MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN IN THE PROVIDED BLUE BOOK.** Point values appear at the beginning of each problem and total 200. Only minimal credit will be awarded for answers without supporting work. **DO NOT USE A CALCULATOR.**

1. (6 pts each) Let \( f(x) = \frac{4}{x-1} \) and \( g(x)=3+6x \). Find and simplify each of the following.

   (a) \( (f \circ g)(x) \)
   
   (b) \( f^{-1}(x) \)

2. (12 pts) Let \( f(x) = \sqrt{3x-2} - \sqrt{x+3} \). Find all values of \( x \), if any, such that \( f(x) = 1 \).

3. (12 pts) Sketch a graph of the function \( f(x) = \begin{cases} 1-x & \text{if } x \leq -3 \\ [x+2] & \text{if } -3 < x \leq -1 \\ 6 & \text{if } x > 1 \end{cases} \)

4. (14 pts) Find the domain of the function \( f(x) = \frac{\ln(81-x^2)}{\sqrt{x-2}} \). State your answer in interval notation.

5. (14 pts) Find the average rate of change of the function \( g(x) = \frac{5}{x^2} \) from \( x = 1 \) to \( x = 1 + h \) and simplify your answer so that no single factor of \( h \) is left in the denominator.

6. Consider the polynomial function \( p(x) = 2x^3 + 7x^2 + 20x - 12 \).
   
   (a) (4 pts) List all the possible rational zeros of \( p(x) \). (You do NOT need to check if any of them work.)
   
   (b) (8 pts) Given that \( x = \frac{1}{2} \) is a zero of \( p(x) \), find all other zeros. Simplify your answers.

7. (12 pts) A wire 6 cm long is cut into two pieces. Each piece is bent into the shape of a square. Find a function that models the total area enclosed by the two squares.

8. (14 pts) Graph the function \( f(x) = \frac{(x-3)^2}{4-x^2} \), labeling all intercepts and asymptotes.
9. (12 pts) Solve the logarithmic equation \( \log_4 (x + 2) + \log_4 3 = \log_4 5 + \log_4 (2x - 3) \).

10. (6 pts each) Find the exact value of each expression.

   (a) \( \log_4 \left( \sqrt{8} \right) \)  
   (b) \( e^{-4 \ln(\sqrt{e})} \)

11. (12 pts) A culture of 100 bacteria is growing exponentially according to the function \( P(t) = P_0 e^{rt} \). If the culture triples in size every two days, find the exact amount of time it will take for there to be 2000 bacteria. **Simplify your answer as much as possible.**

12. Find the exact value of each, if it exists.

   (a) (6 pts) \( \cos \left( \frac{16\pi}{3} \right) \)  
   (b) (8 pts) \( \tan^{-1} \left( 2 \cdot \sin \left( \frac{7\pi}{6} \right) \right) \)

13. (12 pts) Given that \( \tan(\theta) = -\frac{\sqrt{95}}{7} \) and \( \sin(\theta) < 0 \), find the exact value of \( \cos \left( \theta + \frac{\pi}{4} \right) \).

14. (12 pts) Graph the function \( g(x) = 4\cos \left( \frac{x}{2} + \pi x \right) \) over one complete period, labeling all high and low points.

15. (12 pts) Verify that the trigonometric equation is an identity.

   \[ \tan x - \cot x = \frac{1 - 2 \cos^2 x}{\sin x \cos x} \]

16. (12 pts) Find all primary solutions (i.e. \( 0 \leq \theta < 2\pi \)) of the equation \( 2\sin(3\theta) + \sqrt{3} = 0 \).