Write your solutions in a blue book. To receive full credit you must show all work. You are allowed to use an approved graphing calculator unless otherwise indicated. Simplify your answer when possible, but use the precise value rather than an approximation when you have a choice. (Example: If the actual answer is π, then write π, not 3.14.) The 12 problems are worth a total of 150 points. The time limit is 2 hours [120 minutes].

**Cell phones are strictly prohibited!**

1. (10 points) Use the definition of the derivative to differentiate the following function.

   \[ f(x) = \sqrt{2 - x} \]

2. (7 points each) Find the exact value of each of the following limits. Write “∞,” “−∞,” or “does not exist” if appropriate. It is particularly important to show your work on this problem.

   \( \lim_{t \to 3} \frac{|t - 3|}{t^2 - 2t - 3} \)

   \( \lim_{x \to \infty} \frac{x^4 + 5x}{2x^2 - 5} \)

   \( \lim_{x \to 0} x^2 \ln(x) \)

3. (7 points each) Differentiate the following functions. Simplify your answer.

   \( f(x) = \frac{\cos(x)}{\arctan(x)} \)

   \( h(x) = [\sec(3x)]^7 \)

4. Evaluate. Simplify your answer.

   \( \int \left[ \sec^2 x + \frac{1}{\sqrt{1-x^2}} \right] dx \)

   \( \int_1^4 \left[ \frac{1}{x} + \frac{1}{\sqrt{x}} \right] dx \)
5. (10 points) Find $\frac{dy}{dx}$ for

$$\sin(xy) + \pi = e^{x-y}$$

6. (10 points) Find the point(s) on the curve $y = x^3 - 3x^2 + x + 3$ where the tangent is horizontal.

7. (10 points) Find the derivative of

$$y = (\tan x)^x$$

8. (10 points) The graph of a function $f(x)$ is given below. Sketch the graph of the derivative $f'(x)$ clearly showing the intervals where $f'(x) > 0$, where $f'(x) < 0$, and the $x$-values where $f'(x) = 0$.

9. (10 points) Find two positive real numbers $x$ and $y$ such that they add to 120 and the quantity $x^2y$ is maximum.

10. (10 points) On a typical day, a city consumes water at the rate of $r(t) = 100 + 72t - 3t^2$ (in thousands of gallons per hour), where $t$ is the number of hours past midnight. How much water is consumed between 6 pm and midnight?
11. (10 points) The graph of a function \( f(x) \) is shown below.

Define a new function \( g(x) = \int_0^x f(t) \, dt \). Find \( g(8) \).

12. (20 points) Sketch the graph of a single function \( f(x) \) on the interval \([0, 2\pi]\), which satisfies all of the following conditions. Indicate and label all local maxima and minima, intervals of increase and decrease, points of inflection, concavity, and asymptotes.

   (i) \( f(x) \) is defined for all real numbers
   (ii) \( f'(x) = \cos x + \frac{\sqrt{3}}{2} \)
   (iii) \( f''(x) = -\sin(x) \)
   (iv) \( f(0) = 0 \)
   (v) \( f(\pi) \approx 2.7 \)
   (vi) \( f(2\pi) \approx 5.4 \)