MAT 2010 Final Exam – Fall 2016

1. Use the **definition** of derivative to differentiate the following function.

$$f(x) = \sqrt{x} - 1$$

Answer: $f'(x) = \frac{1}{2\sqrt{x}}$

Solution: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \Rightarrow \lim_{h\to 0} \frac{\sqrt{(x+h)}-1-(\sqrt{x}-1)}{h} \Rightarrow \lim_{h\to 0} \frac{\sqrt{(x+h)}-1-\sqrt{x}+1}{h} \Rightarrow \lim_{h\to 0} \frac{\sqrt{(x+h)}-\sqrt{x}}{h}$

$\Rightarrow \lim_{h\to 0} \frac{\sqrt{(x+h)}-\sqrt{x}}{h} \cdot \frac{\sqrt{(x+h)}+\sqrt{x}}{\sqrt{(x+h)}+\sqrt{x}} \Rightarrow \lim_{h\to 0} \frac{x+h-x}{h\left(\sqrt{(x+h)}+\sqrt{x}\right)} \Rightarrow \lim_{h\to 0} \frac{h}{h\left(\sqrt{(x+h)}+\sqrt{x}\right)} \Rightarrow \lim_{h\to 0} \frac{1}{\sqrt{(x+h)}+\sqrt{x}}$

Evaluating the limit by plugging in $h = 0$ gives:

$$\frac{1}{\sqrt{(x+0)}+\sqrt{x}} = \mathbf{\frac{1}{2\sqrt{x}}}$$

2. Find the exact value of each of the following limits. Write "∞," "—∞," or "does not exist" if appropriate. It is particularly important to show your work on this problem.

(a) $\lim_{x\to -1} \frac{(2x-1)^2-9}{x+1}$

Answer: -12

Solution:

$$\lim_{x\to -1} \frac{(2x-1)^2-9}{x+1} \xRightarrow[\text{plugging in } x=-1]{\text{evaluate by}} \frac{(2(-1)-1)^2-9}{(-1)+1} = \frac{0}{0} \quad \text{(indeterminate form)}$$

Via algebra:

$$\lim_{x\to -1} \frac{(2x-1)^2-9}{x+1} \Rightarrow \lim_{x\to -1} \frac{4x^2-4x+1-9}{x+1} \Rightarrow \lim_{x\to -1} \frac{4x^2-4x-8}{x+1} \Rightarrow \lim_{x\to -1} \frac{4(x^2-x-2)}{x+1}$$

$$\Rightarrow \lim_{x\to -1} \frac{4(x-2)(x+1)}{x+1} \Rightarrow \lim_{x\to -1} 4(x-2)$$

Evaluate the limit by plugging in $x = -1$:

$$\lim_{x\to -1} 4(x-2) = 4(-1-2) = \mathbf{-12}$$

(b) $\lim_{x\to\infty} x^2e^{-x}$

Answer: 0

Solution: Rewrite using algebra: $\lim_{x\to\infty} x^2e^{-x} \Rightarrow \lim_{x\to\infty} \frac{x^2}{e^x}$

Evaluate the limit by plugging in $x = \infty$: $\frac{\infty^2}{e^\infty} \Rightarrow \frac{\infty}{\infty}$ (Indeterminate form)

Use l'hospital's rule by taking the derivative of the numerator and denominator separately:

$$\lim_{x\to\infty} \frac{x^2}{e^x} \overset{L}{\Rightarrow} \lim_{x\to\infty} \frac{2x}{e^x}$$

Evaluate the limit by plugging in $x = \infty$:

$$\frac{2(\infty)}{e^\infty} \Rightarrow \frac{\infty}{\infty} \quad \text{(Indeterminate form)}$$

Use l'hospital's rule again, since we are still in indeterminate form:

$$\lim_{x\to\infty} \frac{2x}{e^x} \overset{L}{\Rightarrow} \lim_{x\to\infty} \frac{2}{e^x}$$

Evaluate the limit again by plugging in $x = \infty$:

$$\frac{2}{e^\infty} = \mathbf{0}$$

(c) $\lim_{x\to 3^+} \frac{x-1}{x-3}$

Answer: ∞

Solution: $\lim_{x\to 3^+} \frac{x-1}{x-3} \xRightarrow[x=3]{\textit{evaluate by plugging in}} \frac{3-1}{3-3} = \frac{2}{0}$

Notice that this is NOT indeterminate form since the numerator is equal to 2. When dividing any number by zero, the limit approaches positive infinity. Also, notice since the limit is specifically approaching 3 from the right, the denominator will always be a very small positive number. Therefore,

$$\lim_{x\to 3^+} \frac{x-1}{x-3} = \frac{2}{0} = \infty$$

3. Find the derivative $\frac{\mathrm{d}}{\mathrm{dx}}$ for each of the following functions.

(a) $f(x) = \sqrt[3]{x^2 - 2x + 2}$

Answer: $f'(x) = \frac{1}{3}(x^2 - 2x + 2)^{-\frac{2}{3}} \cdot (2x - 2)$

Solution: First, notice that we can rewrite our function as: $f(x) = \sqrt[3]{x^2 - 2x + 2} = (x^2 - 2x + 2)^{\frac{1}{3}}$

Use chain rule: $f(x) = g\big(h(x)\big) \ \rightarrow \ f'(x) = g'(h(x)) \cdot h'(x)$

$$\text{Let } g(x) = x^{\frac{1}{3}} \ \Rightarrow \ g'(x) = \frac{1}{3}x^{-\frac{2}{3}} \ \text{ and } h(x) = x^2 - 2x + 2 \ \Rightarrow \ h'(x) = 2x - 2$$

$$\text{Then, } f'(x) = g'\big(h(x)\big) \cdot h'(x) \ \Rightarrow \ f'(x) = \boldsymbol{\frac{1}{3}(x^2 - 2x + 2)^{-\frac{2}{3}} \cdot (2x - 2)}$$

(b) $g(x) = e^x \cdot \tan(x)$

Answer: $g'(x) = e^x \tan(x) + e^x sec^2(x)$

Solution: Use product rule: $\frac{d}{dx}(uv) = u'v + uv'$, and let $u = e^x \ \Rightarrow u' = e^x$ and $v = \tan(x) \ \Rightarrow v' = \sec^2(x)$

Then, $g'(x) = (e^x)(\tan(x)) + (e^x)(\sec^2(x)) = \boldsymbol{e^x \tan(x) + e^x sec^2(x)}$

(c) $h(x) = \frac{x^2 - 7x}{\ln(x)}$

Answer: $h'(x) = (2x^3 - 7x^2)\ln(x) - x^3 - 7x^2$

Solution: Use quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$, and let $u = x^2 - 7x \ \Rightarrow u' = 2x - 7$ and $v = \ln(x) \ \Rightarrow v' = \frac{1}{x}$

Then, $h'(x) = \frac{(2x-7)(\ln(x)) - (x^2 - 7x)(\frac{1}{x})}{\left(\frac{1}{x}\right)^2} = x^2[(2x - 7)\ln(x) - x + 7] = \boldsymbol{(2x^3 - 7x^2)\ln(x) - x^3 + 7x^2}$

(d) $k(x) = \int_0^x cos(t^2)\, dt$

Answer: $k'(x) = \cos(x^2)$

Solution: From the Fundamental Theorem of Calculus, we know

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = \frac{d}{dx}\big(F(x) - F(a)\big) = f(x)$$

Note that $F(a) =$ constant, so it's derivative is 0.

Since $f(t) = \cos(t^2)$ and $f(x) = \cos(x^2)$, we get that $\boldsymbol{k'(x) = \cos(x^2)}$

4. Evaluate the following integrals.

(a) $\int_1^2 2x(1-x^{-3})dx$

Answer: 2

Solution: $\int_1^2 2x(1-x^{-3})dx \; = \; \int_1^2 2x - 2x^{-2}dx \; = \; \frac{2x^2}{2} - \frac{2x^{-1}}{-1}\Big|_1^2 = \; x^2 + 2x^{-1}\Big|_1^2$

$$= \; (2)^2 + 2(2)^{-1} - [1^2 + 2(1)^{-1}] = \mathbf{2}$$

(b) $\int \frac{1}{5}\sec(x)\tan(x) - 11 \; dx$

Answer: $\frac{1}{5}\sec(x) - 11x + C$

Solution: $\int \frac{1}{5}\sec(x)\tan(x) - 11 = \boldsymbol{\frac{1}{5}\sec(x) - 11x + C}$

(c) $\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)dx$

Answer: $\cos\left(\frac{1}{x}\right) + C$

Solution: For this integral, we must use a substitution:

Let $u = \frac{1}{x} = x^{-1}$, and $du = -x^{-2}dx \; \Rightarrow -du = \frac{1}{x^2}dx$ Then, we get:

$$\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)dx = -\int \sin(\mathrm{u})\, du \; = -(-\cos(u)) + C = \boldsymbol{\cos\left(\frac{1}{x}\right) + C}$$

5. Find the slope of the tangent line to the curve given by $\tan(xy) = x + y$ at the point $(0, 0)$.

Answer: $y = x$

Solution: Using implicit differentiation and solve for $\frac{dy}{dx}$:

$$\frac{d}{dx}(\tan(xy) = x + y) \Rightarrow \sec^2(xy)\left(y + x\cdot\frac{dy}{dx}\right) = 1 + \frac{dy}{dx} \Rightarrow y\sec^2(xy) + x\sec^2(xy)\left(\frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$$

$$\Rightarrow \; x\sec^2(xy)\left(\frac{dy}{dx}\right) - \frac{dy}{dx} = 1 - y\sec^2(xy) \;\; \Rightarrow \;\; (x\sec^2(xy) - 1)\frac{dy}{dx} = 1 - y\sec^2(xy) \;\; \Rightarrow \; \frac{dy}{dx} = \frac{1 - y\sec^2(xy)}{x\sec^2(xy) - 1}$$

Now, plug in (0,0) for x and y in the $\frac{dy}{dx}$ equation to obtain the slope, m:

$$\frac{dy}{dx} = \frac{1 - (0)\sec^2(0)}{(0)\sec^2(0) - 1} \; = \frac{1}{1} = 1 = m$$

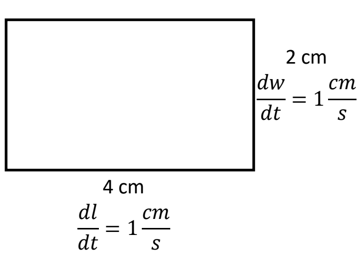
Use the point-slope equation to obtain the equation of the tangent line:

$$y - y_1 = m(x - x_1) \; \rightarrow \; y - (0) = (1)\big(x - (0)\big) \Rightarrow \boldsymbol{y = x}$$

6. A rectangle initially has dimensions 2 cm by 4 cm. All four sides begin increasing in length at a rate of $1\,\frac{\text{cm}}{\text{s}}$. At what rate is the area of the rectangle increasing after 20 seconds?

Answer: $46\,\frac{\text{cm}^2}{\text{s}}$

Solution: A sketch of the situation is shown below. Recall that the area of a rectangle is given by $A = lw$. Differentiating



this equation with respect to time gives: $\frac{d}{dt}(A = lw) \;\; \Rightarrow \;\; \frac{dA}{dt} = \frac{dl}{dt}w + l\frac{dw}{dt}$

Plugging in $\frac{dl}{dt} = \frac{dw}{dt} = 1,\; l = 24,$ and $w = 22$ (looking at the length and width after 20 seconds) gives:

$$\frac{dA}{dt} = 1(22) + 1(24) = 46\,\frac{\text{cm}^2}{\text{s}}$$

Therefore, the area is increasing at a rate of $46\,\frac{cm^2}{s}$

7. Sketch the graph of a single function $f(x)$ which satisfies all of the following conditions. Label all local maxima and minima, intervals of increase and decrease, points of inflection, concavity, and asymptotes.

(i) $f(x)$ is defined for all real numbers.

(ii) $f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}$

(iii) $f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3}$

(iv) $\lim_{x\to\infty} f(x) = 1$

(v) $\lim_{x\to-\infty} f(x) = 1$

(vi) $f(-1) = 0$

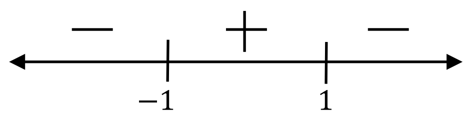
(vii) $f(1) = 2$

Solution: We know from condition (i) that we must have a point for every value of x, since x is defined for all real numbers.

Condition (ii) tells us about intervals of increase and decrease. Since we are already given the first derivative, we need to solve for the zeros and critical numbers, and then make a sign chart. Positive values will indicate areas of increasing, while negative values will indicate areas of decreasing. Zeros will indicate a maximum or minimum. The critical point will represent the vertical asymptote.

$$f'(x) = \frac{2(1-x^2)}{(1+x^2)^2} = 0 \;\Rightarrow\; 2(1-x^2) = 0 \;\Rightarrow\; 1 - x^2 = 0 \;\Rightarrow\; x^2 = 1 \;\Rightarrow\; x = -1, 1$$

Setting the denominator equal to 0 gives the critical value: $1 + x^2 = 0 \;\Rightarrow\; x^2 = -1$
Since the solution would be imaginary, there are no "bad" points.

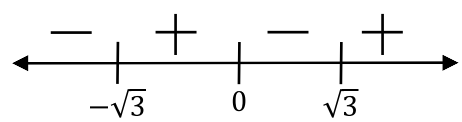


Therefore, the function is **increasing from $(-1, 1)$ and decreasing from** $(-\infty, -1) \cup (1, \infty)$. There is a **maximum at $x = 1$ and a minimum at $x = -1$.**

Condition (iii) tells us about concavity. Since we are already given the second derivative, we need to solve for the zeros and critical numbers, and then make a sign chart. Positive values will indicate concave up, while negative values will indicate concave down. Zeros will be points of inflection, while the critical number will again be our vertical asymptote.

$f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3} = 0 \;\Rightarrow\; 4x(x^2 - 3) = 0$

Setting each piece equal to zero and solving for x gives:

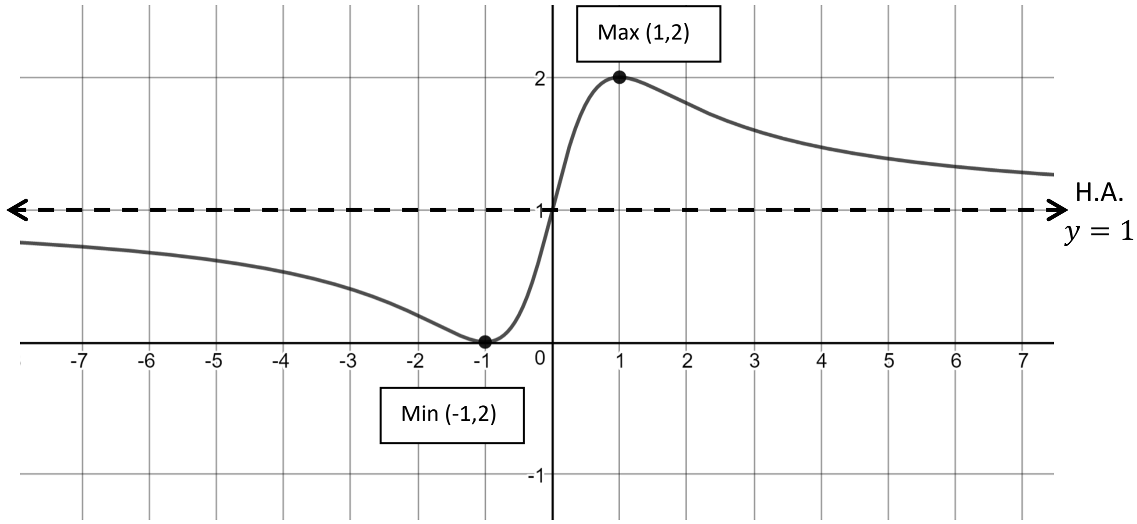


$4x = 0 \;\Rightarrow\; x = 0$

$x^2 - 3 = 0 \;\Rightarrow\; x^2 = 3 \;\Rightarrow\; x = \pm\sqrt{3}$

Again, the denominator will never be zero.
Therefore, the function is **concave up from $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$, and concave down from $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$. Points of infection are at $x = -\sqrt{3}$, $x = 0$, and $x = \sqrt{3}$**

Conditions (iv) and (v) tell us the end behavior of our function. Since both limits approach 1, we know that there must be a horizontal asymptote at $y = 1$. Conditions (vi) and (vii) give points on the graph.



8. What is the largest possible product of two nonnegative real numbers whose sum is 23?
Answer: $x = 11.5$ and $y = 11.5$
Solution: We have two positive numbers x and y such that $x + y = 23$, and a product of $z = xy$. We want to maximize the product, z. First, we must get z in terms of x by plugging in $y = 23 - x$:

$$z = xy \xrightarrow{y=23-x} z = x(23 - x) \Rightarrow z = 23x - x^2$$

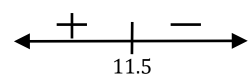
Now that the expression is in terms of only one variable, we are ready to maximize it. In order to do so, we must first take the derivative with respect to x:

$$\frac{d}{dx}(23x - x^2) = 23 - 2x$$

Set the derivative equal to 0, and solve for x:

$$23 - 2x = 0 \Rightarrow 23 = 2x \Rightarrow x = \frac{23}{2} = 11.5$$

To verify that this is indeed a maximum, we can use a sign chart:
Since the function increases and then decreases, it is indeed a maximum.



Now, we need to solve for y:

$$y = 23 - x \xrightarrow{y=11.5} x = 23 - 11.5 \Rightarrow x = 11.5$$

Therefore, the values of x and y that maximize the product and satisfy $x + y = 23$ are

$$\boldsymbol{x = 11.5 \text{ and } y = 11.5}$$

9. A person with full lungs begins to exhale at $t = 0$. The rate at which air flows out of their lungs is $\frac{\pi}{2}\sin(t)$ liters per second. How much air flows out after exhaling for 2 seconds?
Answer: 2.2 liters of air flows out after 2 seconds
Solution: Let $f(t)$ = the amount of air flowing out of lungs at time t. Then, $f(t) = \int_0^t r(t)dt$. Since we know $t = 2$ and $r(t) = \frac{\pi}{2}\sin(t)$, we get:

$$f(2) = \int_0^2 \frac{\pi}{2}\sin(t)\ dt = -\frac{\pi}{2}\cos(t)\Big|_0^2 = -\frac{\pi}{2}\cos(2) - \left[-\frac{\pi}{2}\cos(0)\right] = -\frac{\pi}{2}\cos(2) + \frac{\pi}{2}$$

$\approx$ **2.2 liters of air flows out after 2 seconds.**

10. Let b represent the base diameter of a conifer tree measured in centimeters and h the height of the tree in meters. Then h is related to b by:

$$h = 5.67 + 0.70b + 0.0067b^2$$

(a) Find the total change in the height of the tree when the diameter of the base increases from 10 cm to 15 cm. Give your answer to four decimal places, including proper units.

Answer: 4.3375 meters

Solution: To find the total change in height when the diameter of the base increases from 10 cm to 15 cm, we need to subtract the final height from the initial height:

$$h(15) - h(10) = [5.67 + 0.70(15) + 0.0067(15)^2] - [5.67 + 0.70(10) + 0.0067(10)^2]$$
$$= \mathbf{4.3375\ meters}$$

(b) Find the average rate at which the height of the tree changes when the diameter of the base increases from 10 cm to 15 cm. Give your answer to four decimal places, including proper units.

Answer: 0.8675 meters/cm

Solution: Using the formula,

$$Average\ rate\ of\ change\ from\ a\ to\ b = \frac{h(b) - h(a)}{b - a}$$

with $a = 10$ and $b = 15$, we get:

$$\frac{h(b)-h(a)}{b-a} = \frac{h(15)-h(10)}{15-10} = \frac{4.3375}{5} = \mathbf{0.8675\ meters/cm}$$

(c) Find the instantaneous rate of change in the height of the tree when the diameter of the base is 10 cm. Give your answer to four decimal places, including proper units.

Answer: $0.8340 \frac{meters}{cm}$

Solution: To find the instantaneous rate of change, first take the derivative of $h = 5.67 + 0.70b + 0.0067b^2$

$$h' = 0.70 + 0.0134b$$

Now, plug in $b = 10$:

$$h'(10) = 0.70 + 0.0134(10) = \mathbf{0.8340} \frac{\boldsymbol{meters}}{\boldsymbol{cm}}$$

11. A drag racer, starting from a standstill, can reach a velocity of 330 mph in 4.45 seconds. In other words, if $v(t)$ is the velocity of the car at time t, then $v(0) = 0$ and $v(4.45) = 330$. Assuming that $v(t)$ is continuous and differentiable, use the Mean Value Theorem to find a value for the acceleration $v'(t)$ of the car (in mph/s) that you know must be attained somewhere between $t = 0$ and $t = 4.45$.

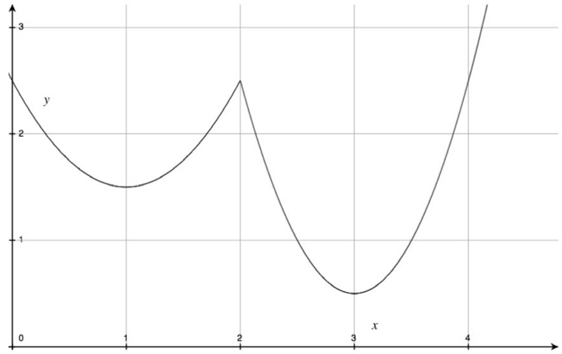
Answer: $v'(t) = 74.1573 \frac{\text{mph}}{\text{s}}$

Solution: Recall the Mean Value Theorem: *If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is at least one number c on (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.*

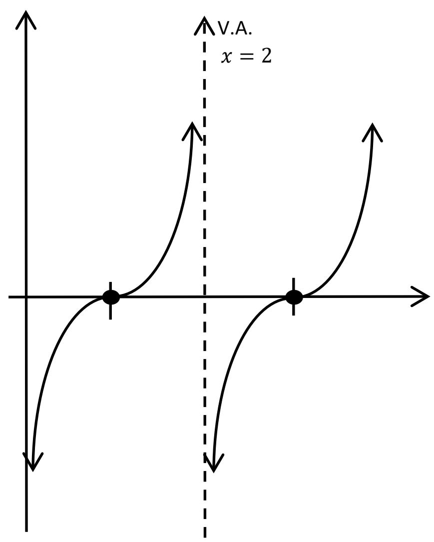
Since our function is continuous and differentiable, we need to find the value of $v'(c)$.

$$f'(c) = \frac{f(b) - f(a)}{b - a} \Rightarrow v'(c) = \frac{v(4.45) - v(0)}{4.45 - 0} \Rightarrow v'(t) = \frac{330 - 0}{4.45} \Rightarrow \boldsymbol{v'(t) = 74.1573 \frac{\text{mph}}{\text{s}}}$$

12. The graph of a function $f(x)$ is shown below. Sketch the graph of the derivative $f'(x)$, showing clearly where $f'(x)$ is positive and negative, and intervals where $f'(x)$ increases or decreases.



Solution: Recall that the graph of the derivative, $f'(x)$, is a graph of the slopes of the function $f(x)$. Since the slope of $f(x)$ is positive from $(1,2) \cup (3,4)$, we know that the values of $f'(x)$ must be positive in that interval. Since the slope of $f(x)$ is negative from $(0,1) \cup (2,3)$, we know the values of $f'(x)$ must be negative on those intervals. The concavity of $f(x)$ tells us where the function $f'(x)$ is increasing or decreasing. Since $f(x)$ is concave up from $(0,2) \cup (2,4)$, the graph of $f'(x)$ should be increasing on those intervals. Notice that $f(x)$ is never concave down, so the slope on the graph of $f'(x)$ should never be decreasing. Since $f'(x)$ has a corner at $x = 2$, the graph of $f'(x)$ should have a vertical asymptote there. Since there are minimums at $x = 1$ and $x = 3$, the graph of $f'(x)$ will have x-intercepts at those points. A graph is given below:

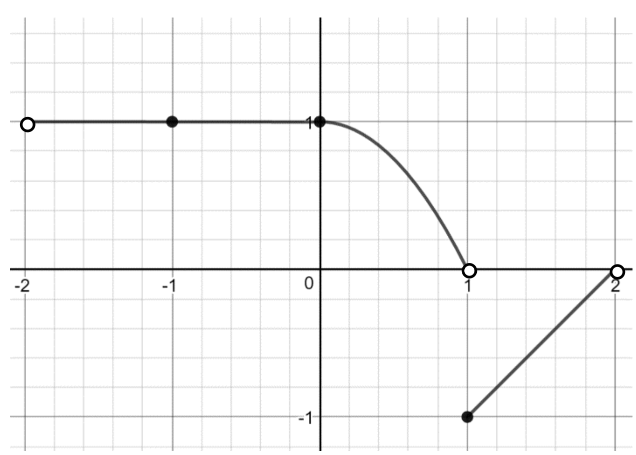


13. Let

$$f(x) = \begin{cases} 1 & if \ -2 < x \le 0 \\ 1 - x^2 & if \ \ 0 < x < 1 \\ x - 2 & if \ \ 1 \le x < 2 \end{cases}$$

(a) Give a careful graph of f on the interval $(-2, 2)$.
Solution: In order to graph the function, plot points:

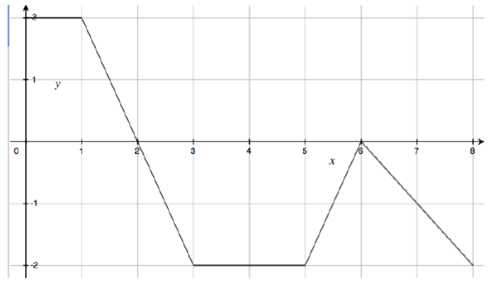
$f(x) = 1$	
-2	1
-1	1
0	1
$f(x) = 1 - x^2$	
0	1
1	0
$f(x) = x - 2$	
1	-1
2	0



(b) Find all values of x in $(-2, 2)$ at which f is not continuous.
Answer: $x = 1$
Solution: The function is not continuous at $\boldsymbol{x = 1}$ because $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$.

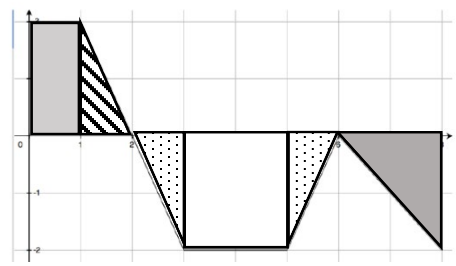
(c) Find all values of x in $(-2, 2)$ at which f is not differentiable, i.e., for which $f'(x)$ does not exist.
Answer: $x = 1$
Solution: the function is not differentiable at $\boldsymbol{x = 1}$ because it is discontinuous there.

14. The graph of the function f is shown below



The function g is defined by

$$g(x) = \int_0^x f(t)\,dt, \quad 0 \le x \le 8$$



a. Find $g(6)$.
Answer: $g(6) = -3$
Solution:

$$g(6) = \int_0^6 f(t)dt$$
$$= A_{rectangle} + A_{striped\ triangle} + A_{square} + 2A_{dotted\ triangle}$$
$$= (1)(2) + \frac{1}{2}(1)(2) + (2)(-2) + 2[\frac{1}{2}(1)(-2)] = \mathbf{-3}$$

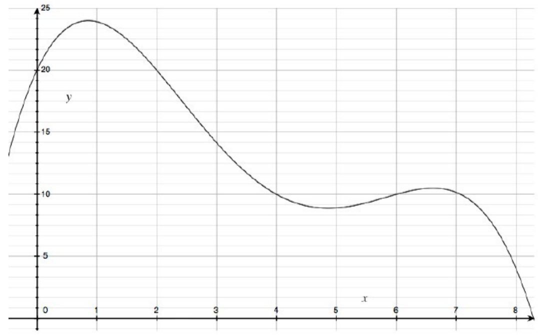
b. For which values on the interval $(0,8)$ does $g(x)$ have a local maximum (if any)?
Answer: $x = 2$
Solution: Since $g(x)$ is the antiderivative of $f(t)$, any places on $f(t)$ which are x-intercepts would correspond to local maxima on $g(x)$. Therefore, there must be local maxima at $x = 2$ and $x = 6$. Since $f(t)$ is the graph of the derivative of $g(x)$, it is depicting the slopes of $g(x)$. We know a maximum occurs when the function goes from increasing to decreasing – or, in terms of our graph, from positive to negative values. Therefore, there is a local maximum at $\boldsymbol{x = 2}$**.**

c. For which values of x in the interval $(0, 8)$ is $g(x) = 0$ (if any)?
Answer: $x = 4$
Solution: In order for the integral to be zero, the area above must be equal to the area below. This happens on the interval from $(0, 4)$. Notice how there would be a rectangle of area 2 plus a triangle of area 1 on the top, and a rectangle of area 2 plus a triangle of area one on the bottom.

$$g(4) = \int_0^4 f(t)dt = A_{rectangle} + A_{striped\ triangle} + \frac{1}{2}A_{square} + A_{dotted\ triangle}$$
$$= 2(1) + \frac{1}{2}(1)(2) + \frac{1}{2}(2)(-2) + \frac{1}{2}(1)(-2) = 2 + 1 - 2 - 1 = 0$$

Therefore, $g(x) = 0$ when $\boldsymbol{x = 4}$**.**

15. The graph of a function $f(x)$ is shown below. Using a Riemann sum with four terms and right endpoints (also known as R_4) estimate the value of $\int_0^8 f(x)\ dx$.



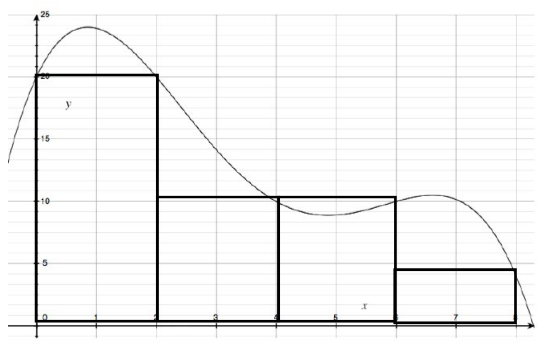
Answer: $R_4 = 88$

Solution: First, we need to find the width of our rectangles using the formula $\Delta x = \frac{b-a}{n}$. Since we are interested in using four terms, $n = 4$. The limits on the integral give $a = 0$ and $b = 8$. Therefore, $\Delta x = \frac{b-a}{n} = \frac{8-0}{4} = 2$.

Now, we can make a table with values needed to calculate the Riemann sum:

i	$x_i = a + i\Delta x$	$f(x_i)$
0	$x_0 = 0 + 0(2) = 0$	$f(0) = 20$
1	$x_1 = 0 + 1(2) = 2$	$f(2) = 20$
2	$x_2 = 0 + 2(2) = 4$	$f(4) = 10$
3	$x_3 = 0 + 3(2) = 6$	$f(6) = 10$
4	$x_4 = 0 + 4(2) = 8$	$f(8) = 4$

We are told to use right endpoints. Therefore, we get:



$$\int_0^8 f(x)\, dx \approx R_4 = \Delta x[f(x_1) + f(x_2) + f(x_3) + f(x_4)] = 2[20 + 10 + 10 + 4] = \mathbf{88}$$