FINAL EXAMINATION, MAT 2010
April 24, 2020

NOTE: Write your solutions on clean sheets of paper using a ball pen. To receive full credit you must show **all** work. You are allowed to use an **approved** graphing calculator unless otherwise indicated. Simplify your answer when possible, but use the precise value rather than an approximation when you have a choice. (Example: If the actual answer is π, then write π, not 3.14.) The 12 problems are worth a total of 150 points. The time limit is 2 hours [120 minutes].

Cell phones are strictly prohibited!

1. (10 points) Use the **definition** of the derivative to differentiate the following function.

$$f(x) = \frac{3}{\sqrt{x+1}}$$

2. (7 points each) Find the exact value of each of the following limits. Write "∞," "$-\infty$," or "does not exist" if appropriate. It is particularly important to show your work on this problem.

   (a) $\displaystyle\lim_{t\to\infty} \frac{1+e^{2t}}{1-e^{2t}}$

   (b) $\displaystyle\lim_{x\to 2^-} \frac{|x-2|}{x^2-4}$

3. (7 points each) Differentiate the following functions. Simplify your answer.

   (a) $f(x) = \arcsin(x^3)$

   (b) $g(x) = \dfrac{3-2x}{x+5}$

   (c) $h(x) = \tan x \cdot \ln(2x+3)$

4. Evaluate. Simplify your answer.

   (a) (7 points) $\displaystyle\int \left[\frac{1}{\sqrt[3]{t^2}} + \frac{1}{1+t^2} - \sec t \tan t\right] dt$

   (b) (8 points) $\displaystyle\int_1^2 \left[\frac{1}{x^2} - \frac{1}{x}\right] dx$

5. (10 points) Find $\dfrac{dy}{dx}$ for the curve given by the equation $\quad e^{y-x} = \sin y - x^2$.

6. (10 points) The value V (in dollars) of a painting t years after it is purchased is modeled by the function

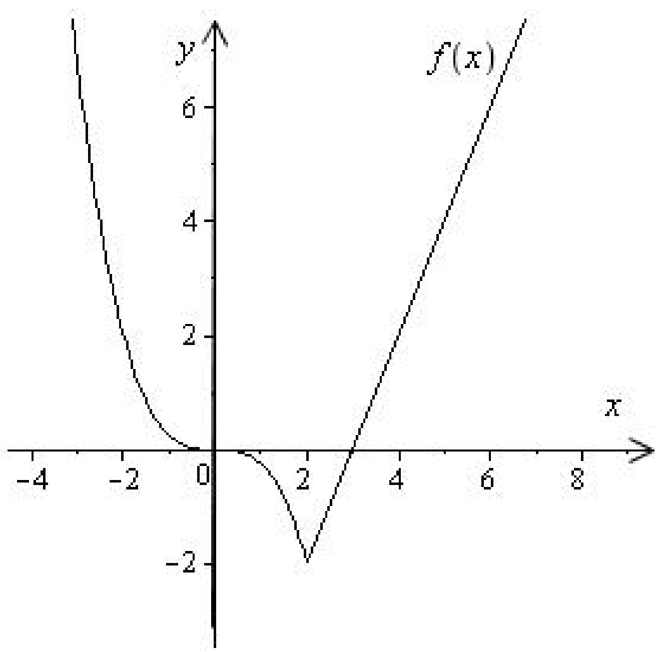
$$V = \frac{100\,t^2 + 50}{t} + 400 \qquad 1 \le t \le 5$$

(a) Find the total change in the value of the painting between the first $(t = 1)$ and fifth $(t = 5)$ years. Give proper units.

(b) Find the average rate of change in the value of the painting between the first $(t = 1)$ and fifth $(t = 5)$ years. Give proper units.

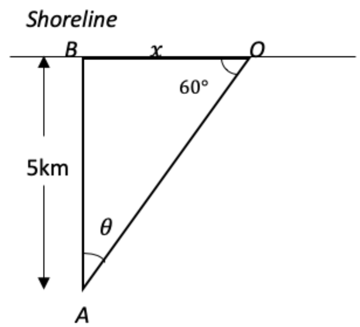
(c) Find the instantaneous rate of change in the value of the painting after year four (t=4). Give proper units.

7. (10 points) The graph of a function $f(x)$ is given below. Sketch the graph of the derivative $f\,'(x)$.

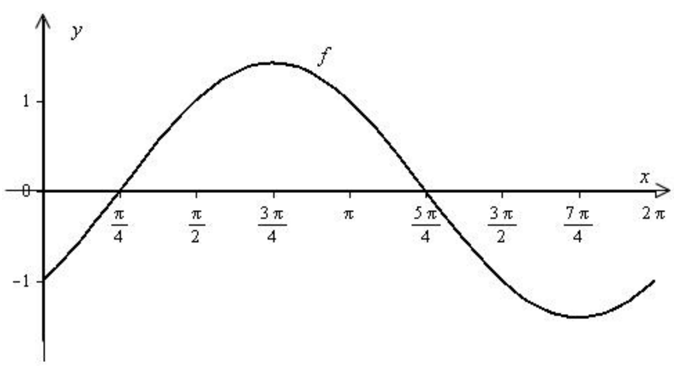


8. (10 points) Find the absolute maximum value and the absolute minimum value of the function $f(x) = x\sqrt{1 - x^2}$ on the interval $[-1,\, 1]$.

9. (10 points) A revolving light located 5 kilometers from a straight shoreline turns with a constant angular speed of 3 rad/min. With what speed is the spot of light moving along the shore when the beam makes an angle of 60° with the shoreline as shown in the figure below?



10. (10 points) The graph of a function f is shown below on the interval $[0,\, 2\pi]$. Define a new



function $g(x) = \displaystyle\int_0^x f(t)\,dt$.

(a) Give all interval(s) in $(0,\, 2\pi)$ where g is decreasing.

(b) Give x value(s) in $(0,\, 2\pi)$ where g has a local maximum value.

(c) Give all interval(s) in $(0,\, 2\pi)$ where g is concave up.

11. (10 points) A particle is moving and its position function at time t (measured in seconds) is given by

$$s(t) = \cos^2 t + \sin t\,, \quad \text{where} \quad 0 \leq t \leq 2\pi$$

(a) Find the velocity $v(t)$ of the particle at any time t.

(b) Find all times t in the interval $[0, 2\pi]$ when the particle will be at rest.

12. (20 points) Sketch the graph of a single function $f(x)$ which satisfies all of the following conditions. Indicate and label all local maxima and minima, intervals of increase and decrease, points of inflection, intervals where f is concave up and where it is concave down. Also give equations of asymptotes and label them.

(i) $f(x)$ is defined for all real numbers

(ii) $f'(x) = -\dfrac{8(x^2-4)}{(x^2+4)^2}$

(iii) $f''(x) = \dfrac{16x(x^2-12)}{(x^2+4)^3}$

(iv) $f(0) = 0$

(v) $f(1) = 1.6$

(vi) $\lim\limits_{x \to \infty} f(x) = 0$

(vii) $\lim\limits_{x \to -\infty} f(x) = 0$