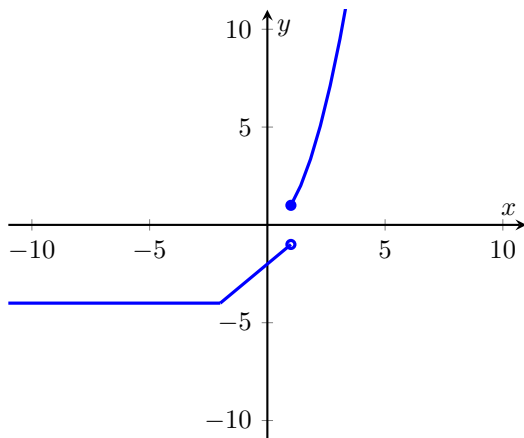


# Final Exam

MAT 1800 Elementary Functions

Fall 2022

1.



2.  $f(x) = \frac{\log_2(x^2 - 25)}{x - 8}$  has domain restrictions  $x^2 - 25 > 0$  and  $x - 8 \neq 0$ .  
 $x^2 - 25 = (x + 5)(x - 5) > 0$  on the interval  $(-\infty, -5) \cup (5, \infty)$ .  
 $x \neq 8$ , so the domain of  $f(x)$  is  $(-\infty, -5) \cup (5, 8) \cup (8, \infty)$

3. (a)

$$\begin{aligned}g(f(x)) &= \frac{(\sqrt[3]{4x} - 3) + 2}{(\sqrt[3]{4x} - 3) - 2} \\&= \frac{\sqrt[3]{4x} - 1}{\sqrt[3]{4x} - 5} \\g(f(2)) &= \frac{\sqrt[3]{4 * 2} - 1}{\sqrt[3]{4 * 2} - 5} = -\frac{1}{3} \\(f + g)(x) &= \sqrt[3]{4x} + \frac{x + 2}{x - 2} \\(f + g)(0) &= \sqrt[3]{4 * 0} + \frac{0 + 2}{0 - 2} = -4\end{aligned}$$

Finally,

$$\frac{(g \circ f)(2)}{(f + g)(0)} = \frac{-\frac{1}{3}}{-4} = \frac{1}{12}$$

(b)  $f^{-1}(1)$  occurs when  $f(x) = 1$ .

$$\sqrt[3]{4x} - 3 = 1$$

$$\sqrt[3]{4x} = 4$$

$$4x = 4^3$$

$$x = 16$$

So  $f^{-1}(1) = 16$ .

4.  $V = 48\text{ft}^3$ .  $V = lwh$  and  $l = x$ ,  $w = x$ , and  $h = y$ , so  $V = x^2y$ . Thus  $48 = x^2y$  and  $y = \frac{48}{x^2}$ .

$$\begin{aligned} SA &= 2x^2 + 4xy \\ &= 2x^2 + 4x\left(\frac{48}{x^2}\right) \\ &= 2x^2 + \frac{192}{x} \end{aligned}$$

5.  $h(t) = -16t^2 + 64t + 145$

The vertex is  $(x, y)$ , where  $x = \frac{-b}{2a}$  and  $y = h(x)$ .

$$x = \frac{-64}{-32} = 2$$

$$y = h(2) = 209$$

So the maximum height of the rocket is 209 feet.

6. We know  $-2$  is a solution to  $p(x)$ , or that  $p(-2) = 0$ . Long division of  $p(x)$  by  $x - (-2) = x + 2$  gives

$$p(x) = (x + 2)(x^2 - 6x + 5)$$

Factoring  $x^2 - 6x + 5$  gives

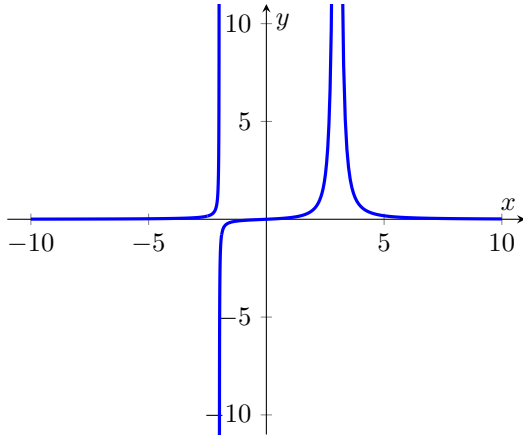
$$p(x) = (x + 2)(x - 1)(x - 5)$$

So  $p(x)$  has roots  $-2, 1$ , and  $5$ .

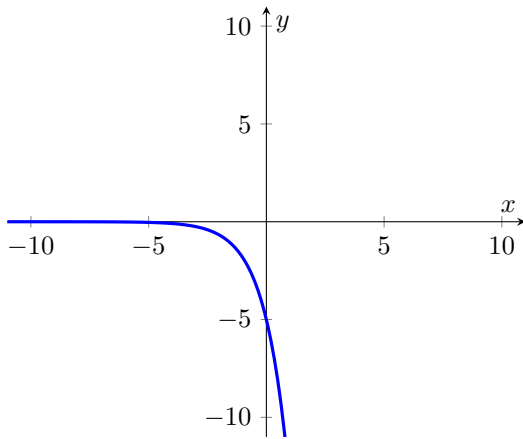
7.

$$\begin{aligned} \frac{g(2+h) - g(2)}{(2+h) - 2} &= \frac{\frac{1}{(2+h)^2} - \frac{1}{2^2}}{h} \\ &= \frac{\frac{1}{h^2+4h-4} - \frac{1}{4}}{h} \\ &= \frac{4 - (h^2+4h+4)}{4(h^2+4h+4)} \\ &= \frac{-h^2-4h}{4(h^2+4h+4)} \\ &= \frac{-h-4}{4h^2+16h+16} \end{aligned}$$

8.



9.



10. (a)

$$\begin{aligned}
 \log_5 \frac{1}{\sqrt[3]{25}} &= \log_5(1) - \log_5(25^{\frac{1}{3}}) \\
 &= \log_5(1) - \frac{1}{3} \log_5(25) \\
 &= 0 - \frac{1}{3} \log_5(5^2) \\
 &= -\frac{2}{3} \log_5(5) \\
 &= -\frac{2}{3}
 \end{aligned}$$

(b)

$$\begin{aligned}
 e^{2\ln(10) - \ln(4)} &= e^{\ln(10^2) - \ln(4)} \\
 &= e^{\ln\left(\frac{100}{4}\right)} \\
 &= e^{\ln(25)} \\
 &= 25
 \end{aligned}$$

11.

$$\begin{aligned}Q(t) &= Q_0 e^{rt} \\ &= 80e^{rt}\end{aligned}$$

Using the point (5, 60) gives

$$\begin{aligned}60 &= 80e^{5r} \\ \frac{3}{4} &= e^{5r} \\ 5r &= \ln\left(\frac{3}{4}\right) \\ r &= \frac{\ln\left(\frac{3}{4}\right)}{5}\end{aligned}$$

Updating  $Q$  to

$$Q(t) = 80e^{\frac{\ln\left(\frac{3}{4}\right)}{5}t}$$

Plugging in 10 gives

$$\begin{aligned}Q(10) &= 80e^{\frac{\ln\left(\frac{3}{4}\right)}{5} \cdot 10} \\ &= 80e^{2\ln\left(\frac{3}{4}\right)} \\ &= 80e^{\ln\left(\frac{9}{16}\right)} \\ &= 80 \cdot \frac{9}{16} \\ &= 45\end{aligned}$$

There are 45 bacteria after 10 hours.

12. (a)

$$\begin{aligned}\tan\left(\frac{2\pi}{3}\right) &= \frac{\sin\left(\frac{2\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)} \\ &= \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} \\ &= -\sqrt{3}\end{aligned}$$

(b)

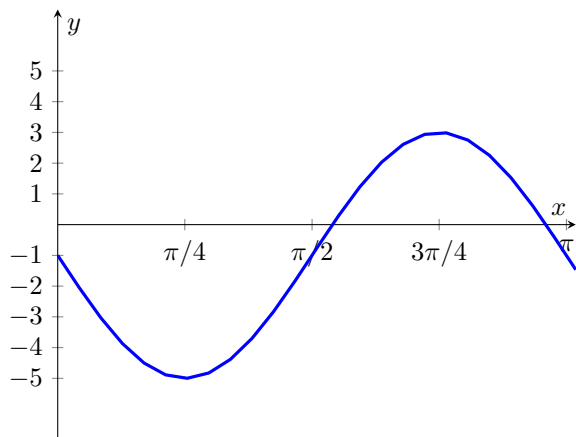
$$\begin{aligned}\csc\left(\frac{13\pi}{4}\right) &= \csc\left(\frac{5\pi}{4}\right) \\ &= \frac{1}{\sin\left(\frac{5\pi}{4}\right)} \\ &= \frac{1}{-\frac{\sqrt{2}}{2}} \\ &= -\frac{2}{\sqrt{2}} \\ &= -\sqrt{2}\end{aligned}$$

(c)

$$\begin{aligned}\cos^{-1}\left(\cos\left(-\frac{\pi}{4}\right)\right) &= \cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right) \\ &= \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\pi}{4}\end{aligned}$$

13. (a) The period is  $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$ . The amplitude is  $|-4| = 4$

(b)



14.  $\tan(\theta) = -\frac{3}{5}$  and  $\cos(\theta) > 0$ .

$$\begin{aligned}\sin\left(\frac{\pi}{4} - \theta\right) &= \sin\left(\frac{\pi}{4}\right)\cos(\theta) - \cos\left(\frac{\pi}{4}\right)\sin(\theta) \\ &= \frac{\sqrt{2}}{2}\cos(\theta) - \frac{\sqrt{2}}{2}\sin(\theta)\end{aligned}$$

By the pythagorean theorem,  $\cos(\theta) = \frac{5\sqrt{34}}{34}$  and  $\sin(\theta) = -\frac{3\sqrt{34}}{34}$ . After substituting we get

$$\begin{aligned}\sin\left(\frac{\pi}{4} - \theta\right) &= \frac{\sqrt{2}}{2} \cdot \frac{5\sqrt{34}}{34} - \frac{\sqrt{2}}{2} \cdot -\frac{3\sqrt{34}}{34} \\ &= \frac{4\sqrt{17}}{17}\end{aligned}$$

15.  $\cos^2(x) - 2\cos(x) = 0$  can be rewritten as  $(\cos(x))^2 - 2\cos(x) = 0$ . Let  $u = \cos(x)$ . Then we have

$$u^2 - 2u = 0$$

The solutions to this equation are  $u = 0, u = 2$ . So  $\cos(x) = 0, \cos(x) = 2$ . Since  $\cos(x) = 2$  is an extraneous solution.  $\cos(x) = 0$  means that  $x = \frac{\pi}{2}$  or  $x = \frac{3\pi}{2}$ .

16.

$$\begin{aligned}\frac{\sec(x) \sin(x)}{\tan(x) + \cot(x)} &= \frac{\frac{\sin(x)}{\cos(x)}}{\frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)}} \\ &= \frac{\frac{\sin(x)}{\cos(x)}}{\frac{\sin^2(x) + \cos^2(x)}{\sin(x) \cos(x)}} \\ &= \frac{\frac{\sin(x)}{\cos(x)}}{\frac{1}{\sin(x) \cos(x)}} \\ &= \frac{\sin(x)}{\cos(x)} \cdot \sin(x) \cos(x) \\ &= \sin^2(x)\end{aligned}$$