FINAL EXAMINATION, MAT 2010
April 30, 2021

INSTRUCTIONS

1. This final exam has 12 problems worth 150 points.
2. Time allowed is 2 hours. You have 20 extra minutes to scan and upload the exam as a pdf file on canvas.
3. To receive full credit, you must show all work neatly and clearly.
4. Write your solutions on blank sheets of paper.
5. Use a ballpoint pen or a 2.0 pencil to write solutions so that your work is easily readable.
6. Leave spaces between solutions and between steps.
7. Simplify your answer when possible but use the precise value rather than an approximation when you have a choice. (Example: If actual answer is π then write π, not 3.14.).
8. You are allowed to use an approved graphing calculator if needed.
9. This test is open book/open notes test. This means you may use the textbook *Calculus: Early Transcendentals* 9th edition by James Stewart and your own notes from this class. You are not allowed to use any other text book or notes from other students or from the internet.

GOOD LUCK!

1. (7 points each) Find the exact value of each of the following limits. Write "∞," "$-\infty$," or "does not exist" if appropriate. It is particularly important to show your work on this problem. Numerical approximations do not constitute an acceptable solution.

   (a) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{2x^2 - 7x}}{5x+1}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\arctan x}{\arcsin x}$

2. (7 points each) Differentiate the following functions.

   (a) $f(x) = 3^x \cdot \sec x$

   (b) $g(x) = \dfrac{3x^2 - 5x}{\tan x}$

   (c) $h(x) = \ln(e^{3x} + e^{-3x})$

3. Evaluate.

   (a) (7 points) $\displaystyle\int \left(\frac{\sqrt{x^5}}{x^3} - e^x - \cos 3\right) dx$

   (b) (8 points) $\displaystyle\int_0^{\pi/4} \Big(1 + 2\cos\theta\Big)\, d\theta$
   NOTE: Give exact answer in (b). Do not convert to decimals.

4. (10 points) Use the **definition** of the derivative to differentiate the following function.

$$f(x) = \sqrt{x+5}$$

   (No credit will be awarded for calculating the derivative *without* using the definition of the derivative)

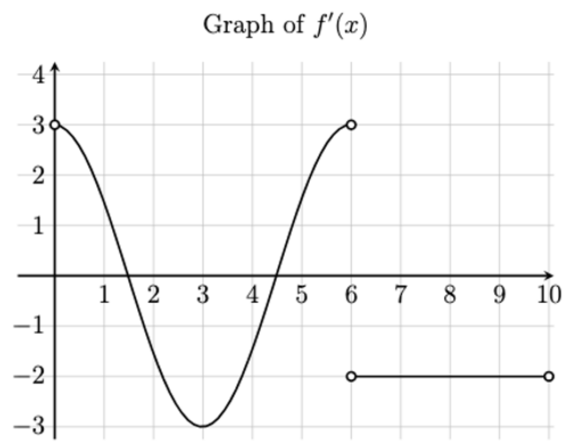
5. (10 points) For the curve $x^2y - 2y^3 + x + 3y = -7$

   (a) Find $\dfrac{dy}{dx}$.

   (b) Find the equation of the tangent line to the curve at the point $(1, 2)$. Write your answer in slope-intercept form.

6. (10 points) Evaluate $\displaystyle\int_0^3 \left(|x-1| + \sqrt{9-x^2}\right) dx$ by interpreting in terms of areas.

7. (10 points) The graph of the derivative $f'(x)$ of some function $f(x)$ is given below.



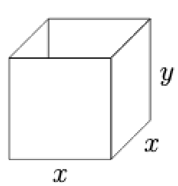
(a) Identify the subinterval(s) of $(0, 10)$ where $f(x)$ has a constant slope.
(b) Identify the subinterval(s) of $(0, 10)$ where $f(x)$ is increasing.
(c) Identify the x-coordinate(s) where $f(x)$ attains a local minimum value (if any).
(d) Identify the subinterval(s) of $(0, 10)$ where $f(x)$ is concave up

8. (10 points) A function $f(x)$ is defined as below,

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } 0 < x \leq 4 \\ -1 & \text{if } x > 4 \end{cases}$$

(a) Sketch the graph of $f(x)$.
(b) Find all values of x at which f is not continuous.
(c) Find all values of x at which f is not differentiable.

9. (10 points) An open rectangular box with a square base must have a volume of $13,500\,\text{cm}^3$. Find the dimensions of the box that minimize the amount of material used.



10. (10 points) A large, 8-ft high decorative mirror is placed on a wood floor and leaned against a wall. Let the angle between the mirror and the floor be θ, and the distance from the floor to the top of the mirror be y. The weight of the mirror and the slickness of the floor cause the mirror to slip. If the top of the mirror is slipping down at a rate of 1.5 ft/min, what is the rate at which the angle θ is changing when θ is $2\pi/9$ radians? (Give answer correct to two decimal places.)

11. (10 points) A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = 6$ cm/s, and its initial displacement is $s(0) = 9$ cm. Find the position function $s(t)$.

12. (20 points) The following information is given about a function $f(x)$.

$f(x)$ is defined for all real numbers except at $x = 0$.
$f(x)$ has a vertical asymptote at $x = 0$.

$$f'(x) = \frac{3(x^2 - 1)}{x^4} \qquad \text{and} \qquad f''(x) = \frac{6(2 - x^2)}{x^5}$$

$$\lim_{x \to -\infty} f(x) = 2 \qquad \text{and} \qquad \lim_{x \to \infty} f(x) = 2$$

with $f(\sqrt{2}) \approx 0.23$ and $f(-\sqrt{2}) \approx 3.77$

(a) Find critical numbers for f' and make a sign chart for f'.

(b) Give intervals of increase and decrease and x-coordinates of any local minima and local maxima of f

(c) Find critical numbers for f'' and make a sign chart for f''.

(d) Give intervals of concavity and x-coordinates of any linflection points of f.

(e) State the equations of horizontal and vertical asymptotes if any.

(f) Using the above information sketch the graph of $f(x)$