MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN.** Only minimal credit will be awarded for answers without supporting work. Each question is worth 12 points except where indicated. **DO NOT USE A CALCULATOR.**

1) If $f(x) = x^2 + x + 1$ and $g(x) = x - 4$, find and simplify:

a) $f(x) - g(x)$

b) $(f \circ g)(x)$

c) $(g \circ f)(3)$

2) Let $f(x) = \frac{x-5}{3x+2}$. Find $f^{-1}(x)$, where $f^{-1}(x)$ is the inverse of the function of f.

3) Sketch a graph of the function $f(x) = \begin{cases} -x & \text{if} \quad x \leq 0 \\ 9 - x^2 & \text{if} \quad 0 < x \leq 3 \\ x - 3 & \text{if} \quad x > 3 \end{cases}$

4) A right triangle has one leg that is four times as long as the other leg. Find a function that models the triangle's perimeter, P, in terms of x, the length of the shorter leg.

5) Find the domain of the following functions:

a) $f(x) = \sqrt{7x - 3}$

b) $h(x) = \log(x^2 - 16)$

6) Find the average rate of change of the function $f(t) = \frac{3}{t}$ from $t = a$ to $t = a + h$ and simplify your answer so that no single factor of h is left in the denominator.

7) (14 points) A ball is thrown straight up in the air. The height, in feet, of the ball t seconds after being thrown is given by the function $h(t) = -3t^2 + 12t + 6$.

a) What is the maximum height reached by the ball?

b) How many seconds after being thrown does the ball reach this height?

8) Find all zeros of the polynomial $p(x) = x^3 - 3x^2 - 8x - 10$. Express any non-real zeros in the form $a + bi$.

9) (16 points) Let $f(x) = \frac{x^2-2x-8}{x^2+5x}$

a) Graph f(x), labeling all intercepts and asymptotes.

b) State the domain and range of f(x).

10) Find the exact value of each expression.

a) $\ln(e^6)$

b) $\log_3(\sqrt{27})$

c) $\log_2(80) - \log_2(5)$

11) The half-life of Strontium-90 is 28 years. How long will it take a 40mg sample to decay to a mass of 16mg?

12) Find all values of x, if any, such that $\log_8(x+2) + \log_8(3) = \log_8(9) + \log_8(2x-11)$.

13) Find the exact value of each.

a) $\sec\left(\frac{7\pi}{4}\right)$

b) $\cos\left(\frac{-2\pi}{3}\right)$

c) $\tan\left(\frac{-7\pi}{6}\right)$

14) (14 points) Graph the function $f(x) = \cos(4\pi x) - 2$ over one complete period. Show each transformation and label all high and low points.

15) Given that $\tan(\theta) = \frac{12}{5}$, θ is in Quadrant III, and $\sin(\alpha) = \frac{-\sqrt{10}}{10}$, α is in Quadrant IV, find and simplify $\sin(\theta - \alpha)$.

16) Find all primary solutions $(0 \leq \theta < 2\pi)$ of $\tan^3(\theta) = \tan(\theta)$.