Instructions:

- Please read the directions to each problem carefully.
- Each problem is worth 12 points except where indicated.
- Solutions should be written clearly and concisely on blank sheets of paper. **All work must be shown to receive full credit. Answers without correct supporting work will receive minimal credit.**
- **No outside assistance of any kind is allowed.** This includes using the internet to find answers, using your notes, having another person look at your work before submission, looking at another person’s work before submission, and/or sharing information in any way while completing the assessment.
- Calculators are not permitted.
- Webcams are required to be on for the duration of the exam.
- You will have 120 minutes to complete the Final Exam and an additional 15 minutes to upload your work on Canvas.

1. (8 pts each) Let \( f(x) = 1 - 7x^2 \) and \( g(x) = \frac{x}{2x+1} \). Find and simplify each of the following.

   (a) \( (g \circ f)(x) \)  
   (b) \( g^{-1}(x) \)

2. Sketch a graph of the function \( g(x) = \begin{cases} |x - 1| & \text{if } x \leq 2 \\ -(x - 3)^2 & \text{if } 2 < x < 4 \\ -2 & \text{if } x \geq 4 \end{cases} \)

3. Find the domain of the function \( f(x) = \frac{\log_3(5+4x-x^2)}{2x-3} \). State your answer in **interval notation**.

4. Find the average rate of change of the function \( g(x) = \frac{3}{x+1} \) from \( x = a \) to \( x = a + h \) and **simplify your answer** so that no single factor of \( h \) is left in the denominator.

5. Consider the polynomial function \( p(x) = x^4 - 2x^3 + 3x^2 + 6x - 18 \). Given that \( x^2 - 3 \) is a factor of \( p(x) \), find all the zeros of the polynomial.
6. Let \( f(x) = \frac{4(x-2)^2}{x(x+3)}. \)
   (a) Find all intercepts and asymptotes for \( f(x) \).
   (b) Sketch the graph of \( f(x) \).

7. Solve the logarithmic equation: \( \log(x - 1) + \log(5) = 2 - \log(x) \).

8. Graph \( f(x) = 4^{-x^2} - 1 \) using transformations. Label all asymptotes and intercepts.

9. A cardboard box has a square base and a square top with each edge of the squares measuring \( x \) inches. The total length of all 12 edges of the box is 112 inches. Express the volume \( V \) of the box as a function of \( x \).

10. (8 pts each) Find the exact value of each expression.
    (a) \( \log_6(7\sqrt{6}) + \log_6\left(\frac{1}{7}\right) \)  
    (b) \( 4^{\log_4(24) - 3\log_4(2)} \)

11. The population of a certain rare animal species increases exponentially according to the function \( P(t) = P_0 e^{rt} \), where \( P_0 \) is the initial population, \( t \) is time measured in years, and \( r \) is a constant. If the population increases from 20 to 80 in 4 years, what will the population be in another 2 years? **Simplify** your answer as much as possible.

12. Find the exact value of each trigonometric function at the given real number if it exists.
    (a) \( \sin\left(-\frac{17\pi}{6}\right) \)  
    (b) \( \tan^{-1}(\cos(\pi)) \)

13. Given that \( \csc \theta = 3 \) and \( \cos \theta < 0 \), find the exact value of \( \cos\left(\frac{4\pi}{3} - \theta\right) \).
14. State the amplitude and period length for the function $f(x) = 2\sin\left(\frac{1}{3} \pi x \right) - 1$. Graph this function over one complete period, labeling all high points and low points.

15. Verify that the trigonometric equation is an identity.

$$\sin x = (1 - \cos x)(\csc x + \cot x)$$

16. Find all primary solutions ($0 \leq x < 2\pi$) of the equation: $\cos(2x) + \sin(x) = 1$. 