MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN IN THE PROVIDED BLUE BOOK.** Only minimal credit will be awarded for answers without supporting work. Each problem is worth 12 points except where indicated. **NO CALCULATORS ALLOWED.**

1. Let $f(x) = \log_3(x-4) - 2$ and $g(x) = (x-1)^2 + 1$.

   a) (6 points) Find and simplify $\dfrac{(f \circ g)(3)}{g(3)+1}$.

   b) (10 points) Find $f^{-1}(0)$.

2. Sketch a graph of the function $f(x) = \begin{cases} |x+2| & \text{if} & x < 1 \\ -3 & \text{if} & 1 \le x \le 4 \\ \sqrt{x} - 6 & \text{if} & x > 4 \end{cases}$.

3. A rectangle has a perimeter of 20 feet. Find a function that models the area of the rectangle in terms of the length x of one of its sides.

4. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x-5} - 2} + \ln(x+5)$.

5. Find the average rate of change of the function $g(x) = \sqrt{6+x}$ from $x = a$ to $x = a+h$. Simplify your answer so that no single factor of h is left in the denominator.

6. Find the minimum value of the function $g(x) = 7 - 12x + 3x^2$.

7. Given that $x = 1$ is a zero of the polynomial $p(x) = 2x^3 - x^2 + 4x - 5$, find all other zeros. Express any non-real zeros in the form $a + bi$.

8. Graph the function $f(x) = \dfrac{(x+4)^2}{x^2 - 3x - 4}$, labeling all intercepts and asymptotes.

9. (8 points each) Simplify each expression completely.

a) $\ln(2e^3) + \ln(\tfrac{1}{2})$

b) $3 \bullet 4^{2\log_4(5)}$

10. The amount of money in an account grows exponentially according to the function $A(t) = A_0 e^{kt}$. If the initial amount in the account is \$200 and there is \$300 in the account after 5 years, how long will it take for the account to grow to \$450? **Simplify** your answer as much as possible.

11. Graph $f(x) = -2^{x-3} + 1$. Label all intercepts and asymptotes.

12. (6 points each) Evaluate each of the following.

a) $\sec\left(-\dfrac{3\pi}{4}\right)$

b) $\sin^{-1}\left(\sin\left(\dfrac{5\pi}{3}\right)\right)$

13. Graph one complete period of the function $f(x) = -4\cos(3x) + 1$, labeling the highest and lowest points.

14. Verify the identity: $\cot(x)\cos(x) = \csc(x) - \sin(x)$

15. Given that $\sin(\theta) = -\dfrac{5}{13}$ and $\sec(\theta) < 0$, find $\sin(2\theta)$.

16. Find all primary solutions ($0 \le \theta < 2\pi$) of $2\cos^2(\theta) = 5\cos(\theta) + 3$.