PRACTICE FOR PLACEMENT EXAM - PART C
For students trying to place in MAT 2010 (Calculus) or MAT 2210 (Probability and Statistics)

Problems in PART C are based on Pre-calculus (MAT 1800, Elementary Functions). Those wishing to place into MAT 2010 should have this material mastered (as well as the material from Parts A and B.) To study, take the Practice Exam Part C. It contains 35 questions which cover the same topics as the 12 questions on the Placement Exam Part C. The actual placement exam is multiple choice.

For help with specific questions, visit the Mathematics Resource Center (MRC) in Room 1198 FAB. The lab is free of charge and an appointment is not necessary. Assistance may also be received from Math Literacy Services, which is a free service and by appointment. You can search either of these services on http://wayne.edu for further information.

For additional practice, take the previous final exams for MAT 1800, found on the Mathematics Department website (http://clasweb.clas.wayne.edu/math). Click the yellow circular **"How Do I?"** button on the homepage, then scroll down the subsequent menu to the appropriate location and links.

For a general review, consult any Pre-calculus textbook. You may borrow one from the WSU libraries.

1. Graph $f(x) = \begin{cases} -x^2 & \text{if } x < -1 \\ |x-2| & \text{if } -1 < x \leq 3, \\ 2x-2 & \text{if } x > 3 \end{cases}$ and find the range.

2. Graph $f(x) = \begin{cases} 5 & \text{if } x \leq -2 \\ 1-2x & \text{if } -2 < x < 0. \\ \sqrt{(x)} & \text{if } x \geq 0 \end{cases}$

3. Find the domain of the function given by $f(x) = \sqrt{16 - x^2}$

4. Find the domain of the function given by $f(x) = \sqrt{x-6} + \dfrac{1}{x^2 - 8x - 9}$

5. Find the domain of the function given by $f(x) = \dfrac{\ln(2x+3)}{\sqrt{x-2}}$

6. Let $f(x) = 2x^2 - 3x + 1$. Find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

7. Let $f(x) = x^2 - x - 1$. Find and simplify $\dfrac{f(3+h) - f(3)}{h}$.

8. Let $f(x) = \sqrt{x+3}$. Find and simplify $\dfrac{f(x+h) - f(x)}{h}$, so that no factor of h is present in your answer.

9. Graph $f(x) = \dfrac{2x^2 + x}{x^2 - 1}$, finding and labeling all intercepts and asymptotes, if any.

10. Graph $f(x) = \dfrac{x-2}{x^2 - 4x}$, finding and labeling all intercepts and asymptotes, if any.

11. Find all roots of the equation $(x^2 - 4)(x^2 + x) + (x^2 - 4) = 0$, expressing any non-real solutions in the form $a + bi$.

12. Given that $2i$ is a root of the $x^4 - x^3 + 5x^2 - 4x + 4 = 0$, find all roots of this equation and express any non-real roots in the form $a + bi$.

13. Given that -3 is a root of the $x^3 + 5x^2 + 9x + 9 = 0$, find all roots of this equation and express any non-real roots in the form $a + bi$.

14. Graph $f(x) = -\log_3(x-1)$, finding and labeling all intercepts and asymptotes, if any.

15. Graph $f(x) = e^{-x} + 3$, finding and labeling all intercepts and asymptotes, if any.

16. Solve $e^{x+3} = \pi^x$.

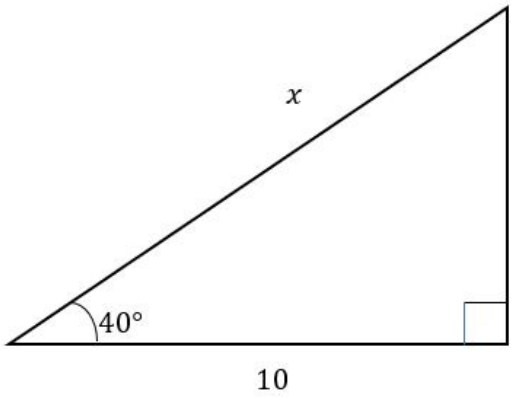
17. Solve $2^{3x-5} = 4$.

18. Solve $\ln(x) + \ln(x-2) = \ln(x+10)$.

19. Solve $\log_2(x) - \log_2(x+2) = 3$.

20. Find the exact values of: a) $\ln\left(e^5 \cdot 3^{2\log_3(3)}\right)$ b) $3 \cdot 2^{\log_2(3)}$.

21. In the right triangle shown, find x:



22. A safety regulation states that the maximum angle of elevation for a rescue ladder is $72°$. A fire department's longest ladder is 110 feet. What is the maximum safe rescue height?

23. You are standing 45 meters from the base of the Empire State Building. You estimate that the angle of elevation to the top of the 86th floor(the observatory) is $82°$. If the total height of the building is another 123 meters above the 86th floor, what is the total height of the building?

24. Find the exact value, if it exists: a) $\sin\left(\frac{23\pi}{6}\right)$ b) $\cot(-3\pi)$

25. Find the exact value, if it exists: a) $\cos\left(\frac{17\pi}{6}\right)$ b) $\csc\left(-\frac{3\pi}{4}\right)$

26. Find the exact value, if it exists: a) $\sin^{-1}\left[\sin\left(\frac{3\pi}{2}\right)\right]$ b) $\cos\left[\cos^{-1}\left(\frac{1}{7}\right)\right]$

27. Find the exact value, if it exists: a) $\arccos(3)$ b) $\arcsin\left(\frac{1}{2}\right)$

28. If $\cot(\theta) = -\sqrt{5}$ and $\sec(\theta) < 0$, find $\sin(\theta)$

29. Given that $\sin(\alpha) = \dfrac{2}{3}$ and that $\dfrac{\pi}{2} \leq \alpha \leq \dfrac{3\pi}{2}$, find $\cot(\alpha)$

30. Let $g(x) = \sin(\pi\, x) + 2$. Graph g over one complete cycle, labeling the intercepts and highest and lowest points.

31. Let $g(x) = 3\cos\left(x + \dfrac{\pi}{4}\right)$. Graph g over one complete cycle, labeling the intercepts and highest and lowest points.

32. Let $g(x) = -\tan\left(\dfrac{1}{2}x\right)$. Graph g over one complete cycle, labeling the intercepts and highest and lowest points.

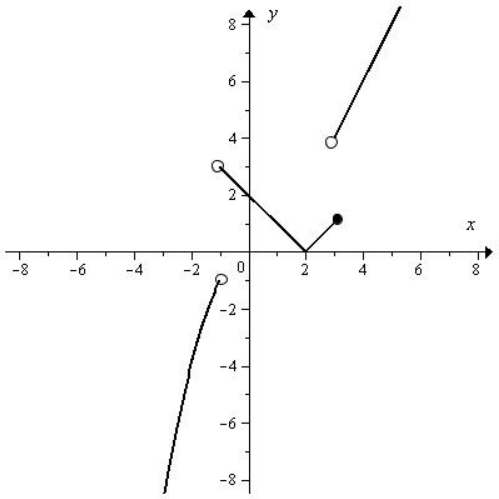
33. Prove the identity: $\dfrac{[1 + \tan(x)]^2}{\sec^2(x)} = 1 + 2\cos(x)\sin(x)$

34. Prove the identity: $\dfrac{\sin(x)}{1 + \cos(x)} + \dfrac{1 + \cos(x)}{\sin(x)} = 2\csc(x)$

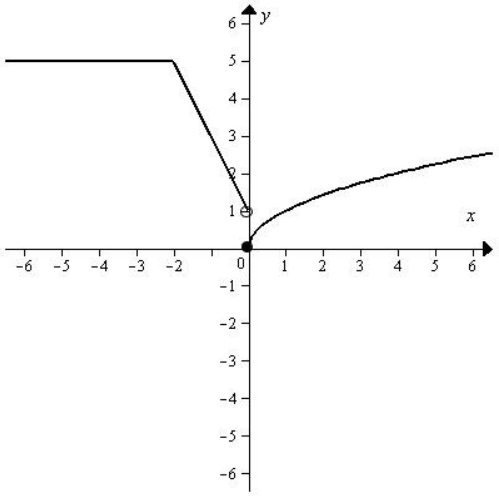
35. Prove the identity: $\dfrac{\sin(\theta)}{1 + \cos(\theta)} = \dfrac{1 - \cos(\theta)}{\sin(\theta)}$

PRACTICE FOR PLACEMENT EXAM PART C - ANSWER KEY

1. Range: $(-\infty, -1) \cup [0, 3) \cup (4, \infty)$



2.



3. $[-4, 4]$

4. $[6, 9) \cup (9, \infty)$

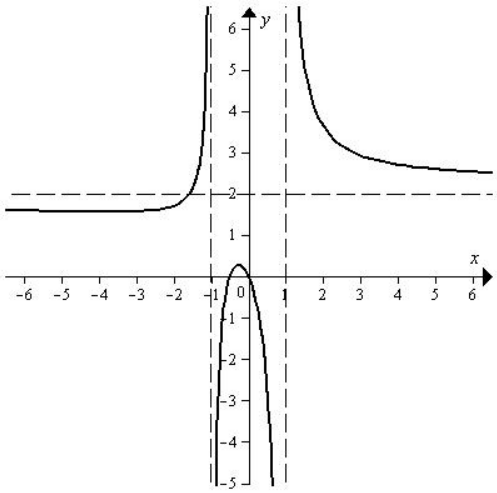
5. $\left(-\dfrac{3}{2}, 2\right) \cup (2, \infty)$

6. $4x + 2h - 3$

7. $h + 5$

8. $\dfrac{1}{\sqrt{x+h+3} + \sqrt{x+3}}$

9.



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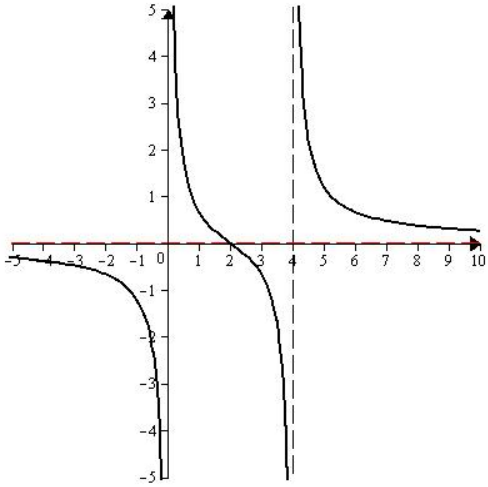
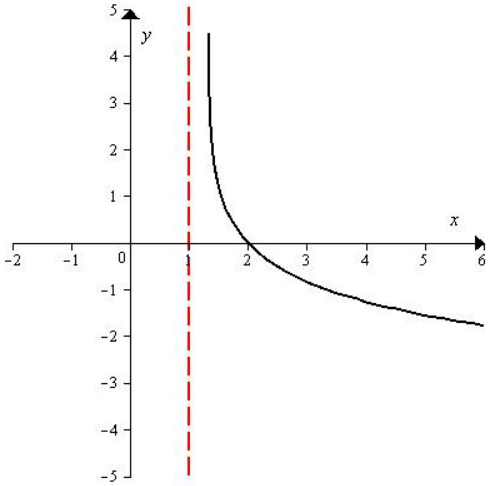


11. $\left\{ -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i, \pm 2 \right\}$

12. $\left\{ \frac{1}{2} \pm \frac{\sqrt{3}}{2}i, \pm 2i \right\}$

13. $\{-1 \pm \sqrt{5}i, -3\}$

14.



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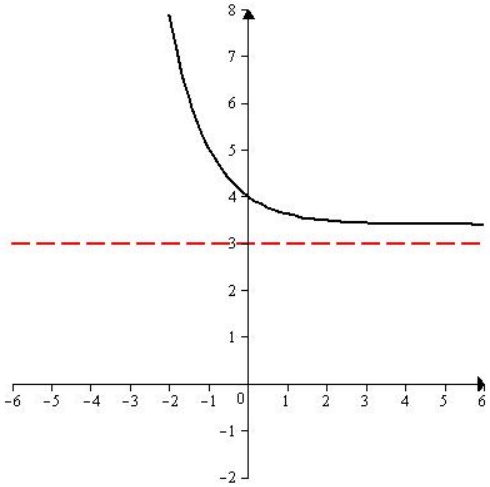


16. $\left\{\dfrac{3}{\ln(\pi)-1}\right\}$ or $\left\{\dfrac{3\log_{\pi}(e)}{1-\log_{\pi}(e)}\right\}$

17. $\left\{\dfrac{7}{3}\right\}$

18. $\{5\}$

19. ϕ or No Solution

20. a) $5+\ln(9)$ b) 9

21. $\dfrac{10}{\cos(40^\circ)}$

22. $110\sin(72^\circ)$ feet

23. $[50\tan(82^\circ) + 123]$ meters

24. a) $-\dfrac{1}{2}$ b) DNE

25. a) $-\dfrac{\sqrt{3}}{2}$ b) $-\sqrt{2}$

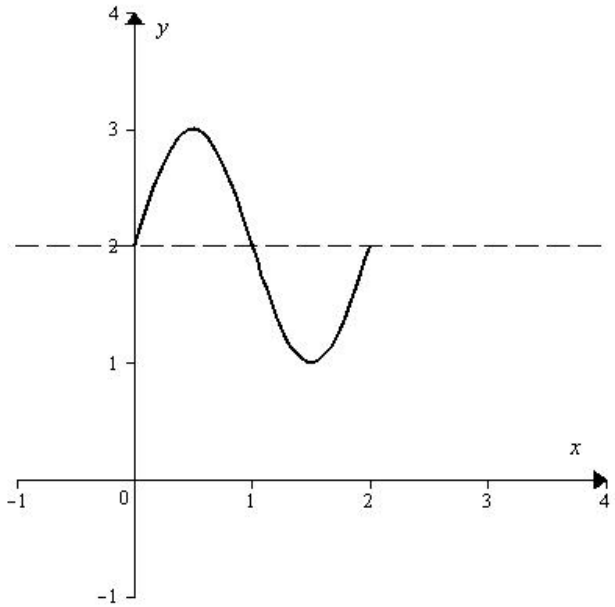
26. a) $-\dfrac{\pi}{2}$ b) $\dfrac{1}{7}$

27. a) DNE b) $\dfrac{\pi}{6}$

28. $\dfrac{\sqrt{6}}{6}$

29. $-\dfrac{\sqrt{5}}{2}$

30.



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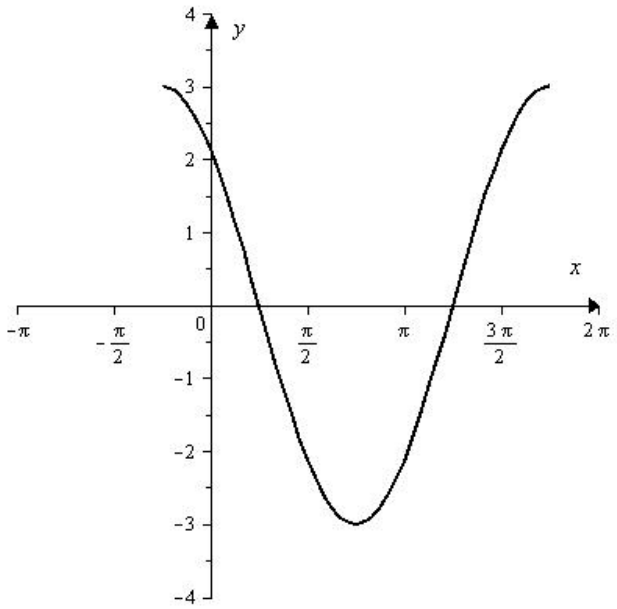

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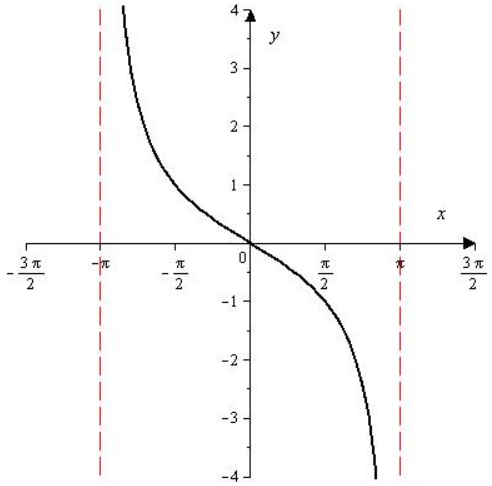

33.

$$\begin{aligned} LHS &= \frac{[1+\tan(x)]^2}{\sec^2(x)} \\ &= \frac{1+2\tan(x)+\tan^2(x)}{\sec^2(x)} \\ &= \frac{\sec^2(x)+2\tan(x)}{\sec^2(x)} \\ &= 1+\frac{2\tan(x)}{\sec^2(x)} \\ &= 1+\frac{\frac{2\sin(x)}{\cos(x)}}{\frac{1}{\cos^2(x)}} \\ &= 1+\frac{2\sin(x)}{\cos(x)}\cdot\frac{\cos^2(x)}{1} \\ &= 1+2\sin(x)\ \cos(x) \quad = \quad RHS \end{aligned}$$

34.

$$\begin{aligned} LHS &= \frac{\sin(x)}{1+\cos(x)}+\frac{1+\cos(x)}{\sin(x)} \\ &= \frac{\sin^2(x)+(1+\cos(x))^2}{(1+\cos(x))\sin(x)} \\ &= \frac{\sin^2(x)+1+2\cos(x)+\cos^2(x)}{(1+\cos(x))\sin(x)} \\ &= \frac{2+2\cos(x)}{(1+\cos(x))\sin(x)} \\ &= \frac{2(1+\cos(x))}{(1+\cos(x))\sin(x)} \\ &= \frac{2}{\sin(x)} \\ &= 2\csc(x) \quad = \quad RHS \end{aligned}$$

35.

$$\begin{aligned} LHS &= \frac{\sin(\theta)}{1+\cos(\theta)} \cdot \frac{1-\cos(x)}{1-\cos(\theta)} \\ &= \frac{1-\cos^2(\theta)}{(1-\cos(\theta))\sin(\theta)} \\ &= \frac{\sin^2(\theta)}{(1-\cos(\theta))\sin(\theta)} \\ &= \frac{\sin(\theta)}{(1-\cos(\theta))} \quad = \quad RHS \end{aligned}$$