FINAL EXAMINATION

MAT 2010

April 24 2020

MRC

1.

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{\dfrac{3}{\sqrt{x+\Delta x+1}} - \dfrac{3}{\sqrt{x+1}}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3(x+1-x-\Delta x-1)}{\Delta x(\sqrt{x+\Delta x+1}+\sqrt{x+1})(\sqrt{x+\Delta x+1}(\sqrt{x+1}} \\
&= \lim_{\Delta x \to 0} \frac{-3\Delta x}{\Delta x(\sqrt{x+\Delta x+1}+\sqrt{x+1})\sqrt{x+\Delta x+1}\sqrt{x+1}} \\
&= \lim_{\Delta x \to 0} \frac{-3}{(\sqrt{x+\Delta x+1}+\sqrt{x+1})\sqrt{x+\Delta x+1}\sqrt{x+1}} \\
&= \frac{-3}{2(x+1)\sqrt{x+1}}
\end{aligned}
$$

2.

(a)

$$
\begin{aligned}
\lim_{t \to \infty} \frac{1+e^{2t}}{1-e^{2t}} &= \lim_{t \to \infty} \frac{e^{-2t}+1}{e^{-2t}-1} \\
&= \frac{1}{-1} = -1
\end{aligned}
$$

(b)

$$
\begin{aligned}
\lim_{x \to 2^-} \frac{|x-2|}{x^2-4} &= \lim_{x \to 2^-} \frac{2-x}{(x-2)(x+2)} \\
&= \lim_{x \to 2^-} \frac{-1}{x+2} = \frac{-1}{4}
\end{aligned}
$$

3.

(a)

$$
\begin{aligned}
f'(x) = (\arcsin(x^3))' &= (x^3)' \cdot (\arcsin)'(x^3) \\
&= 3x^2 \cdot \frac{1}{\sqrt{1-(x^3)^2}} \\
&= \frac{3x^2}{\sqrt{1-x^6}}
\end{aligned}
$$

(b)

$$\begin{aligned} g'(x) &= \left(\frac{3-2x}{x+5}\right)' \\ &= \frac{(3-2x)'(x+5)-(x+5)'(3-2x)}{(x+5)^2} \\ &= \frac{-2(x+5)-(3-2x)}{(x+5)^2} \\ &= \frac{-13}{(x+5)^2} \end{aligned}$$

(c)

$$\begin{aligned} h'(x) &= (\tan x \cdot \ln(2x+3))' \\ &= \tan'(x) \cdot \ln(2x+3) + \tan x \cdot \ln'(2x+3) \\ &= \sec^2 x \cdot \ln(2x+3) + \tan x \cdot \frac{2}{2x+3} \end{aligned}$$

4.

(a)

$$\begin{aligned} A &= \int \left[\frac{1}{\sqrt[3]{t^2}} + \frac{1}{1+t^2} - \sec t \cdot \tan t\right] dt \\ &= \int \frac{dt}{\sqrt[3]{t^2}} + \int \frac{dt}{1+t^2} - \int \sec t \cdot \tan t dt \end{aligned}$$

$$\begin{aligned} &\int \frac{dt}{\sqrt[3]{t^2}} = \int t^{-2/3} dt = 3t^{1/3} = 3\sqrt[3]{t} \\ &\int \frac{dt}{1+t^2} = \arctan t \\ &\int \sec t \cdot \tan t dt = \int \frac{\sin t}{\cos^2 t} dt = \int \frac{-(\cos t)'}{\cos^2 t} dt = \frac{1}{\cos t} \end{aligned}$$

$$A = 3\sqrt[3]{t} + \arctan t - \frac{1}{\cos t} \quad \text{+C}$$

(b)

$$\int_1^2 \left[\frac{1}{x^2} - \frac{1}{x}\right] dx = \left(-\frac{1}{x} - \ln x\right)\Big|_1^2 = \frac{1}{2} - \ln 2$$

5.

$$\begin{aligned} e^{y-x} &= \sin y - x^2 \\ \frac{de^{y-x}}{dx} &= \frac{d(\sin y - x^2)}{dx} \\ \frac{d(y-x)}{dx} e^{y-x} &= \frac{dy}{dx} \cos y - 2x \\ (\frac{dy}{dx} - 1)e^{y-x} &= \frac{dy}{dx} \cos y - 2x \\ \frac{dy}{dx}(e^{y-x} - \cos y) &= e^{y-x} - 2x \\ \frac{dy}{dx} &= \frac{e^{y-x} - 2x}{e^{y-x} - \cos y} \end{aligned}$$

6.

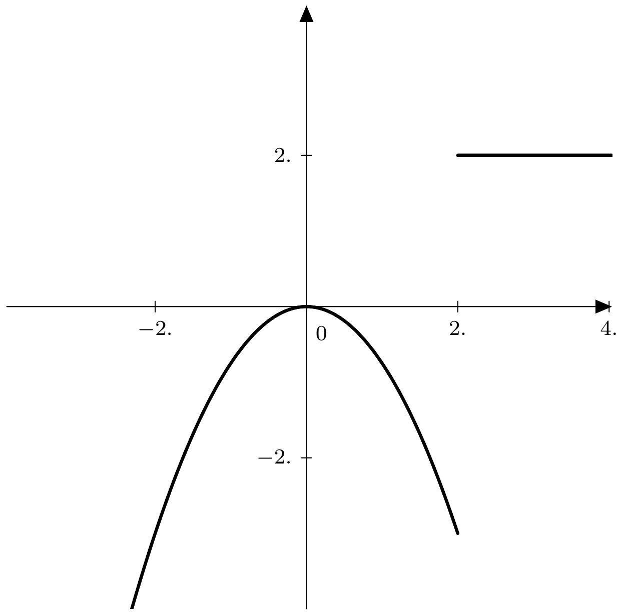
$$V(t) = \frac{100t^2 + 50}{t} + 400$$

(a) total change: $V(5) - V(1) = 510 - 150 = 360$ \$.

(b) average rate of change $\dfrac{V(5) - V(1)}{5 - 1} = \dfrac{360}{4} = 90.$

(c) $V'(t) = 100 - \dfrac{50}{t^2}$, $V'(4) = 96,875$ \$/year.

7.



8.

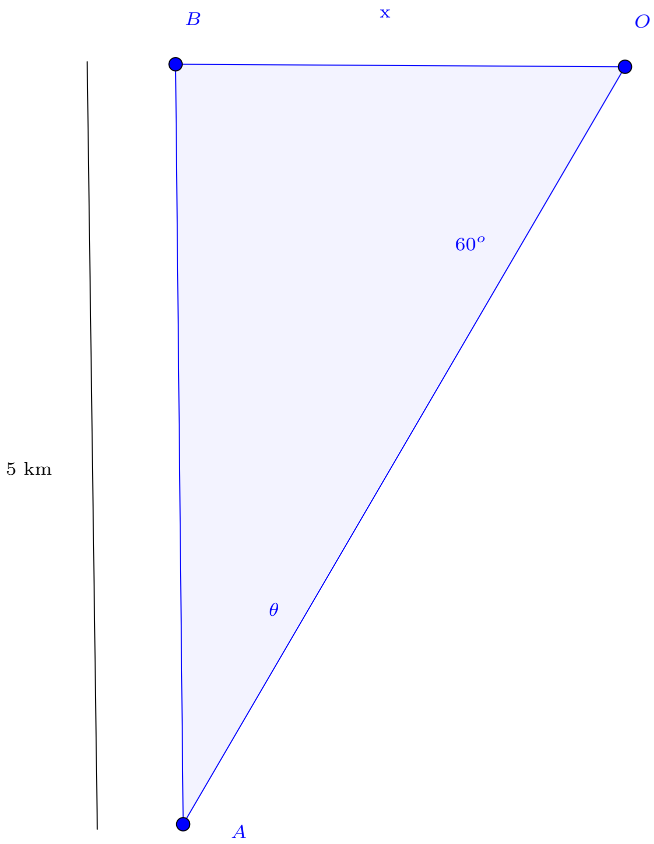
$$\begin{aligned} f(x) &= x\sqrt{1-x^2} \\ f'(x) &= \sqrt{1-x^2} + \frac{x.(-2x)}{2\sqrt{1-x^2}} \\ &= \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} \\ f'(x) = 0 &\leftrightarrow \ x = \frac{1}{\sqrt{2}} \quad \text{or} \quad x = -\frac{1}{\sqrt{2}} \end{aligned}$$

x	-1		$\frac{-1}{\sqrt{2}}$		$\frac{1}{\sqrt{2}}$		1
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$		$\searrow$		$\nearrow$		$\searrow$	

$$\begin{aligned} &f(-1) = f(1) = 0 \\ &f\left(\frac{-1}{\sqrt{2}}\right) = \frac{-1}{2} \\ &f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} \end{aligned}$$

$\rightarrow \min \frac{-1}{2}, \quad \max \frac{1}{2}.$

9.



$$\tan\theta = \frac{x}{5}$$
$$\frac{dtan\theta}{dt} = \frac{dx}{5dt}$$
$$\frac{d\theta}{dt}\sec^2\theta = \frac{1}{5}\frac{x}{dt}$$
$$\frac{dx}{dt} = 5\cdot 3\cdot \sec^2 t(30^o)$$

10.

(a) g decreases on $(0, \pi/4) \cup (5\pi/4, 2\pi)$

(b) g has local max at $x = 5\pi/4$

(c) g is concave up on $(0, 3\pi/4)$ and $(7\pi/4, 2\pi)$

11. $s(t) = \cos^2 t + \sin t$, where $0 \le t \le 2\pi$

(a) $v(t) = -2\cos t \sin t + \cos t$

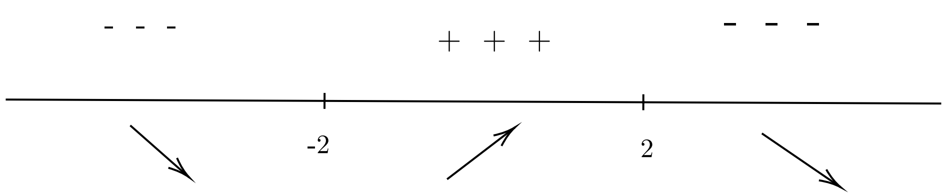
(b) $-2\cos t \sin t + \cos t = 0$

$\cos t = 0 \Longrightarrow \boxed{t = \frac{\pi}{2}, \frac{3\pi}{2}}$

OR $-2\sin t + 1 = 0 \Longrightarrow \sin t = \frac{1}{2} \Longrightarrow \boxed{t = \frac{\pi}{6}, \frac{5\pi}{6}}$

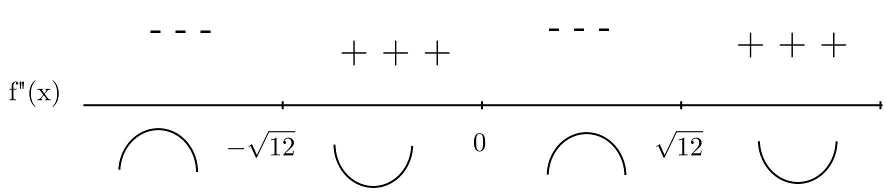
12. Sign chart for $f'(x)$:

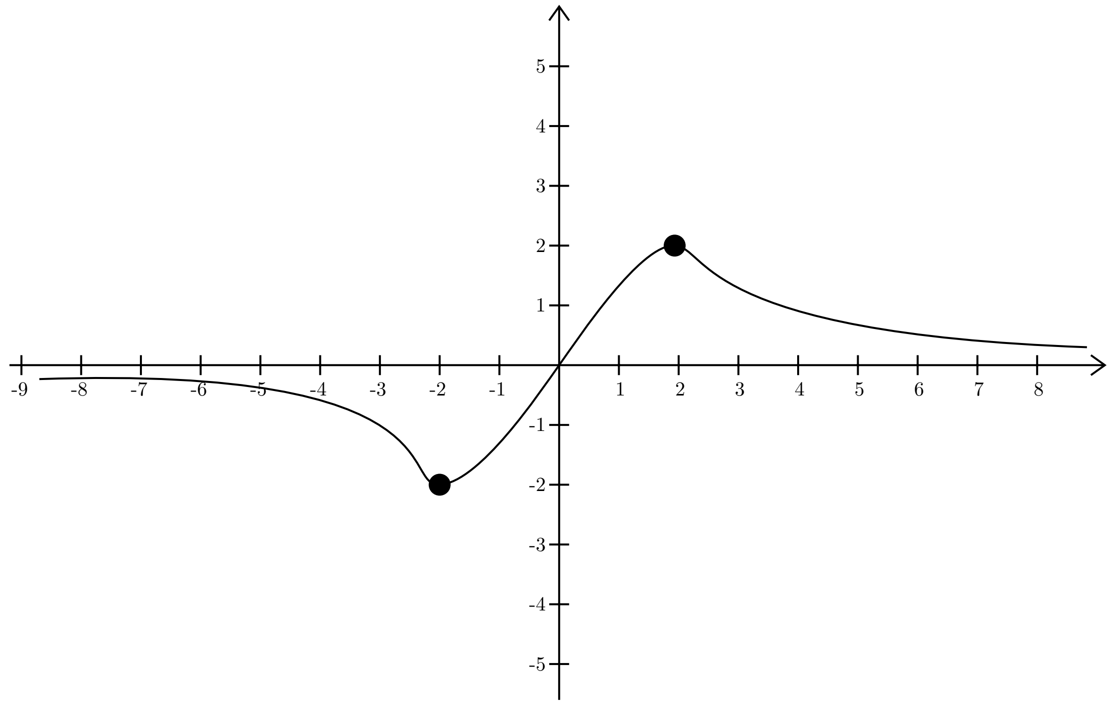
$f'(x) = \dfrac{-8(x^2-4)}{(x^2+4)^2} \longrightarrow x = -2, 2$ critical numbers.



Sign chart for $f''(x)$:

$f''(x) = \dfrac{16x(x^2-12)}{(x^2+4)^3} \Longrightarrow x = -\sqrt{12}, 0, \sqrt{12}$ critical numbers.

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-4
-5
-9
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-6
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2
3
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8