FINAL EXAMINATION, MAT 2010
April 26, 2019

Write your solutions in a blue book. To receive full credit you must show *all* work. You are allowed to use an *approved* graphing calculator unless otherwise indicated. Simplify your answer when possible, but use the precise value rather than an approximation when you have a choice. (Example: If the actual answer is π, then write π, not 3.14.) The 12 problems are worth a total of 150 points. The time limit is 2 hours [120 minutes].

Cell phones are strictly prohibited!

1. (10 points) Use the **definition** of the derivative to differentiate the following function.

$$f(x) = \sqrt{2 - x}$$

2. (7 points each) Find the exact value of each of the following limits. Write "∞," "$-\infty$," or "does not exist" if appropriate. It is particularly important to show your work on this problem.

   (a) $\lim\limits_{t \to 3} \dfrac{|t - 3|}{t^2 - 2t - 3}$

   (b) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^4 + 5x}}{2x - 5}$

   (c) $\lim\limits_{x \to 0} x^2 \ln(x)$

3. (7 points each) Differentiate the following functions. Simplify your answer.

   (a) $f(x) = \dfrac{\cos(x)}{\arctan(x)}$

   (b) $h(x) = [\sec(3x)]^7$

4. Evaluate. Simplify your answer.

   (a) (7 points) $\displaystyle\int \left[\sec^2 x + \frac{1}{\sqrt{1 - x^2}}\right] dx$

   (b) (8 points) $\displaystyle\int_1^4 \left[\frac{1}{x} + \frac{1}{\sqrt{x}}\right] dx$

5. (10 points) Find $\dfrac{dy}{dx}$ for

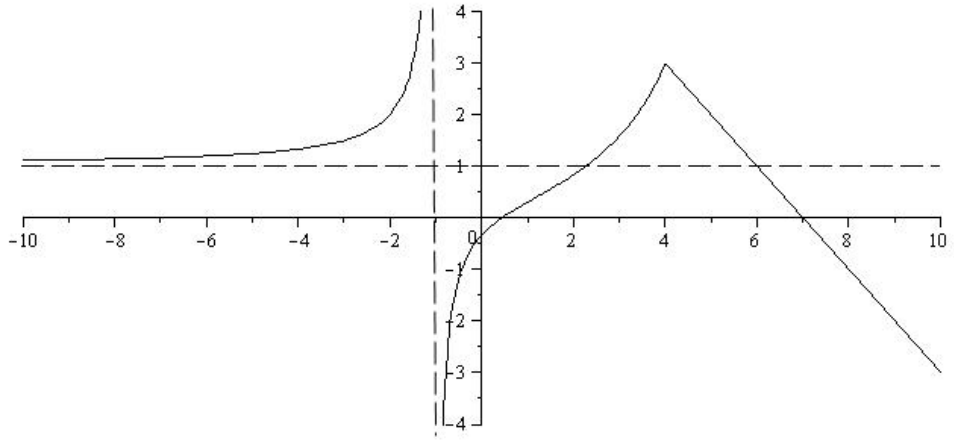
$$\sin(xy) + \pi = e^{x-y}$$

6. (10 points) Find the point(s) on the curve $y = x^3 - 3x^2 + x + 3$ where the tangent is horizontal.

7. (10 points) Find the derivative of

$$y = (\tan x)^{x^3}$$

8. (10 points) The graph of a function $f(x)$ is given below. Sketch the graph of the derivative

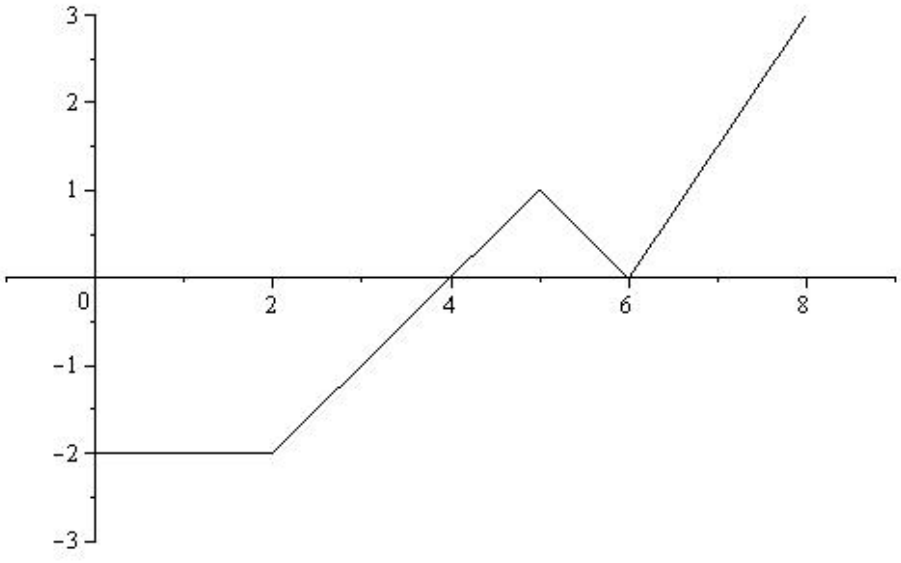


$f'(x)$ clearly showing the intervals where $f'(x) > 0$, where $f'(x) < 0$, and the x-values where $f'(x) = 0$.

9. (10 points) Find two positive real numbers x and y such that they add to 120 and the quantity x^2y is maximum.

10. (10 points) On a typical day, a city consumes water at the rate of $r(t) = 100 + 72t - 3t^2$ (in thousands of gallons per hour), where t is the number of hours past midnight. How much water is consumed between 6 pm and midnight?

11. (10 points) The graph of a function $f(x)$ is shown below.



Define a new function $g(x) = \int_0^x f(t)\,dt$. **Find** $g(8)$.

12. (20 points) Sketch the graph of a single function $f(x)$ on the interval $[0, 2\pi]$, which satisfies all of the following conditions. Indicate and label all local maxima and minima, intervals of increase and decrease, points of inflection, concavity, and asymptotes.

(i) $f(x)$ is defined for all real numbers

(ii) $f'(x) = \cos x + \dfrac{\sqrt{3}}{2}$

(iii) $f''(x) = -\sin(x)$

(iv) $f(0) = 0$

(v) $f(\pi) \approx 2.7$

(vi) $f(2\pi) \approx 5.4$