FINAL EXAMINATION, MAT 2010
December 17, 2021

Write your solutions in a blue book. To receive full credit you must show *all* work. You are allowed to use an *approved* graphing calculator unless otherwise indicated. Simplify your answers, when possible but use the precise value rather than an approximation when you have a choice. (Example: If the actual answer is π, then write π, not 3.14.) The 12 problems are worth a total of 150 points. The time limit is 2 hours [120 minutes].

Cell phones are strictly prohibited!

1. (10 points) Use the **definition** of the derivative to differentiate the following function.

$$f(x) = \frac{3}{3x-2}$$

NOTE: No credit will be given if the definition of the derivative is not used.

2. (7 points each) Find the exact value of each of the following limits. Write "∞," "$-\infty$," or "does not exist" if appropriate. It is particularly important to show your work on this problem. Numerical approximations do not constitute an acceptable solution.

(a) $\displaystyle\lim_{x\to 2} \frac{\dfrac{1}{x+4} - \dfrac{1}{3x}}{x-2}$

(b) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{2x^2+3}}{x+7}$

(c) $\displaystyle\lim_{x\to 2} \frac{e^{x^2} - e^4}{x-2}$

3. (7 points each) Differentiate the following functions. Simplify your answer.

   (a) $f(x) = \left[\sin(3x^2 + x)\right]^4$

   (b) $g(x) = \cos(2x)\,\ln(x - 1)$

4. Evaluate. Simplify your answer.

   (a) (7 points) $\int \left(\frac{5}{t^2+1} - \frac{2}{\sqrt{1-t^2}} + \sqrt{2}\right) dt$

   (b) (8 points) $\int_1^2 \left[\frac{1}{x} - \frac{2}{x^3}\right] dx$

5. (10 points) Find $\frac{dy}{dx}$ for the curve given by the equation

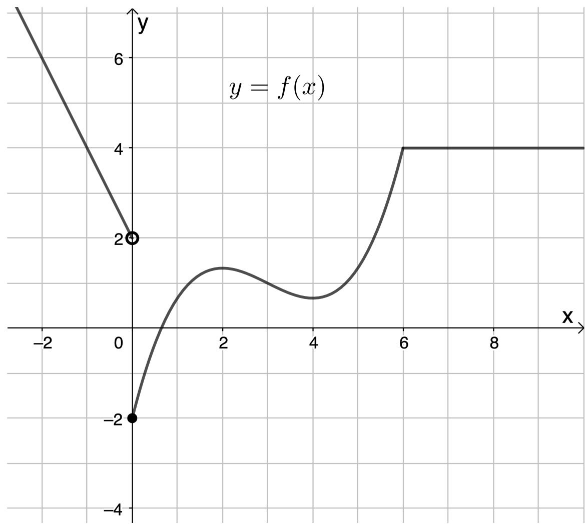
$$e^{x-y} = 2x^2 - y^2$$

6. (10 points) Find the critical numbers (if any) of the function $f(x) = \frac{x^2}{x-1}$.

7. (10 points) A cylindrical container with a top and bottom is required to have a volume of 16π m^3. What is the minimum amount of material (surface area) needed to make such a container?
   [For a cylinder with base radius r and height h, the volume is given by $V = \pi r^2 h$, and the surface area is given by $S = 2\pi r^2 + 2\pi r h$.]

8. (10 points) The gate of a dam is opened and water is released from the reservoir at a rate of $r(t) = 100 + \sqrt{t}$ gallons per minute, where t is measured in minutes since the gate has been opened. If the gate is opened at 7 a.m. and is left open until 9:24 a.m., how much water is released during these 144 minutes?

9. (10 points) The graph of a function $f(x)$ is given below. Sketch the graph of the derivative function $f'(x)$.



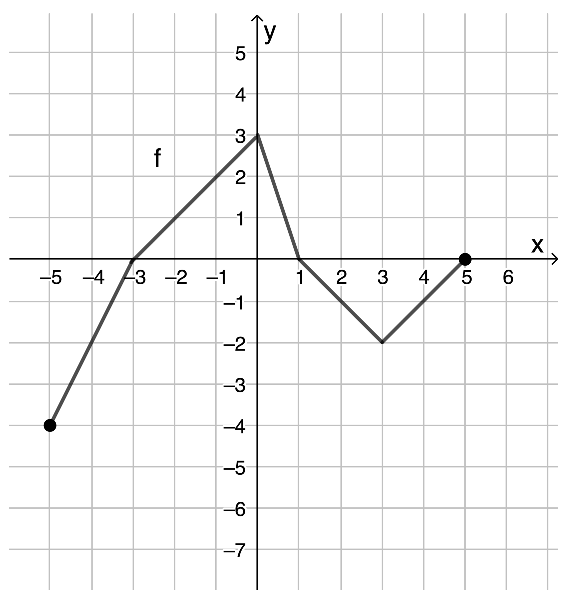
10. (10 points) The graph of a function $f(x)$ is given below. Define a new function $g(x)$ as

$$g(x) = \int_{-5}^{x} f(t)\,dt, \quad -5 \le x \le 5.$$

(a) (6 points) Find $g(5)$

(b) (2 points) Find $g'(2)$ and $g''(2)$ (State DNE if a value does not exist).

(c) (2 points) Find $g'(1)$ and $g''(1)$ (State DNE if a value does not exist).



11. (10 points) Using differntials or linear approximation estimate the value of $(1.2)^{5/3}$. Give answer correct to two decimal places.

12. (20 points) Sketch the graph of a single function $f(x)$ which satisfies all of the following conditions. Label all local maxima and minima, intervals of increase and decrease, points of inflection, concavity, and asymptotes.

    (i) $f(x)$ is defined for all real numbers

    (ii) $f'(x) = -\dfrac{3(x^2-3)}{2(x^2+1)}$

    (iii) $f''(x) = -\dfrac{12x}{(x^2+1)^2}$

    (iv) $f(0) = 0$

    (v) $f(3) \approx 3$

    (vi) $f(-3) \approx -3$

    (vii) $\lim_{x\to\infty} f(x) = -\infty$

    (viii) $\lim_{x\to-\infty} f(x) = \infty$