MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN IN THE PROVIDED BLUE BOOK.** Only minimal credit will be awarded for answers without supporting work. Each problem is worth 12 points except where indicated. **NO CALCULATORS ALLOWED.**

1. Sketch a graph of the function $f(x) = \begin{cases} -4 & if \quad x < -2 \\ x-2 & if -2 \leq x < 1 \\ x^2 & if \quad x \geq 1 \end{cases}$

2. Find the domain of the function $f(x) = \dfrac{\log_2(x^2 - 25)}{x - 8}$

3. (8 points each) Let $f(x) = \sqrt[3]{4x} - 3$ and $g(x) = \dfrac{x+2}{x-2}$.

    a) Find and simplify $\dfrac{(g \circ f)(2)}{(f+g)(0)}$.

    b) Find $f^{-1}(1)$.

4. A rectangular box with a square base has a volume of 48 ft³. Find a function that models its surface area, S, in terms of the length of its base, x.

5. The height, h, in meters of a model rocket being launched off a building is given by the function $h(t) = -16t^2 + 64t + 145$, where t is the number of seconds after launch. What is the maximum height reached by the rocket?

6. Given that -2 is a root of the polynomial $p(x) = x^3 - 4x^2 - 2x + 20$, find all solutions to the equation $x^3 - 4x^2 - 2x + 20 = 0$. Express any non-real solutions in the form $a + bi$.

7. Find the average rate of change of the function $g(x) = \dfrac{1}{x^2}$ from $x = 2$ to $x = 2 + h$. Simplify your answer completely.

8. (14 points) Graph the function $f(x) = \dfrac{4x}{(x-3)^2(x+2)}$. **Label all intercepts and asymptotes.**

9. Graph $g(x) = -3^{x+1} - 2$. **Label all intercepts and asymptotes.**

10. (4 points each) Simplify each expression completely.

   a) $\log_5 \dfrac{1}{\sqrt[3]{25}}$

   b) $e^{2\ln(10) - \ln(4)}$

11. The number of bacteria in a culture decreases exponentially according to the function $Q(t) = Q_0 e^{rt}$. Suppose the culture initially contains 80 bacteria and decreases to 60 bacteria after 5 hours. What is the size of the culture after 10 hours?

12. (6 points each) Evaluate each of the following.

   a) $\tan\left(\dfrac{2\pi}{3}\right)$

   b) $\csc\left(\dfrac{13\pi}{4}\right)$

   c) $\cos^{-1}\left(\cos\left(-\dfrac{\pi}{4}\right)\right)$

13. Consider the function $f(x) = -4\sin(2x) - 1$.

   a) (4 points) State the period and amplitude of the function.

   b) (8 points) Graph one period of the function, labeling the highest and lowest points.

14. Given that $\tan(\theta) = -\dfrac{3}{5}$ and $\cos(\theta) > 0$, find $\sin\left(\dfrac{\pi}{4} - \theta\right)$. Simplify your answer completely.

15. Find all primary solutions $(0 \le \theta < 2\pi)$ of the trigonometric equation

$$\cos^2(x) - 2\cos(x) = 0$$

16. Verify the identity: $\dfrac{\sec(x)\sin(x)}{\tan(x) + \cot(x)} = \sin^2(x)$