Qualifying Exam Syllabi
Graduate Committee
Department of Mathematics
Wayne State University
Fall 2023

These are 3-hour written examinations covering undergraduate-level material from a sophisticated point of view. Students are required to pass a qualifying exam in Algebra or Analysis, as well as one additional exam from the following four choices: Algebra; Analysis; Applied Mathematics; and Probability and Statistics.

Students are expected to demonstrate mastery of the subject matter, as well as to demonstrate mathematical sophistication and maturity. Solutions should use appropriate terminology and notation, and they should contain organized, well-structured logical arguments.

Students may choose to take exams in any of their first three semesters in the Ph. D. program, in which case they must have passed both exams by the end of the following semester (this allows for at most one retake of a failed exam). Students must select their exam areas at the beginning of each semester.

Under special circumstances, the Departmental Graduate Committee may approve petitions on an individual basis for exceptions to these rules. In the possible exceptions, every student still needs to fulfill the requirements by the end of the second year in the Ph. D. program.

Algebra Exam. Three sections:

1. Elementary group theory. Topics including, but not limited to: kernels, cokernels, and quotient groups; group actions on sets, fixed points, and orbits; Sylow theorems; the structure of finitely generated abelian groups.

2. Elementary ring and field theory. Topics including, but not limited to: maximal ideals, prime ideals, quotient rings, and integral domains; field of fractions of integral domains; irreducibility of polynomials; algebraic elements and minimal polynomials.

3. Linear algebra. Topics including, but not limited to: eigenvalues, eigenvectors, nullspaces/kernels, and rank of linear operators; inner product spaces; diagonalization; minimal polynomial and characteristic polynomial of matrices; Cayley–Hamilton theorem; and Jordan canonical forms.

The topics in all three sections are similar to the topics covered in MAT 5420, MAT 5430, and MAT 6420.

Typical References:

- Axler, *Linear Algebra Done Right*
- Beachy & Blair, *Abstract Algebra*

- Herstein, *Abstract Algebra*
- Hoffman & Kunze, *Linear Algebra*

Analysis Exam. Elementary point set topology in euclidean and metric spaces; advanced calculus (including the inverse and implicit function theorems); compactness, connectedness, continuity, convergence, etc. Basic complex analysis including power series, Laurent series, residues, Cauchy–Riemann equations, winding numbers, Cauchy integral theorem and integral formula.

The topics covered are similar to the topics covered in MAT 5600, MAT 5610, and MAT 6600.

Typical References:

- Apostol, *Mathematical Analysis*
- Churchill & Brown, *Complex Variables and Applications*
- Goldberg, *Methods of Real Analysis*
- Marsden, *Basic Complex Analysis*
- Rudin, *Principles of Mathematical Analysis*

Applied Mathematics Exam. This exam has three distinct components: Optimization, Differential Equations, and Numerical Methods. Students who take this exam choose two out the three sections of the exam.

Optimization: Optimality conditions in unconstrained optimization; numerical methods of unconstrained optimization (including gradient method, Newton's method, conjugate gradient method); simplex method in linear programming; nonlinear programming with equality and inequality constraints; elements of convex analysis.

The topics covered are similar to the topics covered in MAT 5770 and MAT 5870.

Typical References:

- Chong and Zak, *Introduction to Optimization*
- Hiller and Lieberman, *Introduction to Operations Research*

Differential Equations: Linear systems of differential equations; second-order differential equations; boundary value problems; stability theory; asymptotic solutions; Sturm-Liouville problems; eigenvalues and eigenfunctions; separation of variables; Laplace equations; wave equations; heat equations; Green's functions; variational principles.

The topics covered are similar to the topics covered in MAT 5280 and MAT 5220.

Typical References:

- Waltman, *Second Course in Elementary Differential Equations*
- Haberman, *Elementary Applied Partial Differential Equations*
- R'Malley, *Thinking about Ordinary Differential Equations*

Numerical Methods: Numerical errors; solutions of nonlinear equations; Newton's method, bisection method, secant method; polynomial interpolation; numerical approximation; numerical integration and differentiation; numerical linear algebra topics; direct and iterative solvers, conjugate-gradient method, GMRES method, eigenvalue problems; numerical solutions of ordinary differential equations; Euler's method, Runge–Kutta methods; numerical solutions of partial differential equations; finite difference methods.

The topics covered are similar to the topics covered in MAT 5100 and MAT 5110.

Typical References:

- Cheney and Kincaid, *Numerical Mathematics and Computing*
- Trefethen and Bau, *Numerical Linear Algebra*

Probability and Statistics Exam. Students who take this exam must complete both sections.

Probability: Sample space and axioms of probability, combinatorics. Conditional probabilities, independence, the Bayes' formula. Random variables and their distributions, expectations, discrete distributions (binomial, Poisson, geometric, negative binomial, hypergeometric); continuous distributions (uniform, normal, exponential, beta, gamma, Cauchy). Joint distributions, covariance, correlation, sums of independent random variables (convolution formula). Conditional densities and expectations. Moment generating functions. The Chebyshev and Markov inequalities, law of large numbers, central limit theorem.

The topics covered are similar to the topics covered in MAT 5700.

Typical References:

- Sheldon Ross, *A First Course in Probability*, 10th edition, Chapters 1–8.
- John Kinney, *Probability: An Introduction with Statistical Applications.*
- Paul G. Hoel, Sidney C. Port, and Charles J. Stone, *Introduction to Probability Theory.*

Statistics: Probability distributions (chi-square, t, F); sample mean and sample variance; parameters and statistics; standard error; point estimation (method of moments, maximum likelihood); Fisher information matrix; Cramer–Rao lower bound; unbiasedness; consistency; efficiency; confidence intervals; Bayes estimation; sufficiency; factorization theorem; Rao–Blackwell Theorem; hypothesis testing; Neyman–Person Lemma; UMP tests, (generalized) likelihood ratio tests; two

sample tests (t–tests, Wilcoxon tests); analysis of variance (ANOVA); analysis of categorical data (chi–square tests); linear regression.

The topics covered are similar to the topics covered in STA 5800.

Typical References:

- John Rice, *Mathematics Statistics and Data Analysis.*
- R. Hogg, J. McKean, and A. Craig, *Introduction to Mathematical Statistics.*
- George Casella, Roger L. Berger, *Statistical Inference.* (*The book includes materials beyond the scope of the exam.)