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MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN IN THE PROVIDED BLUE BOOK.** Only minimal credit will be awarded for answers without supporting work. Each problem is worth 10 points. **DO NOT USE A CALCULATOR.**

1. Let $f(x)=\dfrac{9}{6x-x^2}$. Find all the values of x, if any exist, such that $f(x)<1$. State your answer in **interval notation**.

2. Sketch a graph of the function $h(x)=\begin{cases} -5 & \text{if} \quad x\le -2 \\ |2-x| & \text{if} \quad -2<x\le 3 \\ \sqrt{1+x} & \text{if} \quad x>3 \end{cases}$

3. Let $f(x)=\dfrac{1}{5+x}$ and $g(x)=\dfrac{1}{x}$. Find and simplify each of the following.

(a) $\dfrac{5\cdot f(5)}{(g\circ f)(5)}$ (b) $(f\circ f)(x)$

4. Let $f(x)=7\left(e^{4x}\right)-10$. Find $f^{-1}(4)$, where f^{-1} is the inverse function of f.

5. Find the domain of the function $f(x)=\dfrac{\log_8(-6x)-|x|}{\sqrt{x+5}-1}$. State your answer in **interval notation**.

6. Find a polynomial $p(x)$ of degree 4 with **integer coefficients** that has -1 as its only real zero, has $2i$ as a zero, and has a constant term of 8.

7. Let $r(x)=\sqrt{3x}$. Find the average rate of change of $r(x)$ from $x=a$ to $x=a+h$ and **simplify your answer** so that no single factor of h is left in the denominator.

8. Consider the polynomial function $p(x)=x^4+2x^3+2x^2+14x-35$.

(a) List all the possible rational zeros of $p(x)$. (You do **NOT** need to check if any of them work.)

(b) Given that x^2+7 is a factor of $p(x)$, find all the zeros of the polynomial.

9. Graph the function $f(x)=\dfrac{3(x-2)^2}{(3x-4)(x+1)}$, labeling all intercepts and asymptotes.

10. Solve the logarithmic equation $2\log\left(\sqrt{1-x}\right)=2-\log(-5x)$.

11. Graph the function $s(x)=3-\left(\frac{1}{4}\right)^{x-2}$, labeling all asymptotes and **at least one point**.

12. A movie theater holds 1000 people. With the ticket price at \$8 during the week, the attendance at the theater has been 200 people. A market survey indicates that for every dollar the ticket price is lowered, attendance increases by 50.

(a) Find a function that models the revenue R in terms of the ticket price x.
(b) What ticket price will maximize the revenue?

13. Find the exact value of each expression.

(a) $\ln\left(10^{-2\log\left(e^{8}\right)}\right)$ (b) $\log_9\left(\frac{\sqrt[4]{3}}{\sqrt[3]{81}}\right)$

14. A bacteria culture grows exponentially according to the function $P(t)=P_0e^{kt}$. If the culture begins with 3600 bacteria and triples every 25 minutes, how long will it take for the culture to reach 4200 bacteria. **Simplify your answer as much as possible.**

15. Find the exact value of each trigonometric function at the given real number, if it exists.

(a) $\cos\left(\frac{7\pi}{6}\right)$ (b) $\tan\left(-\frac{8\pi}{3}\right)$

16. Find the exact value of each expression, if it exists.

(a) $\tan^{-1}\left(\sin\left(\frac{7\pi}{2}\right)\right)$ (b) $\sec\left(\sin^{-1}\left(\frac{9}{13}\right)\right)$

17. Given that $\tan\theta=-\sqrt{3}$ with θ in Quadrant II and $\cos\phi=\frac{4\sqrt{2}}{9}$ with ϕ in Quadrant IV, find the exact value of $\sin(\theta-\phi)$.

18. State the amplitude and period length of the function $f(x)=3-\cos\left(\pi x-\frac{\pi}{2}\right)$ and then graph one complete period. Be sure to label the highest and lowest points on the graph.

19. Verify that the trigonometric equation is an identity.

$$\frac{1-\cos x}{\sin x}+\frac{\sin x}{1-\cos x}=2\csc x$$

20. Find all primary solutions (i.e. $0\le x<2\pi$) of the equation $\tan(\theta)=\sin(2\theta)$.