FINAL EXAMINATION, MAT 2010
December 18, 2020

INSTRUCTIONS

1. This final exam has 12 problems worth 150 points.
2. Time allowed is 2 hours. You have 20 extra minutes to scan and upload the exam as a pdf file on canvas.
3. To receive full credit, you must show all work neatly and clearly.
4. Write your solutions on blank sheets of paper.
5. Use a ballpoint pen or a 2.0 pencil to write solutions so that your work is easily readable.
6. Leave spaces between solutions and between steps.
7. Simplify your answer when possible but use the precise value rather than an approximation when you have a choice. (Example: If actual answer is $\pi$ then write $\pi$, not $3.14$.)
8. You are allowed to use an approved graphing calculator if needed.

GOOD LUCK

1. (10 points) Use the **definition** of the derivative to differentiate the following function.

$$f(x) = \frac{x^2}{3+x}$$

2. (7 points each) Find the exact value of each of the following limits. Write "$\infty$," "$-\infty$," or "does not exist" if appropriate. It is particularly important to show your work on this problem. Numerical approximations do not constitute an acceptable solution.

   (a) $\displaystyle\lim_{t\to 0} \frac{\sqrt{9+t}-3}{t}$

   (b) $\displaystyle\lim_{x\to -2} \frac{3x^3-12x}{x^2-3x-10}$

3. (7 points each) Differentiate the following functions. Simplify your answer.

   (a) $f(x) = \sqrt[3]{x^4} \cdot \sec x$

   (b) $g(x) = \dfrac{\tan x}{x^3+1}$

   (c) $h(x) = \arcsin(5x)$

4. Evaluate. Simplify your answer.

   (a) (7 points) $\displaystyle\int \left[\frac{5x+3-x\,\sec^2 x}{x}\right] dx$

   (b) (8 points) $\displaystyle\int_0^{\pi/2} \left[e^x - \cos x\right] dx$

5. (10 points) Find $\dfrac{dy}{dx}$ for the curve given by the equation

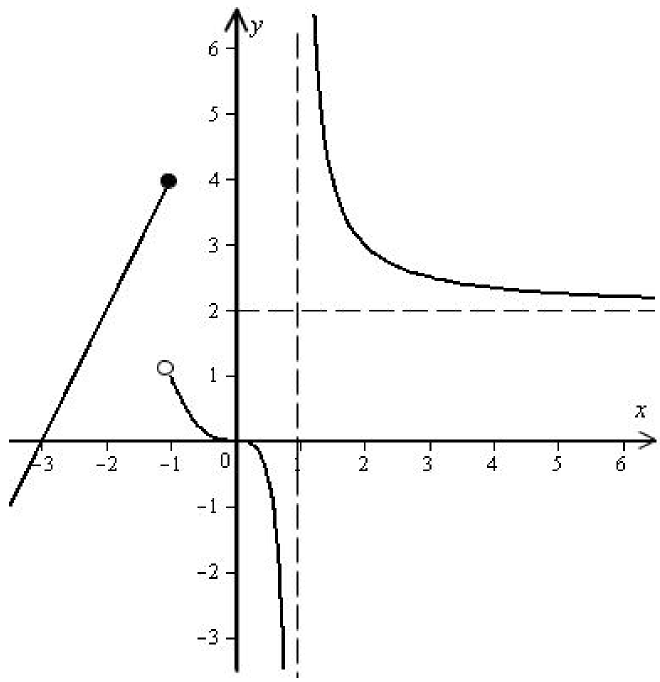
$$\sin(xy) + y^3 = x^2$$

6. (10 points) A particle is moving with the given data. Find the position function $s(t)$ of the particle.

$$a(t) = t^2 - 7t + 6, \qquad s(0) = 0, \qquad s(1) = 5$$

7. (10 points) Using the left-endpoint Riemann sum with 4 equal subintervals, estimate the area bounded above by the curve $f(x) = \dfrac{1}{1+x^2}$ and below by $x$-axis on the interval $[0,2]$. Give answer correct to three decimal places.

8. (10 points) The graph of a function $f(x)$ is given below. Sketch the graph of the derivative $f'(x)$ on the interval $(-\infty, \infty)$.




9. (10 points) A boy is walking toward the base of a pole $20$ m high at the rate of $1$ m/sec. At what rate (in meters per second) is the distance from his feet to the top of the pole changing when he is $5$ m from the pole? (You may give answer correct to two decimal places.)

10. (10 points) The weekly cost $C$, in dollars, of manufacturing $x$ lightbulbs is

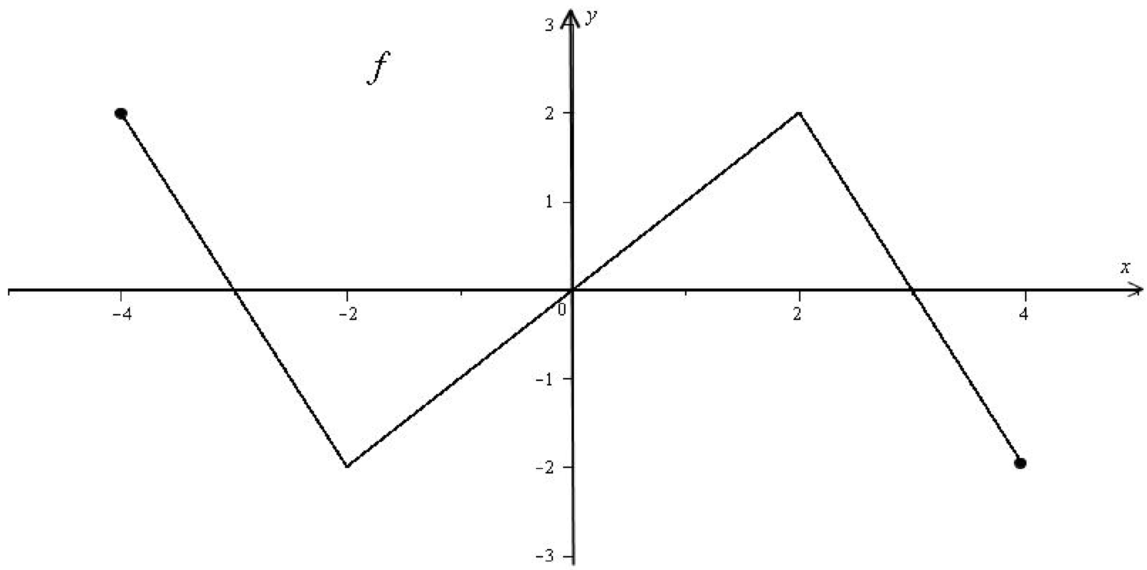
$$C(x) = 7500 + \sqrt{125x}$$

   (a) Find the average rate of change of the weekly cost $C$ of manufacturing from $100$ to $150$ lightbulbs. Give answer correct to two decimal places. Give proper units.

   (b) Find the instantaneous rate of change in the weekly cost when $125$ lightbulbs are manufactured. Give answer correct to two decimal places. Give proper units.

11. (10 points) The graph of a function $f$ is shown below. Define a new function

$$g(x) = \int_{-4}^{x} f(t)\,dt, \qquad -4 \le x \le 4$$




(a) (5 points) Determine $g(2)$. [Show all work on your answer sheet].

(b) (3 points) Determine $g'(-2)$.

(c) (2 points) Determine $g''(0)$.

12. (20 points) Sketch the graph of a single function $f(x)$ which satisfies all of the following conditions. Indicate and label all local maxima and minima, intervals of increase and decrease, points of inflection, intervals where $f$ is concave up and where $f$ is concave down. Also give equations of any asymptotes and label them.

(i) $f(x)$ is defined for all real numbers

(ii) $f'(x) = \frac{1}{2}(x+5)(x-1)\,e^{x/2}$

(iii) $f''(x) = \frac{1}{4}(x^2+8x+3)\,e^{x/2}$

(iv) $f(0) = -5$

(v) $f(-5) \approx 1.64$

(vi) $\lim_{x\to\infty} f(x) = \infty$

(vii) $\lim_{x\to-\infty} f(x) = 0$