FINAL EXAMINATION, MAT 2010
December 13, 2019

NOTE: Write your solutions in a blue book. To receive full credit you must show *all* work. You are allowed to use an *approved* graphing calculator unless otherwise indicated. Simplify your answer when possible, but use the precise value rather than an approximation when you have a choice. (Example: If the actual answer is π, then write π, not 3.14.) The 12 problems are worth a total of 150 points. The time limit is 2 hours [120 minutes].

Cell phones are strictly prohibited!

1. (10 points) Use the **definition** of the derivative to differentiate the following function.

$$f(x) = \frac{x}{x+3}$$

2. (7 points each) Find the exact value of each of the following limits. Write "∞," "$-\infty$," or "does not exist" if appropriate. It is particularly important to show your work on this problem.

   (a) $\lim\limits_{x\to 3} \dfrac{x^3 - 9x}{x^2 - 2x - 3}$

   (b) $\lim\limits_{x\to\infty} \dfrac{3 - 2x^2 + 5x^4}{2x^4 - 5}$

   (c) $\lim\limits_{x\to 0} \dfrac{\tan(\sqrt{x})}{\sqrt{x}}$

3. (7 points each) Differentiate the following functions. Simplify your answer.

   (a) $f(x) = \dfrac{1 + x^2}{\arctan(x)}$

   (b) $g(x) = [\sec(3x)]^5$

4. Evaluate. Simplify your answer.

   (a) (7 points) $\displaystyle\int \sec x \Big[\sec x + \tan x\Big]\, dx$

   (b) (8 points) $\displaystyle\int_0^1 \Big[3\sqrt{t} - 2\, e^t\Big]\, dt$

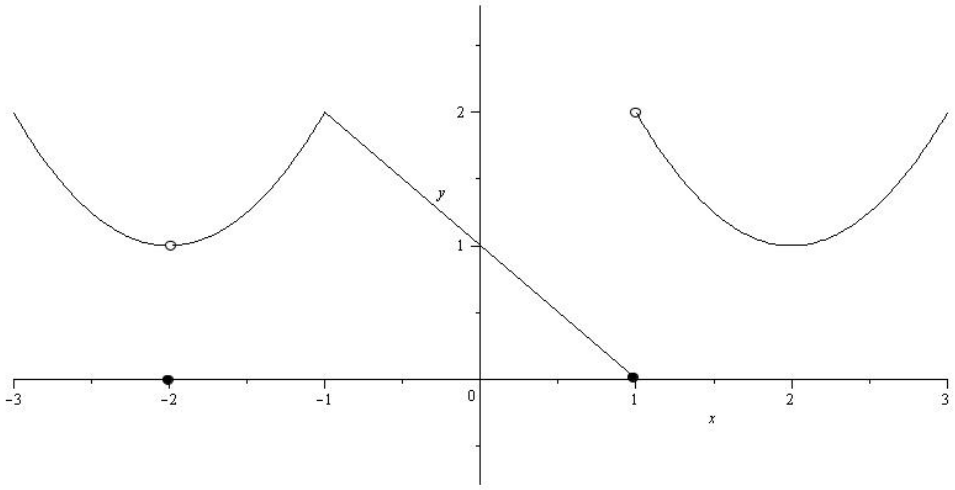
5. (10 points) Consider the equation

$$y^4 + xy = x^3 - x + 2.$$

(a) Find $\dfrac{dy}{dx}$.

(b) Find the equation of the tangent line at the point $(1, 1)$.

6. (10 points) The graph of a function $f(x)$ is given below.



(a) For which values of x in the interval $(-3, 3)$ is f not continuos? Give the name of each discontinuity.

(b) For which values of x in the interval $(-3, 3)$ is f not differentiable?

(c) Give values of (i) $\lim_{x \to 1^-} f(x)$ (ii) $\lim_{x \to 1^+} f(x)$ (iii) $\lim_{x \to -1} f(x)$

7. (10 points) On a typical day, a city consumes water at the rate of $r(t) = 100 + 72t - 3t^2$ gallons per hour, where t is the number of hours past midnight. How much water is consumed between 6 A.M. and 9 A.M.?

8. (10 points) The position function of a spring in motion is given by,

$$s(t) = 2\,e^{-1.5t}\,\sin(2\pi\,t),$$

where s is measured in centimeters and t in seconds.

(a) Find the velocity of the spring after t seconds.

(b) Find the velocity of the spring after 2 seconds. Give your answer correct to three decimal places. Include proper units.

9. (10 points) The base of a triangle is decreasing at a rate of 2 ft/min and the height of the triangle is increasing at the rate of 1.5 ft/min. How fast is the area of the triangle changing when the base of the triangle is 5 ft long and height is 3 ft?

10. (10 points) Find the absolute minimum and absolute maximum values of $f(x) = x - \ln x$ on the interval $\left[\frac{1}{2}, 2\right]$.

11. (10 points) Using a Riemann sum with $n = 4$ subintervals, find the overestimate (i.e. upper Riemann sum) of the area of the region bounded above by the function $f(x) = 2 + \sqrt{2x}$ and below by the x-axis on the interval $[0, 2]$.

12. (20 points) Sketch the graph of the function $f(x)$ which satisfies the following conditions. Using interval notation list all intervals where the function f is decreasing, increasing, concave up, and concave down. List the x- coordinates of all local maxima and minima, and points of inflection. Show asymptotes with dashed lines and give their equations. Label all important points on the graph.

(i) $f(x)$ is defined for all real numbers

(ii) $f'(x) = (x^2 - 2x - 3)\,e^x$

(iii) $f''(x) = (x^2 - 5)\,e^x$

(iv) $f(0) = 3$

(v) $\lim_{x\to\infty} f(x) = \infty$

(vi) $\lim_{x\to -\infty} f(x) = 2$