

## MAT 1800 FINAL EXAM

Instructions:

- Please read the directions to each problem carefully.
- Each problem is worth 12 points except where indicated. The Final Exam is worth a total of 200 points.
- Solutions should be written clearly and concisely on blank sheets of paper. **All work must be shown to receive full credit. Answers without correct supporting work will receive minimal credit.**
- **No outside assistance of any kind is allowed.** This includes using the internet to find answers, using your notes, having another person look at your work before submission, looking at another person's work before submission, and/or sharing information in any way while completing the exam.
- Calculators are not permitted.
- **Please have your camera on and your microphone muted.** Webcams are required to be on for the duration of the exam.
- **Please direct all questions to me privately in the chat window.**
- You will have 2 hours to complete the Final Exam and an additional 15 minutes to upload your work on Canvas.

1. (8 pts each) Let  $f(x) = x^2 + 4x - 13$  and  $g(x) = \frac{1-5x}{8x}$ . Find and simplify each of the following.

(a)  $\frac{g(1)}{(g \circ f)(2)}$

(b)  $g^{-1}(x)$

2. Sketch a graph of the function  $g(x) = \begin{cases} x + 5 & \text{if } x \leq -3 \\ |x + 1| & \text{if } -3 < x \leq 2 \\ \sqrt{x - 2} & \text{if } x > 2 \end{cases}$ .

3. Find the domain of the function  $f(x) = \frac{\sqrt{5x+7}}{\ln(1-x)}$ . State your answer in **interval notation**.

4. Find the average rate of change of the function  $g(x) = \sqrt{3x}$  from  $x = 3$  to  $x = 3 + h$  and **simplify your answer** so that no single factor of  $h$  is left in the denominator.

5. Consider the polynomial function  $p(x) = x^3 - 19x - 30$ . Given that  $x = -2$  is a zero of  $p(x)$ , find all other zeros.

6. Let  $f(x) = \frac{x^2+2x}{x^2-6x+8}$ .

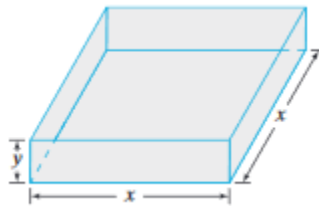
(a) Find all intercepts and asymptotes for  $f(x)$ .

(b) Sketch the graph of  $f(x)$ .

7. Solve the logarithmic equation:  $\log_2(x) - \log_2(1+x) = 3 - \log_2(x+6)$ .

8. Graph  $f(x) = -2^{x+1} + 8$  using transformations. Label all asymptotes and intercepts.

9. The figure below shows an open box (a box with no top) with a square base. The box has a volume of 10 cubic feet. Express the surface area,  $S$ , of the box as a function of the length of a side of its square base,  $x$ .



10.(8 pts each) Find the exact value of each expression.

(a)  $\log_4 \left( \frac{\sqrt[3]{2}}{16} \right)$

(b)  $10^{\frac{1}{2} \log(36) + \log(9)}$

11.If a sample of a radioactive isotope decays from 400 grams to 50 grams in 6 days, find the half-life of the isotope. **Simplify** your answer as much as possible.

12.Find the exact value of each trigonometric expression, if it exists.

(a)  $\cot \left( \frac{19\pi}{3} \right)$

(b)  $\cos^{-1} \left( \sin \left( -\frac{\pi}{4} \right) \right)$

13.Given that  $\sin u = -\frac{2}{5}$ ,  $u$  in quadrant III, and  $\cos v = \frac{3}{4}$ ,  $v$  in quadrant IV, find the exact value of  $\sin(u+v)$ .

**14.** State the amplitude and period length for the function  $f(x) = -3\cos\left(\frac{2}{3}x\right)$ . Graph this function over one complete period, labeling all high points and low points.

**15.** Verify that the trigonometric equation is an identity.

$$\frac{\sin x \cos x}{\tan x} = 1 - \sin^2 x$$

**16.** Find all primary solutions ( $0 \leq x < 2\pi$ ) of the equation  $2\sin^2(x) + 3\cos(x) = 3$ .