MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN IN THE PROVIDED BLUE BOOK.** Only minimal credit will be awarded for answers without supporting work. Each problem is worth 12 points except where indicated. **NO CALCULATORS ALLOWED.**

1. Sketch a graph of the function $f(x) = \begin{cases} x+1 & \text{if} \quad x \leq -3 \\ 2 & \text{if} \ -3 < x < 0 \\ x^2+2 & \text{if} \quad x \geq 0 \end{cases}$

2. Find the domain of the function $f(x) = \frac{e^x}{\log_2(x^2-9)-4}$

3. (8 points each) Let $f(x) = \sqrt{x-5}$ and $g(x) = \frac{3x}{x-2}$

a) Find and simplify $\frac{(f \circ g)(4)}{(g)(4)}$.

b) Find $g^{-1}\left(\frac{9}{4}\right)$.

4. Consider a rectangular box with a square base. If the base has an area of 100 ft^2, express the **surface area** of the entire box, S, as a function of its height h.

5. A small ball is thrown straight up in the air. The height, in feet, of the ball t seconds after being thrown is given by the function $h(t) = -\frac{1}{8}t^2 + 3t$. What is the maximum height of the ball?

6. Given that -3 is a root of the polynomial $2x^3 + 8x^2 + 9x + 9$, find all solutions to the equation $2x^3 + 8x^2 + 9x + 9 = 0$. Express any non-real solutions in the form $a + bi$.

7. Find the average rate of change of the function $f(x) = 3x^2 + 2x$ from $x = 2$ to $x = 2 + h$. Simplify your answer completely.

8. (14 points) Graph the function $f(x) = x^4 - 2x^3 - 24x^2$. **Label all intercepts and asymptotes.**

9. Graph $g(x) = -\log_2(x+1)$. **Label all intercepts and asymptotes.**

10. (4 points each) Simplify each expression completely.

a) $\log_4 \sqrt{8}$

b) $e^{\ln(10)+\frac{1}{2}\ln(4)}$

11. The number of leaves that have fallen to the ground, t hours after a windstorm begins, increases exponentially according to the function $A(t) = A_0 e^{rt}$. There are initially 40 leaves on the ground, and after 2 hours there are 60 leaves on the ground. How many leaves will be on the ground after 6 hours?

12. (6 points each) Evaluate each of the following.

a) $\sec(\frac{11\pi}{4})$ b) $\tan(-\frac{2\pi}{3})$ c) $\sin^{-1}(\sin(\frac{3\pi}{4}))$

13. Consider the function $f(x) = 3\cos(2x) + 1$

a) (4 points) State the period and amplitude of the function.

b) (8 points) Graph one period of the function, labeling the highest and lowest points.

14. Given that $\cot(\theta) = \frac{1}{2}$ and $\cos(\theta) < 0$, find $\sin(2\theta)$.

15. Find all primary solutions $(0 \le \theta < 2\pi)$ of the trigonometric equation

$$2\sin^2(\theta) + \sin(\theta) - 1 = 0$$

16. Prove the identity: $\sec(x) - \cos(x) = \tan(x) \cdot \sin(x)$