Problem 1

Part(a):

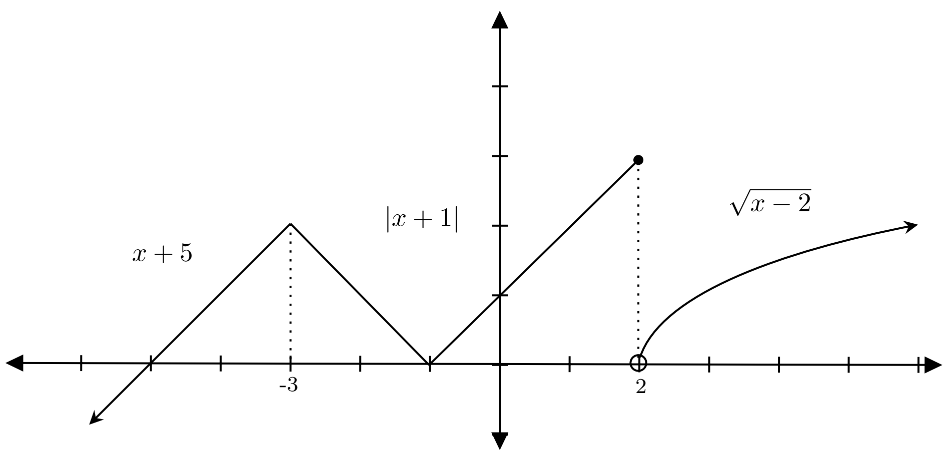
$$\frac{g(1)}{(g\circ f)(2)} = \frac{g(1)}{g(f(2))} = \frac{-4/8}{-6/8} = \frac{4}{6} = \frac{2}{3} \text{ becuase,}$$

$$g(1) = \frac{1-5(1)}{8(1)} = -\frac{4}{8},\ f(2) = 2^2 + 4(2) - 13 = -1,\ g(f(2)) = g(-1) = \frac{1-5(-1)}{8(-1)} = -\frac{6}{8}$$

Part(b): To find the inverse function of $g(x)$, replace the x's with the y's in $y = \frac{1-5x}{8x}$:

$$x = \frac{1-5y}{8y} \implies 8xy = 1-5y \implies 8xy + 5y = 1 \implies y(8x+5) = 1 \implies g^{-1}(x) = \frac{1}{8x+5}$$

Problem 2



Problem 3

The function $f(x)$ is not defined when the denominator is zero. If $\ln(1-x) = 0 \implies 1 - x = 1 \implies x = 0$, so for the domain of $f(x)$, $x \neq 0$. Next, logarithms are only defined for strictly positive values, so we must have $1 - x > 0 \implies x < 1$. Square roots cannot take negative values so we must have $5x + 7 \geq 0 \implies 5x \geq -7 \implies x \geq -\frac{7}{5}4$. Putting this all together we have the domain of $f(x)$ is $-\frac{7}{5} \leq x < 1$ and $x \neq 0$. In interval notation, $[-\frac{7}{5}, 0) \cup (0, 1)$.

Problem 4

$$\frac{g(x+h) - g(x)}{x+h-x} = \frac{\sqrt{3(x+h)} - \sqrt{3x}}{h} = \frac{\sqrt{3(x+h)} - \sqrt{3x}}{h} \cdot \frac{\sqrt{3(x+h)} + \sqrt{3x}}{\sqrt{3(x+h)} + \sqrt{3x}}$$

$$= \frac{3(x+h)-3x}{h\left(\sqrt{3(x+h)}+\sqrt{3x}\right)} = \frac{3h}{h\left(\sqrt{3(x+h)}+\sqrt{3x}\right)} = \frac{3}{\sqrt{3(x+h)}+\sqrt{3x}}$$

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Problem 5

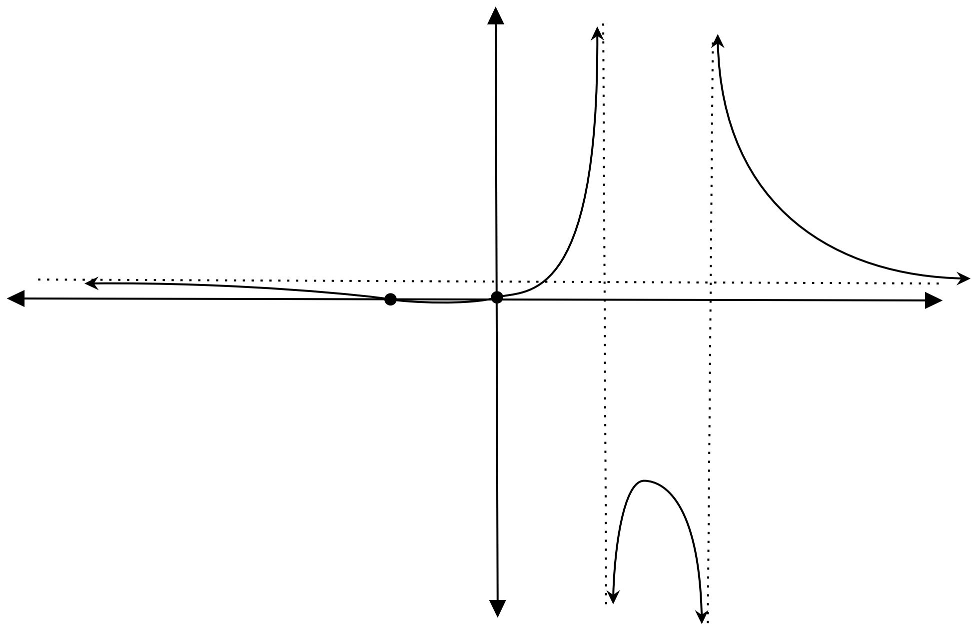
Since $x = -2$ is a zero of the polynomial, $(x+2)$ is a factor. Use polynomial long division to find the other factors.

$$\begin{array}{r|l}
 & \quad x^2 - 2x - 15 \\ \hline
x+2 & \; x^3 + 0x^2 - 19x - 30 \\
 & \underline{-x^3 - 2x^2} \\
 & \qquad -2x^2 - 19x \\
 & \qquad \underline{+2x^2 + 4x} \\
 & \qquad\qquad -15x - 30 \\
 & \qquad\qquad \underline{+15x + 30} \\
 & \qquad\qquad\qquad\quad 0 \quad \checkmark
\end{array}$$

So, $p(x) = x^3 - 19x - 30 = (x+2)(x^2-2x-15) = (x+2)(x-5)(x+3)$ with zeroes $x = -2, -3, 5$.

Problem 6

Part(a): First, factor $f(x)$ as $\frac{x(x+2)}{(x-4)(x-2)}$. There are no removable holes since there is no common factor in the numerator and denominator. So, the vertical asymptotes for this rational function will occur when the denominator is zero. Solving $x^2 - 6x + 8 = 0 \implies (x-4)(x-2) = 0 \implies x = 4, 2$. Thus $f(x)$ has vertical asymptotes at $x = 2$ and $x = 4$. Since the degree of the numerator is equal to the degree of the denominator (highest power of both are 2), there is a horizontal asymptote that is equal to the leading coefficient of the numerator over the leading coefficient of the denominator. So, $f(x)$ has a horizontal asymptote $y = \frac{1}{1} = 1$.
Part(b): Sketch of $f(x)$ from information in part(a) and test values:



Problem 7

$$\log_2(x)-\log_2(1+x) = 3-\log_2(x+6) \implies \log_2(\frac{x}{1+x}) = 3-\log_2(x+6) \implies \log_2(\frac{x}{1+x})+\log_2(x+6) = 3$$

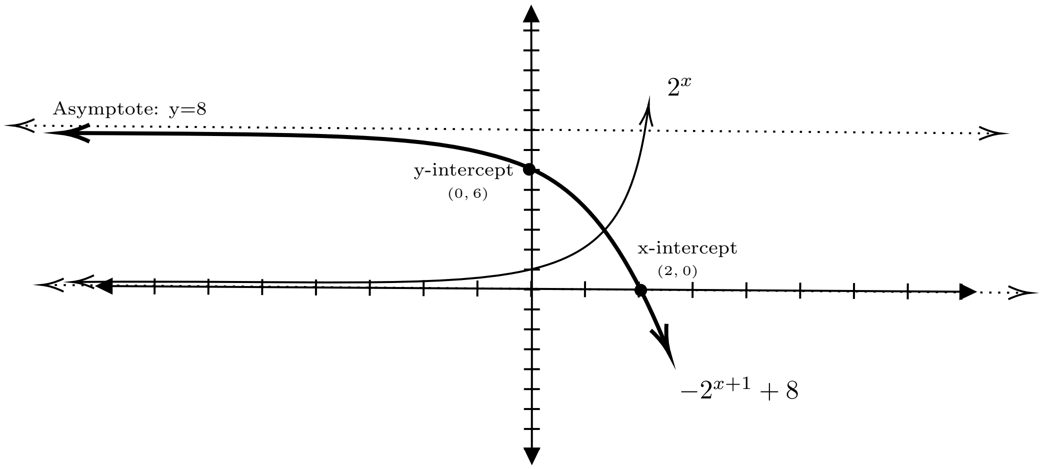
$$\implies \log_2(\frac{x(x+6)}{1+x}) = 3 \implies 2^{\log_2(\frac{x(x+6)}{1+x})} = 2^3 \implies \frac{x(x+6)}{1+x} = 8 \implies x^2+6x = 8+8x$$

$$\implies x^2-2x-8 = 0 \implies (x-4)(x+2) = 0 \implies x = 4, -2$$

Double check that both solutions are defined in the original equation. Notice, if $x = -2$, then $\log_2(-2)$ is not defined. So we get rid of that solution and our final answer is $x = 4$.

Problem 8

The parent function in this problem is 2^x. To graph $f(x)$, graph the parent function and apply all the transformations. First, the graph is shifted 1 unit to the left: 2^{x+1}. Then, it is reflected across the x-axis: -2^{x+1}. Last, it is shifted 8 units up: $-2^{x+1}+8$.



Problem 9

$$V = \text{base} \cdot \text{width} \cdot \text{height} = x^2 y \implies 10 = x^2 y \implies \frac{10}{x^2} = y$$

$$S = \text{bottom area} + \text{side areas} = x^2 + 4xy \implies S = x^2 + 4x\left(\frac{10}{x^2}\right) = x^2 + \frac{40}{x}$$

Problem 10

Part(a):

$$\log_4\left(\frac{2^{\frac{1}{3}}}{16}\right) = \log_4\left(\frac{(\sqrt{4})^{\frac{1}{3}}}{4^2}\right) = \log_4\left(\frac{4^{\frac{1}{3}\cdot\frac{1}{2}}}{4^2}\right) = \log_4\left(\frac{4^{\frac{1}{6}}}{4^2}\right) = \log_4\left(4^{\frac{1}{6}-2}\right)$$

$$= \log_4\left(4^{\frac{1}{6}-\frac{12}{6}}\right) = \log_4\left(4^{-\frac{11}{6}}\right) = -\frac{11}{6}$$

Part(b):

$$10^{\frac{1}{2}\log(36)+\log(9)} = 10^{\log(36^{\frac{1}{2}})+\log(9)} = 10^{\log(\sqrt{36})+\log(9)} = 10^{\log(6)+\log(9)}$$

$$= 10^{\log(6\cdot 9)} = 10^{\log(54)} = 10^{\log_{10}(54)} = 54$$

Problem 11

The general formula for exponential decay is $A = Pe^{r\cdot t}$, where A is the amount of material, P is the amount of material you start with, r is the rate of decay (or growth if exponential growth), and t for time.

$$50 = 400e^{r\cdot 6} \implies \frac{50}{400} = e^{r\cdot 6} \implies \ln\left(\frac{1}{8}\right) = \ln\left(e^{r\cdot 6}\right) \implies \ln\left(\frac{1}{8}\right) = r\cdot 6$$

$$\implies r = \frac{1}{6} \cdot \ln\left(\frac{1}{8}\right)$$

Now we use this r and solve the equation for t when the material is half (200 grams):

$$200 = 400e^{\frac{1}{6}\cdot\ln\left(\frac{1}{8}\right)\cdot t} \implies \frac{1}{2} = e^{\frac{1}{6}\cdot\ln\left(\frac{1}{8}\right)\cdot t} \implies \ln\left(\frac{1}{2}\right) = \ln\left(e^{\frac{1}{6}\ln\left(\frac{1}{8}\right)\cdot t}\right)$$

$$\implies \ln\left(\frac{1}{2}\right) = \frac{1}{6}\ln\left(\frac{1}{8}\right)t \implies \ln\left(\frac{1}{2}\right) = \frac{1}{6}\ln\left(\left(\frac{1}{2}\right)^3\right)t \implies \ln\left(\frac{1}{2}\right) = \ln\left(\left(\frac{1}{2}\right)^{3\frac{1}{6}t}\right)$$

$$\implies \ln\left(\frac{1}{2}\right) = \ln\left(\left(\frac{1}{2}\right)^{\frac{t}{2}}\right) \implies \frac{1}{2} = \left(\frac{1}{2}\right)^{\frac{t}{2}} \implies 1 = \frac{t}{2} \implies t = 2$$

Problem 12

Part(a):

$$\cot\left(\frac{19\pi}{3}\right) = \cot\left(\frac{\pi}{3}\right) = \frac{\cos(\frac{\pi}{3})}{\sin(\frac{\pi}{3})} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$

Part(b):

$$\cos^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right) = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

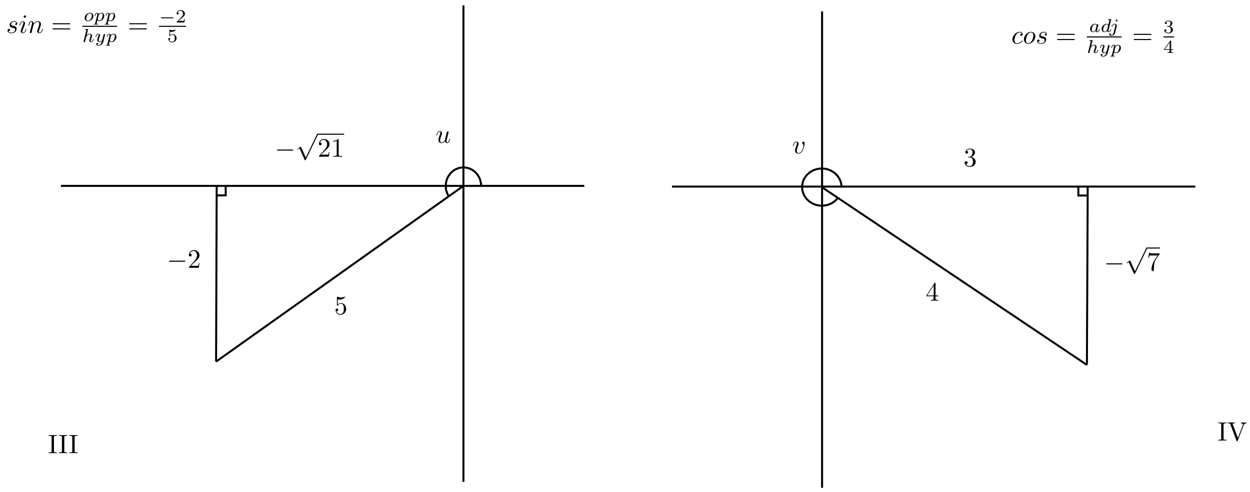
$$\cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right) = \frac{\pi}{2} - \sin^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right) = \frac{\pi}{2} - \left(-\frac{\pi}{4}\right) = \frac{3\pi}{4}$$

Problem 13

Rewrite the problem using the identity for sine angle addition:

$$\sin(u+v) = \sin(u)\cos(v) + \cos(u)\sin(v)$$

Then create triangles to find the values of $\sin(v)$ and $\cos(u)$ (as in the image):

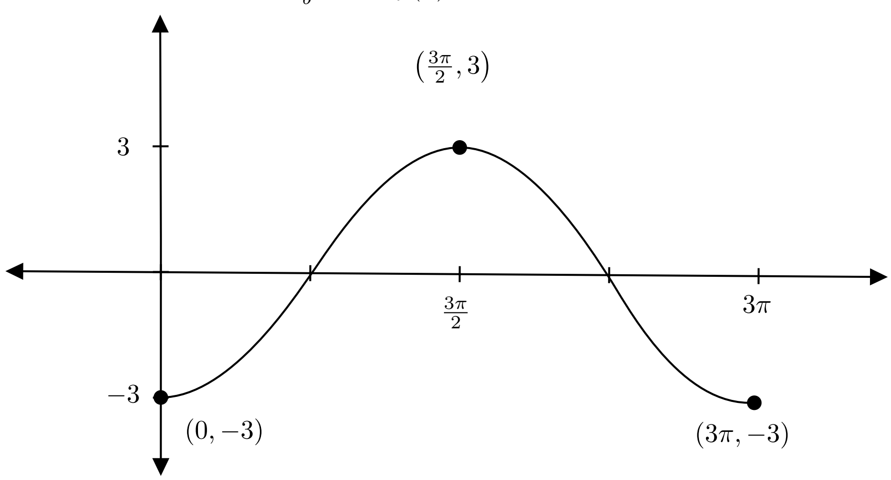


Then using all the information:

$$\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u) = \frac{-2}{5} \cdot \frac{3}{4} + \frac{-\sqrt{21}}{5} \cdot \frac{-\sqrt{7}}{4} = -\frac{3}{10} + \frac{147}{20}$$

Problem 14

In general, a wave function looks like $A\sin(bx) + c$ or $A\cos(bx) + c$, where $|A|$ is the amplitude, and the period is $\frac{2\pi}{b}$. So, $f(x)$ has an amplitude of 3 and a period of $\frac{2\pi}{\frac{2}{3}} = \frac{3}{2} \cdot 2\pi = 3\pi$.



Problem 15

$$\text{Solving the right hand side: } 1 - \sin^2(x) = \cos^2(x) = \cos(x)\cos(x) = \cos(x)\cos(x)\frac{\sin(x)}{\sin(x)}$$

$$= \sin(x)\cos(x)\frac{\cos(x)}{\sin(x)} = \frac{\sin(x)\cos(x)}{\tan(x)}$$

Problem 16

$$2\sin^2(x) + 3\cos(x) = 3 \implies 2(1-\cos^2(x)) + 3\cos(x) - 3 = 0 \implies -2\cos^2(x) + 3\cos(x) - 1 = 0$$

$$\text{substitution } u = \cos(x) \implies -2u^2 + 3u - 1 = 0 \implies (-2u+1)(u-1) = 0 \implies u = 1, \frac{1}{2}$$

$$\text{so, } 1 = \cos(x) \text{ gives } x = 0 \text{ and } \frac{1}{2} = \cos(x) \text{ gives } x = \frac{\pi}{3}, \frac{5\pi}{3}.$$