8

- Speed of of sound

D. Teaney, “Chemical freezeout in heavy ion collisions”, *preprint* arxiv:nucl-th/0204023.
Pasi Huovinen and Peter Petreczky “QCD Equation of State and Hadron Resonance Gas”, Nucl. Phys. A837:26-53 (2010).

- Chemical potentials for stable hadrons below chemical freeze out

[58] T. Hirano and K. Tsuda, “The Effect of early chemical freezeout on radial and elliptic flow from a full 3-D hydrodynamic model”, *preprint* arxiv:nucl-th/0202033.