MAT 2010 Final Exam – Fall 2016

1. Use the **definition** of derivative to differentiate the following function.

$$f(x) = \sqrt{x} - 1$$

2. Find the exact value of each of the following limits. Write "∞," "−∞," or "does not exist" if appropriate. It is particularly important to show your work on this problem.
   (a) $\lim_{x \to -1} \frac{(2x-1)^2 - 9}{x+1}$

   (b) $\lim_{x \to \infty} x^2 e^{-x}$

   (c) $\lim_{x \to 3^+} \frac{x-1}{x-3}$

3. Find the derivative $\frac{\mathrm{d}}{\mathrm{dx}}$ for each of the following functions.
   (a) $f(x) = \sqrt[3]{x^2 - 2x + 2}$

   (b) $g(x) = e^x \cdot \tan(x)$

   (c) $h(x) = \frac{x^2 - 7x}{\ln(x)}$

   (d) $k(x) = \int_0^x cos(t^2)\, dt$

4. Evaluate the following integrals.
   (a) $\int_1^2 2x(1 - x^{-3})dx$

   (b) $\int \frac{1}{5} \sec(x)\tan(x) - 11 \ dx$

   (c) $\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$

5. Find the slope of the tangent line to the curve given by $\tan(xy) = x + y$ at the point $(0, 0)$.

6. A rectangle initially has dimensions 2 cm by 4 cm. All four sides begin increasing in length at a rate of $1 \frac{\mathrm{cm}}{\mathrm{s}}$. At what rate is the area of the rectangle increasing after 20 seconds?

7. Sketch the graph of a single function $f(x)$ which satisfies all of the following conditions. Label all local maxima and minima, intervals of increase and decrease, points of inflection, concavity, and asymptotes.
   (i) $f(x)$ is defined for all real numbers.
   (ii) $f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}$
   (iii) $f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3}$
   (iv) $\lim_{x \to \infty} f(x) = 1$

(v) $\lim_{x \to -\infty} f(x) = 1$

(vi) $f(-1) = 0$

(vii) $f(1) = 2$

8. What is the largest possible product of two nonnegative real numbers whose sum is 23?

9. A person with full lungs begins to exhale at $t = 0$. The rate at which air flows out of their lungs is $\frac{\pi}{2}\sin(t)$ liters per second. How much air flows out after exhaling for 2 seconds?

10. Let b represent the base diameter of a conifer tree measured in centimeters and h the height of the tree in meters. Then h is related to b by:

$$h = 5.67 + 0.70b + 0.0067b^2$$

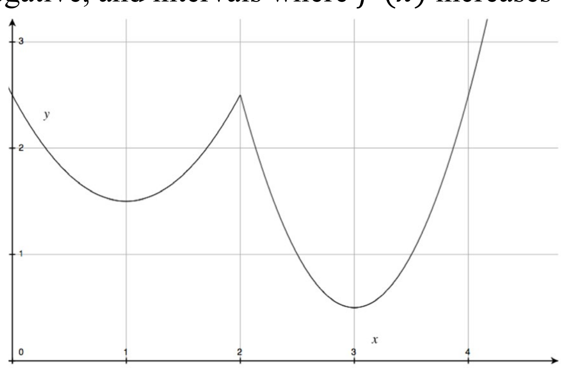
(a) Find the total change in the height of the tree when the diameter of the base increases from 10 cm to 15 cm. Give your answer to four decimal places, including proper units.

(b) Find the average rate at which the height of the tree changes when the diameter of the base increases from 10 cm to 15 cm. Give your answer to four decimal places, including proper units.

(c) Find the instantaneous rate of change in the height of the tree when the diameter of the base is 10 cm. Give your answer to four decimal places, including proper units.

11. A drag racer, starting from a standstill, can reach a velocity of 330 mph in 4.45 seconds. In other words, if $v(t)$ is the velocity of the car at time t, then $v(0) = 0$ and $v(4.45) = 330$. Assuming that $v(t)$ is continuous and differentiable, use the Mean Value Theorem to find a value for the acceleration $v'(t)$ of the car (in mph/s) that you know must be attained somewhere between $t = 0$ and $t = 4.45$.

12. The graph of a function $f(x)$ is shown below. Sketch the graph of the derivative $f'(x)$, showing clearly where $f'(x)$ is positive and negative, and intervals where $f'(x)$ increases or decreases.



13. Let

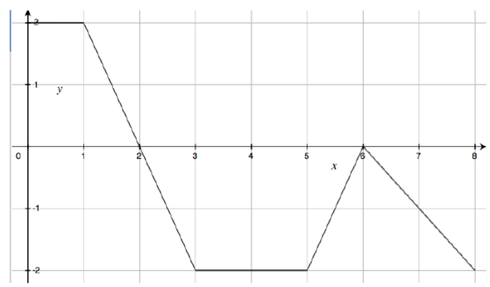
$$f(x) = \begin{cases} 1 & if \quad -2 < x \le 0 \\ 1 - x^2 & if \quad 0 < x < 1 \\ x - 2 & if \quad 1 \le x < 2 \end{cases}$$

(a) Give a careful graph of f on the interval $(-2, 2)$.

(b) Find all values of x in $(-2, 2)$ at which f is not continuous.

(c) Find all values of x in $(-2, 2)$ at which f is not differentiable, i.e., for which $f'(x)$ does not exist.

14. The graph of the function f is shown below



The function g is defined by

$$g(x) = \int_0^x f(t)\, dt, \quad 0 \le x \le 8$$

a. Find $g(6)$.

b. For which values on the interval $(0,8)$ does $g(x)$ have a local maximum (if any)?

c. For which values of x in the interval $(0, 8)$ is $g(x) = 0$ (if any)?

15. The graph of a function $f(x)$ is shown below. Using a Riemann sum with four terms and right endpoints (also known as R_4) estimate the value of $\int_0^8 f(x)\, dx$.