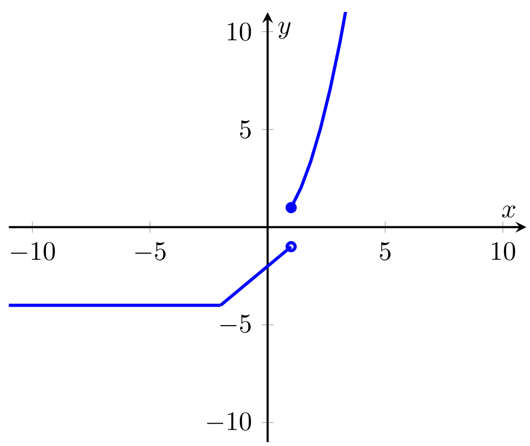
# Final Exam

MAT 1800 Elementary Functions

Fall 2022

1.




2. $f(x) = \dfrac{\log_2(x^2-25)}{x-8}$ has domain restrictions $x^2 - 25 > 0$ and $x - 8 \neq 0$.
   $x^2 - 25 = (x+5)(x-5) > 0$ on the interval $(-\infty, -5) \cup (5, \infty)$.
   $x \neq 8$, so the domain of $f(x)$ is $(-\infty, -5) \cup (5, 8) \cup (8, \infty)$

3. (a)

$$g(f(x)) = \frac{(\sqrt[3]{4x} - 3) + 2}{(\sqrt[3]{4x} - 3) - 2}$$

$$= \frac{\sqrt[3]{4x} - 1}{\sqrt[3]{4x} - 5}$$

$$g(f(2)) = \frac{\sqrt[3]{4 * 2} - 1}{\sqrt[3]{4 * 2} - 5} = -\frac{1}{3}$$

$$(f + g)(x) = \sqrt[3]{4x} + \frac{x+2}{x-2}$$

$$(f + g)(0) = \sqrt[3]{4 * 0} + \frac{0+2}{0-2} = -4$$

Finally,

$$\frac{(g \circ f)(2)}{(f+g)(0)} = \frac{-\frac{1}{3}}{-4} = \frac{1}{12}$$

(b) $f^{-1}(1)$ occurs when $f(x) = 1$.

$$\sqrt[3]{4x} - 3 = 1$$

$$\sqrt[3]{4x} = 4$$

$$4x = 4^3$$

$$x = 16$$

So $f^{-1}(1) = 16$.

4. $V = 48\text{ft}^2$. $V = lwh$ and $l = x$, $w = x$, and $h = y$, so $V = x^2y$. Thus $48 = x^2y$ and $y = \frac{48}{x^2}$.

$$\begin{aligned} SA &= 2x^2 + 4xy \\ &= 2x^2 + 4x\left(\frac{48}{x^2}\right) \\ &= 2x^2 + \frac{192}{x} \end{aligned}$$

5. $h(t) = -16t^2 + 64t + 145$
The vertex is $(x, y)$, where $x = \frac{-b}{2a}$ and $y = h(x)$.

$$x = \frac{-64}{-32} = 2$$

$$y = h(2) = 209$$

So the maximum height of the rocket is 209 feet.

6. We know $-2$ is a solution to $p(x)$, or that $p(-2) = 0$. Long division of $p(x)$ by $x - (-2) = x + 2$ gives

$$p(x) = (x + 2)(x^2 - 6x + 5)$$

Factoring $x^2 - 6x + 5$ gives

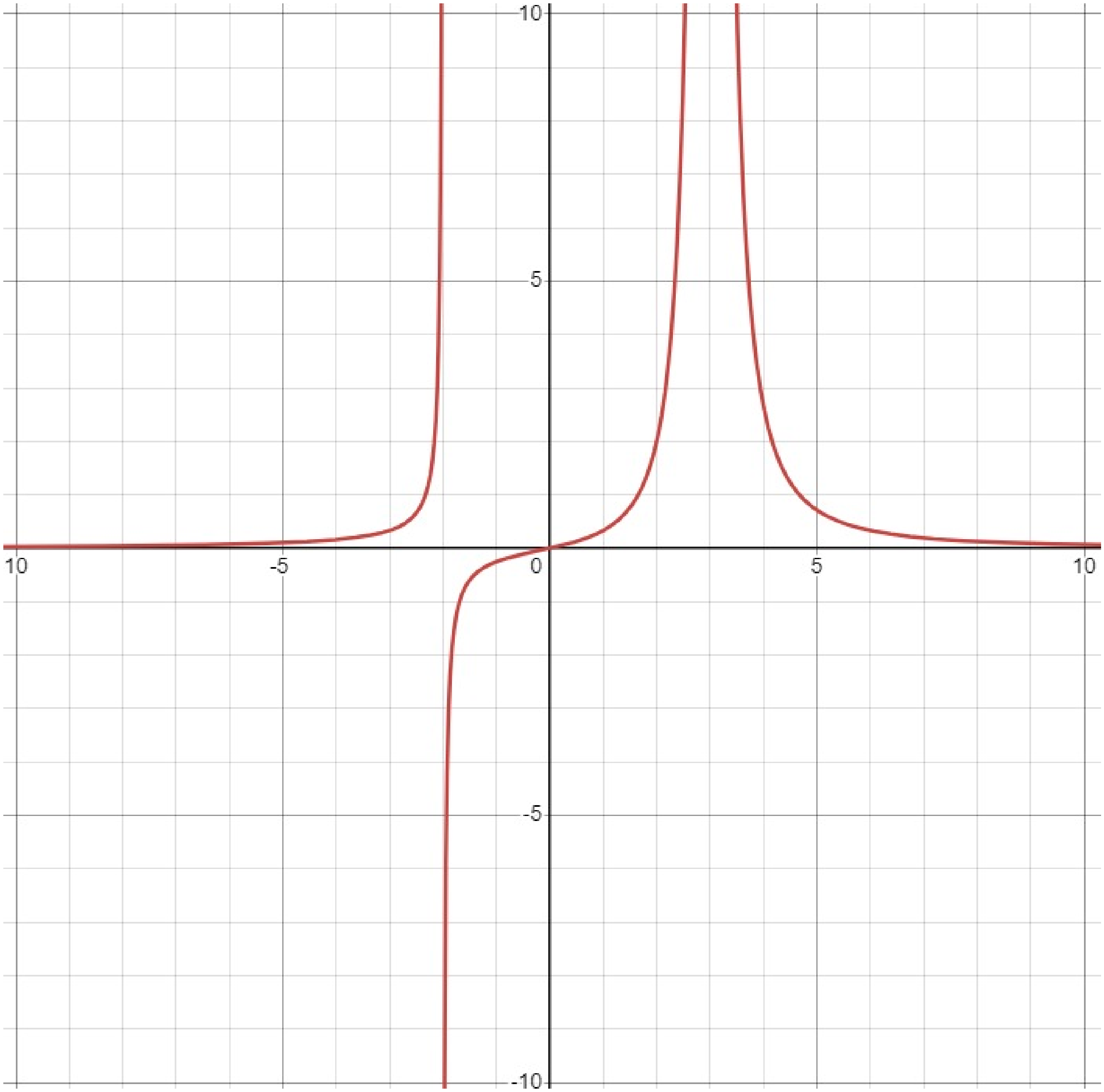
$$p(x) = (x + 2)(x - 1)(x - 5)$$

So $p(x)$ has roots $-2, 1,$ and $5$.

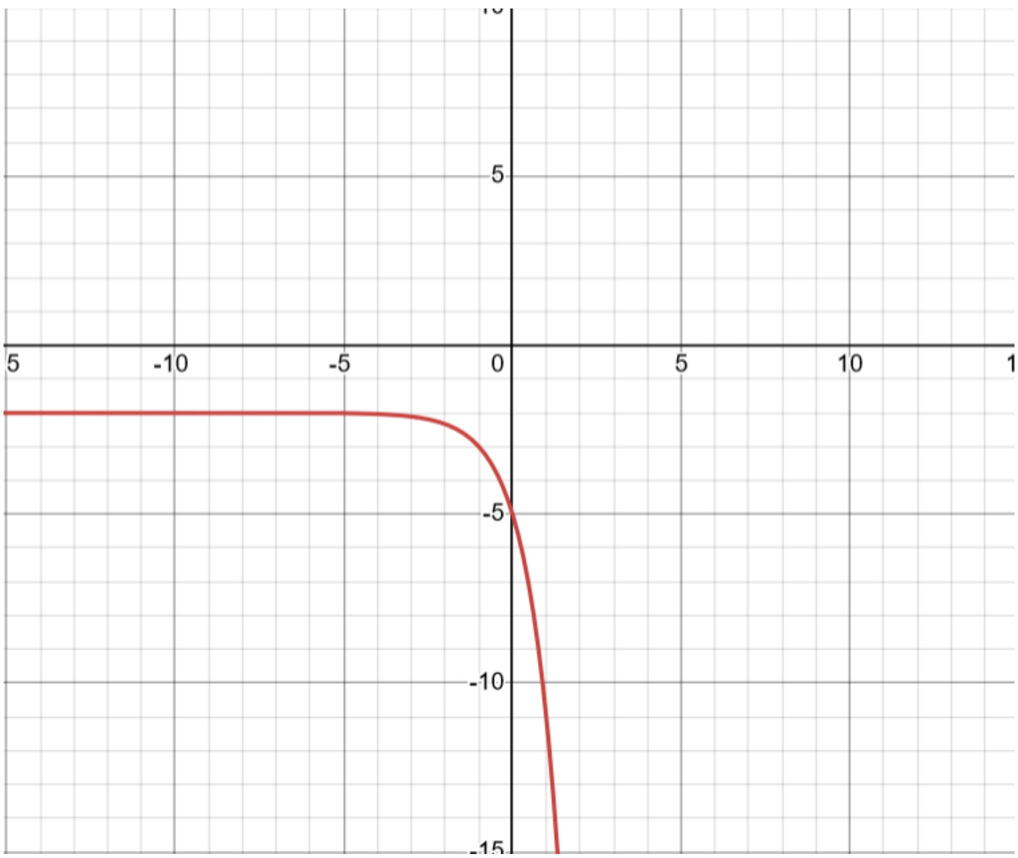
7.

$$\begin{aligned} \frac{g(2+h) - g(2)}{(2+h) - 2} &= \frac{\frac{1}{(2+h)^2} - \frac{1}{2^2}}{h} \\ &= \frac{\frac{1}{h^2+4h-4} - \frac{1}{4}}{h} \\ &= \frac{\frac{4-(h^2+4h+4)}{4(h^2+4h+4}) }{h} \\ &= \frac{\frac{-h^2-4h}{4(h^2+4h+4)}}{h} \\ &== \frac{-h-4}{4h^2+16h+16} \end{aligned}$$

8.



9.




10. (a)

$$
\begin{aligned}
\log_5 \frac{1}{\sqrt[3]{25}} &= \log_5(1) - \log_5(25^{\frac{1}{3}}) \\
&= \log_5(1) - \frac{1}{3}\log_5(25) \\
&= 0 - \frac{1}{3}\log_5(5^2) \\
&= -\frac{2}{3}\log_5(5) \\
&= -\frac{2}{3}
\end{aligned}
$$

(b)

$$
\begin{aligned}
e^{2\ln(10)-\ln(4)} &= e^{\ln(10^2)-\ln(4)} \\
&= e^{\ln\left(\frac{100}{4}\right)} \\
&= e^{\ln(25)} \\
&= 25
\end{aligned}
$$

11.

$$Q(t) = Q_0 e^{rt}$$
$$= 80e^{rt}$$

Using the point $(5, 60)$ gives

$$60 = 80e^{5r}$$
$$\frac{3}{4} = e^{5r}$$
$$5r = \ln\left(\frac{3}{4}\right)$$
$$r = \frac{\ln(\frac{3}{4})}{5}$$

Updating $Q$ to

$$Q(t) = 80e^{\frac{\ln(\frac{3}{4})}{5}t}$$

Plugging in 10 gives

$$Q(10) = 80e^{\frac{\ln(\frac{3}{4})}{5} \cdot 10}$$
$$= 80e^{2\ln\left(\frac{3}{4}\right)}$$
$$= 80e^{\ln\left(\frac{9}{16}\right)}$$
$$= 80 \cdot \frac{9}{16}$$
$$= 45$$

There are 45 bacteria after 10 hours.

12. (a)

$$\tan\left(\frac{2\pi}{3}\right) = \frac{\sin(\frac{2\pi}{3})}{\cos(\frac{2\pi}{3})}$$
$$= \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}$$
$$= -\sqrt{3}$$

(b)

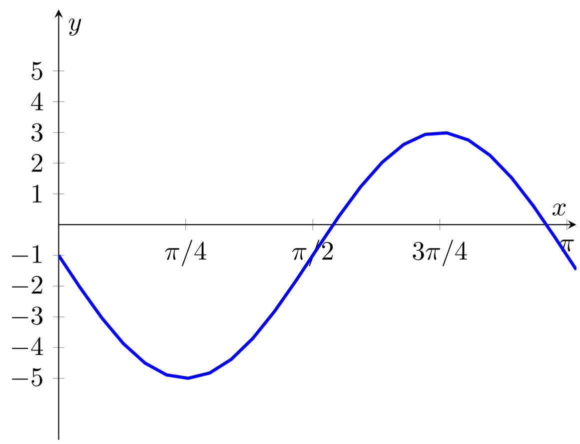
$$\csc\left(\frac{13\pi}{4}\right) = \csc\left(\frac{5\pi}{4}\right)$$
$$= \frac{1}{\sin(\frac{5\pi}{4})}$$
$$= \frac{1}{-\frac{\sqrt{2}}{2}}$$
$$= -\frac{2}{\sqrt{2}}$$
$$= -\sqrt{2}$$

(c)

$$\cos^{-1}\left(\cos\left(-\frac{\pi}{4}\right)\right) = \cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right)$$
$$= \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\pi}{4}$$

13. (a) The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The amplitude is $|-4| = 4$

(b)




14. $\tan(\theta) = -\frac{3}{5}$ and $\cos(\theta) > 0$.

$$\sin\left(\frac{\pi}{4} - \theta\right) = \sin\left(\frac{\pi}{4}\right)\cos(\theta) - \cos\left(\frac{\pi}{4}\right)\sin(\theta)$$
$$= \frac{\sqrt{2}}{2}\cos(\theta) - \frac{\sqrt{2}}{2}\sin(\theta)$$

By the pythagorean theorem, $\cos(\theta) = \frac{5\sqrt{34}}{34}$ and $\sin(\theta) = -\frac{3\sqrt{34}}{34}$. After substituting we get

$$\sin\left(\frac{\pi}{4} - \theta\right) = \frac{\sqrt{2}}{2} \cdot \frac{5\sqrt{34}}{34} - \frac{\sqrt{2}}{2} \cdot -\frac{3\sqrt{34}}{34}$$
$$= \frac{4\sqrt{17}}{17}$$

15. $\cos^2(x) - 2\cos(x) = 0$ can be rewritten as $(\cos(x))^2 - 2\cos(x) = 0$. Let $u = \cos(x)$. Then we have

$$u^2 - 2u = 0$$

The solutions to this equation are $u = 0, u = 2$. So $\cos(x) = 0, \cos(x) = 2$. Since $\cos(x) = 2$ is an extraneous solution. $\cos(x) = 0$ means that $x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$.

16.

$$\frac{\sec(x)\sin(x)}{\tan(x)+\cot(x)} = \frac{\frac{\sin(x)}{\cos(x)}}{\frac{\sin(x)}{\cos(x)}+\frac{\cos(x)}{\sin(x)}}$$

$$= \frac{\frac{\sin(x)}{\cos(x)}}{\frac{\sin^2(x)+\cos^2(x)}{\sin(x)\cos(x)}}$$

$$= \frac{\frac{\sin(x)}{\cos(x)}}{\frac{1}{\sin(x)\cos(x)}}$$

$$= \frac{\sin(x)}{\cos(x)} \cdot \sin(x)\cos(x)$$

$$= \sin^2(x)$$