

## MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN IN THE PROVIDED BLUE BOOK.** Point values appear at the beginning of each problem and total 200. Only minimal credit will be awarded for answers without supporting work. **DO NOT USE A CALCULATOR.**

1. (6 pts each) Let  $f(x) = \frac{4}{x-1}$  and  $g(x) = 3+6x$ . Find and simplify each of the following.

(a)  $(f \circ g)(x)$

(b)  $f^{-1}(x)$

2. (12 pts) Let  $f(x) = \sqrt{3x-2} - \sqrt{x+3}$ . Find all values of  $x$ , if any, such that  $f(x) = 1$ .

3. (12 pts) Sketch a graph of the function  $f(x) = \begin{cases} 1-x & \text{if } x \leq -3 \\ |x+2| & \text{if } -3 < x \leq 1 \\ 6 & \text{if } x > 1 \end{cases}$ .

4. (14 pts) Find the domain of the function  $f(x) = \frac{\ln(81-x^2)}{\sqrt{x-2}}$ . State your answer in **interval notation**.

5. (14 pts) Find the average rate of change of the function  $g(x) = \frac{5}{x^2}$  from  $x=1$  to  $x=1+h$  and **simplify your answer** so that no single factor of  $h$  is left in the denominator.

6. Consider the polynomial function  $p(x) = 2x^3 + 7x^2 + 20x - 12$ .

(a) (4 pts) List all the possible rational zeros of  $p(x)$ . (You do **NOT** need to check if any of them work.)

(b) (8 pts) Given that  $x = \frac{1}{2}$  is a zero of  $p(x)$ , find all other zeros. Simplify your answers.

7. (12 pts) A wire 6 cm long is cut into two pieces. Each piece is bent into the shape of a square. Find a function that models the total area enclosed by the two squares.

8. (14 pts) Graph the function  $f(x) = \frac{(x-3)^2}{4-x^2}$ , labeling all intercepts and asymptotes.

9. (12 pts) Solve the logarithmic equation  $\log_4(x+2) + \log_4 3 = \log_4 5 + \log_4(2x-3)$ .

10. (6 pts each) Find the exact value of each expression.

(a)  $\log_4(\sqrt[6]{8})$

(b)  $e^{-4\ln(\sqrt{\pi})}$

11. (12 pts) A culture of 100 bacteria is growing exponentially according to the function  $P(t) = P_0 e^{rt}$ . If the culture triples in size every two days, find the exact amount of time it will take for there to be 2000 bacteria. **Simplify your answer as much as possible.**

12. Find the exact value of each, if it exists.

(a) (6 pts)  $\cos\left(\frac{16\pi}{3}\right)$

(b) (8 pts)  $\tan^{-1}\left(2 \cdot \sin\left(\frac{7\pi}{6}\right)\right)$

13. (12 pts) Given that  $\tan(\theta) = -\frac{\sqrt{95}}{7}$  and  $\sin(\theta) < 0$ , find the exact value of  $\cos\left(\theta + \frac{\pi}{4}\right)$ .

14. (12 pts) Graph the function  $g(x) = 4\cos\left(\frac{\pi}{2} + \pi x\right)$  over one complete period, labeling all high and low points.

15. (12 pts) Verify that the trigonometric equation is an identity.

$$\tan x - \cot x = \frac{1 - 2\cos^2 x}{(\sin x)(\cos x)}$$

16. (12 pts) Find all primary solutions (i.e.  $0 \leq \theta < 2\pi$ ) of the equation  $2\sin(3\theta) + \sqrt{3} = 0$ .