MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN IN THE PROVIDED BLUE BOOK.** Only minimal credit will be awarded for answers without supporting work. Each problem is worth 12 points except where indicated. **NO CALCULATORS ALLOWED.**

1. Let \( f(x) = \log_3(x-4) - 2 \) and \( g(x) = (x-1)^2 + 1 \).

   a) (6 points) Find and simplify \( \frac{f \circ g(3)}{g(3)+1} \).

   b) (10 points) Find \( f^{-1}(0) \).

2. Sketch a graph of the function \( f(x) = \begin{cases} 
  |x+2| & \text{if } x < 1 \\
  -3 & \text{if } 1 \leq x \leq 4 \\
  \sqrt{x} - 6 & \text{if } x > 4 
\end{cases} \).

3. A rectangle has a perimeter of 20 feet. Find a function that models the area of the rectangle in terms of the length \( x \) of one of its sides.

4. Find the domain of the function \( f(x) = \frac{1}{\sqrt{x-5} - 2} + \ln(x+5) \).

5. Find the average rate of change of the function \( g(x) = \sqrt{6+x} \) from \( x = a \) to \( x = a + h \). Simplify your answer so that no single factor of \( h \) is left in the denominator.

6. Find the minimum value of the function \( g(x) = 7 - 12x + 3x^2 \).

7. Given that \( x = 1 \) is a zero of the polynomial \( p(x) = 2x^3 - x^2 + 4x - 5 \), find all other zeros. Express any non-real zeros in the form \( a + bi \).

8. Graph the function \( f(x) = \frac{(x+4)^2}{x^2 - 3x - 4} \), labeling all intercepts and asymptotes.
9. (8 points each) Simplify each expression completely.
   a) \( \ln\left(2e^3\right) + \ln\left(\frac{1}{2}\right) \)
   b) \( 3 \cdot 4^{\log_5(5)} \)

10. The amount of money in an account grows exponentially according to the function \( A(t) = A_0 e^{kt} \). If the initial amount in the account is $200 and there is $300 in the account after 5 years, how long will it take for the account to grow to $450? Simplify your answer as much as possible.

11. Graph \( f(x) = -2^{x-3} + 1 \). Label all intercepts and asymptotes.

12. (6 points each) Evaluate each of the following.
   a) \( \sec\left(-\frac{3\pi}{4}\right) \)
   b) \( \sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right) \)

13. Graph one complete period of the function \( f(x) = -4\cos(3x) + 1 \), labeling the highest and lowest points.

14. Verify the identity: \( \cot(x)\cos(x) = \csc(x) - \sin(x) \)

15. Given that \( \sin(\theta) = -\frac{5}{13} \) and \( \sec(\theta) < 0 \), find \( \sin(2\theta) \).

16. Find all primary solutions \( 0 \leq \theta < 2\pi \) of \( 2\cos^2(\theta) = 5\cos(\theta) + 3 \).