Problem 1

Part(a):

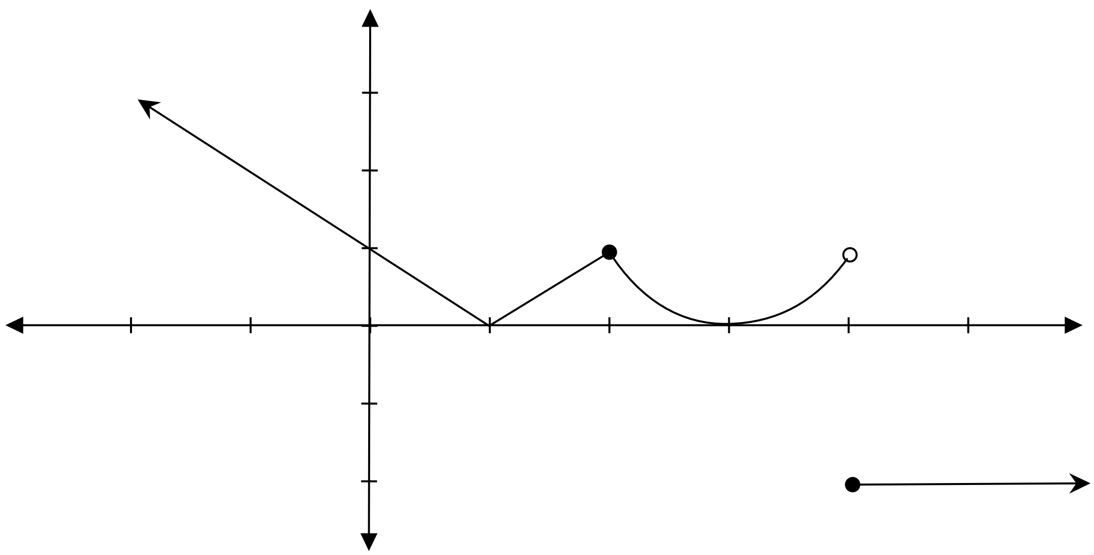
$$(g \circ f)(x) = g(f(x)) = \frac{1-7x^2}{2(1-7x^2)+1} = \frac{1-7x^2}{3-14x^2}$$

Part(b):

$$g(x) = \frac{x}{2x+1} \implies y = \frac{x}{2x+1} \implies x = \frac{y}{2y+1} \implies (2y+1)x = y \implies 2xy + x = y$$

$$\implies y = \frac{x}{1-2x} \implies g^{-1}(x) = \frac{x}{1-2x}$$

Problem 2



Problem 3

Domain of numerator: $5 + 4x - x^2 > 0 \implies (x-5)(x+1) < 0 \implies (-\infty, -2) \cup (5, \infty)$
Domain of denominator: $2x - 3 \neq 0 \implies x \neq \frac{3}{2} \implies (-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$
Domain of $f(x)$: $(-\infty, -2) \cup (5, \infty)$

Problem 4

$$\frac{g(a)-g(a+h)}{(a)-(a+h)} = \frac{\frac{3}{a+1}-\frac{3}{a+h+1}}{h} = \frac{1}{h}\cdot\left(\frac{3(a+h+1)}{(a+1)(a+h+1)} - \frac{3(a+1)}{(a+1)(a+h+1)}\right)$$
$$= \frac{3h}{h(a+1)(a+h+1)} = \frac{3}{(a+1)(a+h+1)}$$

Problem 5

Use polynomial long division to find the other factor of $p(x)$:

$$\begin{array}{r l}
 & x^2 - 2x + 6 \\
x^2 - 3 & \overline{\big)\, x^4 - 2x^3 + 3x^2 + 6x - 18} \\
 + & \underline{-x^4 \qquad\quad + 3x^2} \\
 & -2x^3 + 6x^2 \\
 + & \underline{2x^2 \qquad\quad - 6x} \\
 & 6x^2 \qquad - 18 \\
 + & \underline{-6x^2 \qquad + 18} \\
 & 0 \;\checkmark
\end{array}$$

$p(x) = (x^2-3)(x^2-2x+6) \implies x = \pm\sqrt{3}, 1 \pm i\sqrt{5}.$

Problem 6

Part(a):

y-intercept: $\frac{4(0-2)^2}{0(0+3)} = \frac{16}{0}$ so it does not exist.

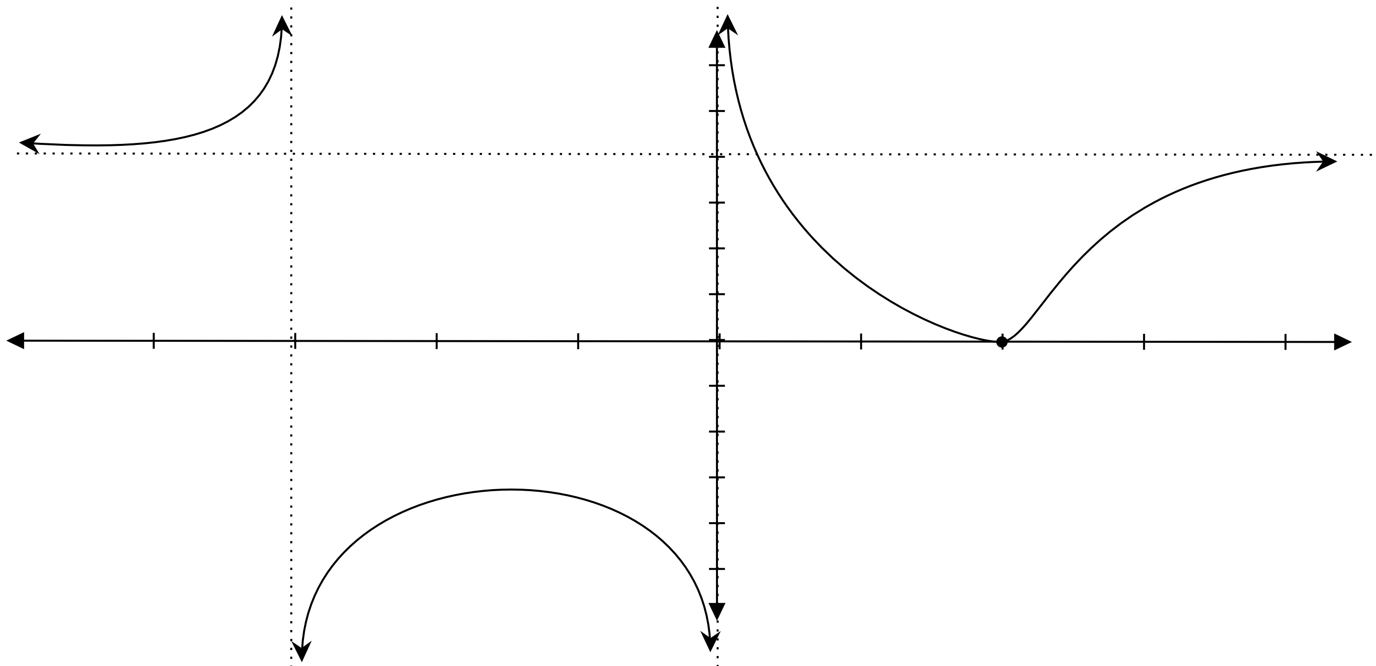
x-intercept: $\frac{4(x-2)^2}{x(x+3)} = 0 \Rightarrow 4(x-2)^2 = 0 \Rightarrow x-2=0 \Rightarrow x = 2$. So x has an intercept at $x = 2$.

vertical asymptote: $x(x+3) = 0 \Rightarrow x = 0, x = -3$

horizontal asymptote: $\frac{4(x-2)(x-2)}{x(x+3)} \Rightarrow$ asymptote at $y = 4$.

Problem 6

Part(b):



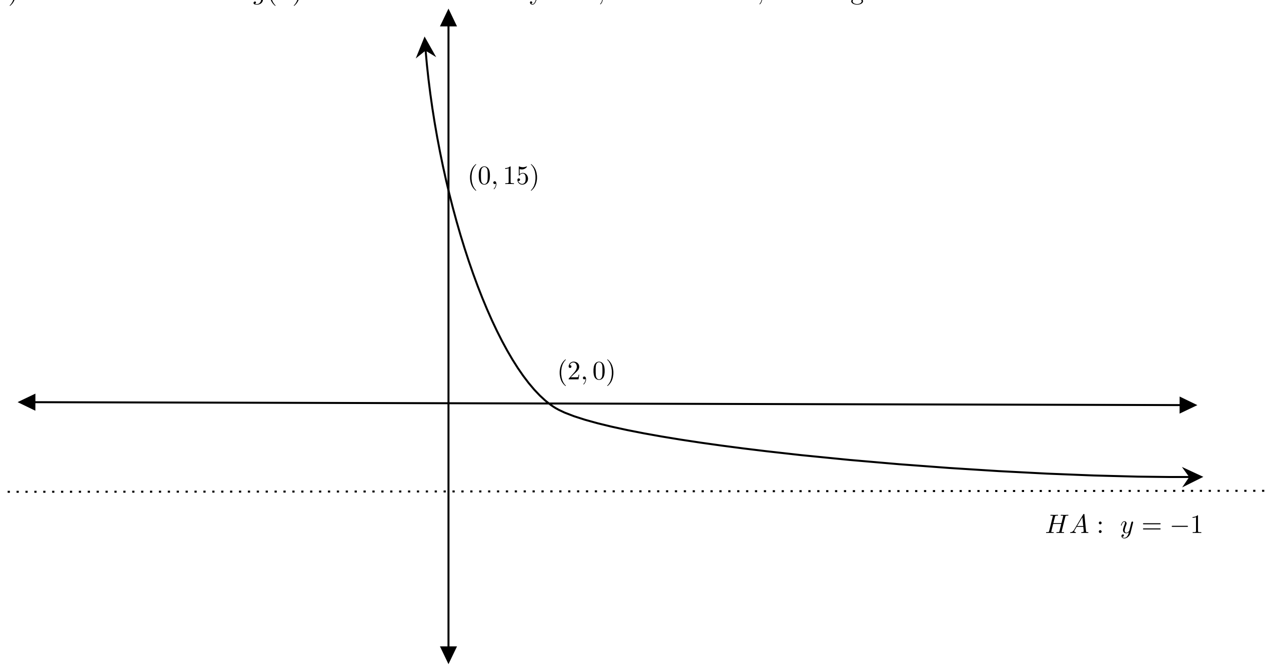
Problem 7

$$\log(x-1)+\log(5)=2-\log(x) \implies \log(5x-5)=2-log(x) \implies \log(5x-5)+\log(x)=2$$
$$\implies \log(5x^2-5x)=2 \implies 10^2=5x^2-5x \implies 100=5(x^2-x) \implies 20=x^2-x$$
$$\implies x^2-x-20=0 \implies (x-5)(x+4)=0 \implies x=5, x=-4$$

We can't have logs of negative numbers, so the only solution is $x=5$.

Problem 8

$f(x) = 4^{-x+2} - 1$. Take $g(x) = 4^x$. Reflect over y axis, shift down 1, shift right 2.



Problem 9

Sides of the top square: $x + x + x + x = 4x$
Sides of the bottom square: $x + x + x + x = 4x$
Let the rectangles have length l and width x. Then the sum of all l values is $l + l + l + l = 4l$
Sum of all edges: $4x + 4x + 4l = 112 \Rightarrow 8x + 4l = 112 \Rightarrow 4l = 112 - 8x \Rightarrow l = 28 - 2x$
Volume: length $\cdot$ width $\cdot$ height $= l \cdot x \cdot x = lx^2 = (28 - 2x)x^2 = 28x^2 - 2x^3$

Problem 10

Part(a):

$$\log_6 7\sqrt{6} + \log_6 \frac{1}{7} = \log_6 \frac{7\sqrt{6}}{7} = \log_6 \sqrt{6} = \frac{1}{2}\log_6 6 = \frac{1}{2} * 1 = \frac{1}{2}$$

Part(b):

$$4^{\log_4 24 - 3\log_4 2} = 4^{\log_4 \frac{24}{2^3}} = 4^{\log_4 3} = 3$$

Problem 11

$(0, 20), (4, 80)$
$P(4) = 20e^{4r} = 80 \Rightarrow 20e^{4r} = 80 \Rightarrow e^{4r} = 4 \Rightarrow 4r = \ln(4) \Rightarrow r = \frac{\ln(4)}{4}$
$P(6) = 20e^{\frac{ln(4)}{4}*6} = 20e^{\frac{3\ln(4)}{2}} = 20e^{\frac{\ln(64)}{2}} = 20 * 8 = 160$ Their population after 6 years is 160.

Problem 12

Part(a):

$$\sin(-\frac{17\pi}{6}) = \sin(-\frac{5\pi}{6}) = \sin(\frac{7\pi}{6}) = -\frac{1}{2}$$

Part(b):

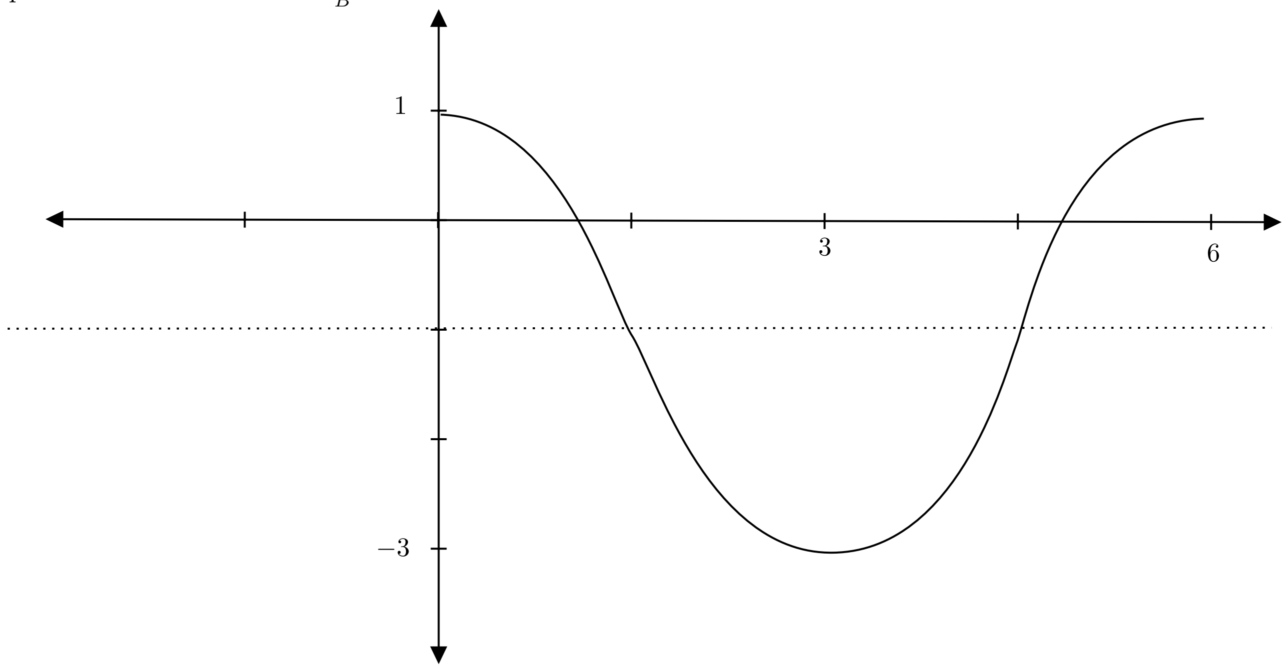
$$\tan^{-1}(\cos(\pi)) = \tan^{-1}(-1) = \frac{\sin^{-1}(-1)}{\cos^{-1}(-1)} = \frac{\frac{3\pi}{2}}{\pi} = \frac{3}{2}$$

Problem 13

$3 = \csc(\theta) = \frac{1}{\sin(\theta)} \Rightarrow \sin(\theta) = \frac{1}{3}$.
$\sin(\theta) = \frac{1}{3} \Rightarrow \cos(\theta) = \frac{2\sqrt{2}}{2}$.
$\cos(\frac{4\pi}{3} - \theta) = \cos(\frac{4\pi}{3})\cos(\theta) + \sin(\frac{4\pi}{3})\sin(\theta) = -\frac{1}{2}\frac{2\sqrt{2}}{3} + -\frac{\sqrt{3}}{2}\frac{1}{3} = -\frac{\sqrt{2}}{3} - \frac{\sqrt{3}}{6}$

Problem 14

Amplitude $A = 2$. Period $P = \frac{2\pi}{B} = 6$.



Problem 15

$$(1-\cos(x))(\csc(x)+\cot(x)) = (1-\cos(x))\left(\frac{1}{\sin(x)}+\frac{\cos(x)}{\sin(x)}\right) = (1-\cos(x))\left(\frac{1+\cos(x)}{\sin(x)}\right)$$

$$= \left(\frac{(1-\cos(x))(1+\cos(x))}{\sin(x)}\right) = \frac{1-\cos^2(x)}{\sin(x)} = \frac{\sin^2(x)}{\sin(x)} = \sin(x)$$

Problem 16

$$\cos(2x)+\sin(x) = 1 \implies \left(1-2\sin^2(x)\right)+\sin(x)-1 = 0 \implies -\sin^2(x)+\sin(x) = 0$$

$$\implies \sin(x)\left(1-\sin(x)\right) = 0$$

$$\sin(x) = 1 \implies x = \frac{\pi}{2}\ ,\ \sin(x) = 0 \implies x = 0, \pi$$

So, the primary solutions are $x = 0, \frac{\pi}{2}, \pi$.