

Preliminary Exam Syllabi  
Graduate Committee  
Department of Mathematics  
Wayne State University  
Winter 2012

These are 2-hour written examinations covering undergraduate-level material from a sophisticated point of view. Students are required to pass a preliminary exam in Algebra or Analysis, as well as one additional exam from the following four choices: Algebra; Analysis; Applied Mathematics; and Probability and Statistics.

Students may choose to take exams in their first semester in the Ph.D. program, in which case they must satisfy the requirements by the end of their second semester; or they may choose to take exams in their second semester, in which case they must satisfy the requirements by the end of their third semester. Students must select exams at the beginning of each semester, to be taken later in that semester.

Under special circumstances, the Graduate Committee may approve petitions on an individual basis for exceptions to these rules.

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**Algebra Exam.** Elementary group theory, ring theory, and field theory. Linear algebra including inner product spaces, diagonalization, and naive construction of canonical forms.

The topics covered are similar to the topics covered in MAT 5420, 5430, and 6420.

Typical References:

- Axler, *Linear Algebra Done Right*
- Beachy & Blair, *Abstract Algebra*
- Herstein, *Abstract Algebra*
- Hoffman & Kunze, *Linear Algebra*

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**Analysis Exam.** Elementary point set topology in euclidean and metric spaces; advanced calculus (including the inverse and implicit function theorems); compactness, connectedness, continuity, convergence, etc. Basic complex analysis including power series, Laurent series, residues, Cauchy-Riemann equations, winding numbers, Cauchy integral theorem and integral formula.

The topics covered are similar to the topics covered in MAT 5600, 5610, and 6600.

Typical References:

- Apostle, *Mathematical Analysis*
  - Churchill & Brown, *Complex Variables and Applications*
  - Goldberg, *Methods of Real Analysis*
  - Marsden, *Basic Complex Analysis*
  - Rudin, *Principles of Mathematical Analysis*
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**Applied Mathematics Exam.** This exam has three distinct components: Optimization, Differential Equations, and Numerical Methods. Students who take this exam choose two out of the three sections of the exam.

*Optimization:* Optimality conditions in unconstrained optimization; numerical methods of unconstrained optimization (including gradient method, Newton's method, conjugate gradient method); simplex method in linear programming; non-linear programming with equality and inequality constraints; elements of convex analysis.

The topics covered are similar to the topics covered in MAT 5770 and 5870.

Typical References:

- Chong and Zak, *Introduction to Optimization*
- Hiller and Lieberman, *Introduction to Operations Research*

*Differential Equations:* Linear systems of differential equation; second-order differential equations; boundary value problems; stability theory; asymptotic solutions; Sturm-Liouville problems; eigenvalues and eigenfunctions; separation of variables; Laplace equations; wave equations; heat equations; Green's functions; variational principles.

The topics covered are similar to the topics covered in MAT 5280 and 5220.

Typical References:

- Waltman, *Second Course in Elementary Differential Equations*
- Haberman *Elementary Applied Partial Differential Equations*

*Numerical Methods:* Numerical errors; solutions of nonlinear equations: Newton's method, bisection method, secant method; polynomial interpolation; numerical approximation; numerical integration and differentiation; numerical linear algebra topics: direct and iterative solvers, conjugate-gradient method, GMRES method, eigenvalue problems; numerical solutions of ordinary differential equations: Euler's method, Runge-Kutta methods; numerical solutions of partial differential equations: finite difference methods.

The topics covered are similar to the topics covered in MAT 5100 and 5110.

Typical References:

- Cheney and Kincaid, *Numerical Mathematics and Computing*
- Trefethen and Bau, *Numerical Linear Algebra*

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**Probability and Statistics Exam.** Students who take this exam must complete both sections.

*Probability:* Sample space and axioms of probability, combinatorics. Conditional probabilities, independence, the Bayes' formula. Random variables and their distributions, expectations, discrete distributions (binomial, Poisson, geometric, negative binomial, hypergeometric); continuous distributions (uniform, normal, exponential, beta, gamma, Cauchy). Joint distributions, covariance, correlation, sums of independent random variables (convolution formula). Conditional densities and expectations. Moment generating functions. The Chebyshev and Markov inequalities, law of large numbers, central limit theorem.

The topics covered are similar to the topics covered in MAT 5700.

Typical references:

- Sheldon Ross, *A First Course in Probability*.
- John Kinney, *Probability: An Introduction with Statistical Applications*.
- Paul G. Hoel, Sidney C. Port, and Charles J. Stone, *Introduction to Probability Theory*.

*Statistics:* Statistical models, observations, loss and risk functions. Sample mean and variance, chi-square, T-Student and Fisher ratio statistics. Estimation theory: method of moments, maximum likelihood estimators. The Bayes estimation, posterior and conjugate prior distributions. Testing of hypotheses: Neyman-Pearson lemma, likelihood ratio tests. Two sample problems. Linear regression and analysis of variance (ANOVA).

The topics covered are similar to the topics covered in MAT 5800.

Typical references:

- John Rice, *Mathematics Statistics and Data Analysis*.
- R. Hogg, J. McKean, and A. Craig, *Introduction to Mathematical Statistics*.