MAT 1050 GROUP FINAL EXAM - Winter 2015

SHOW ALL WORK. DO NOT USE A CALCULATOR.

1. (7 pts.) Simplify by adding (or subtracting) like terms wherever possible:
\[ 8^x + 2^x + x^2 + 2x^2 - \pi \cdot 2^x - 3x \]

2. (7 pts.) Simplify completely: \((-3ab^{-2})^{-1}\left(\frac{27a^4b^{-2}c^0}{a^2b^2c^{-1}}\right)\)

3. (7 pts.) Multiply and simplify: \((5 - \sqrt{3})^2\)

4. (7 pts.) Simplify completely: \(\left(\frac{8^{-1/3}}{8^{-1/3} + 8^{-2/3}}\right)^{-1}\)

5. (6 pts.) Solve: \(-3\left|\frac{x-3}{2}\right| + 4 \leq 13\)

6. (6 pts.) Let \(f(x) = |2x + 5|\) and \(g(x) = |2x - 3|\). Find all \(x\) for which \(f(x) = g(x)\)

7. (7 pts.) Joe’s summer job for the park service has him documenting bird sightings at a particular feeding spot for the same 20 minute period each day. Over the course of 1 month, he has recorded 115 sightings of robins, cardinals and blue jays combined. If the number of robins is three times the number of cardinals, and the number of blue jays is 5 more than one-half the number of robins, how many of each bird did Joe see?

8. (7 pts.) Solve for \(a\): \(\frac{1}{a} + \frac{2}{b} = \frac{3}{c}\)

9. (6 pts.) Let \(h\) be the function given by \(h(x) = \frac{|x-3|}{5} - 4\sqrt{\frac{1}{3}x} + 2\). What is the domain of \(h\)?
10. (6 pts.) Let \( f \) be the function given by \( f(x) = 2x - 4|x^2 - 5| \).
   \[ \text{a) Find and simplify } f\left(\frac{1}{2}\right). \]
   \[ \text{b) Find and simplify } f(-2). \]

11. (7 pts.) Let \( g \) be the function given by \( g(x) = -x^2 + 3x + 5 \).
   \[ \text{Find and simplify } \frac{g(x+h)-g(x)}{h}. \]

12. (6 pts.) Find the equation of the line that is perpendicular to the line \( 3x - 5y = 9 \) and has y-intercept 4.

13. (6 pts.) Find the equation of the line that is parallel to the line \( x = 2 \) and passes through the point (5, 1).

14. (7 pts.) Three consecutive odd numbers are such that the square of the third is 72 more than the square of the first. Find the three numbers.

15. (7 pts.) Solve, writing all non-real solutions in the form \( a + bi \):
   \[ 2x^3 = x^2 - 3x \]

16. (7 pts.) Graph, labeling the vertex and all \( x \) or \( y \) intercepts:
   \[ f(x) = x^2 - 5x \]

17. (7 pts.) Simplify completely:
   \[ \frac{x^2+8x}{2x^2-3x-5} - \frac{1}{x+1} \]

18. (7 pts.) Let \( f(x) = \sqrt{x} + 1 \) and \( g(x) = \sqrt{3x} - 3 \). Find all \( x \) for which \( f(x) = g(x) \).

19. (7 pts.) Solve: \( x^3 \leq 9x \)

20. (7 pts.) Solve: \( \frac{x^2-4}{(x-5)^2} < 0 \)
21. (7 pts.) The graph of a function, \( f \), is shown here.

   a) What is the domain of \( f \)?
   b) What is the range of \( f \)?
   c) What is \( f(0) \)?
   d) Find all numbers, \( x \), such that \( f(x) = 0 \).

22. (7 pts.) Solve: \( \left( \frac{2}{x-2} \right)^2 - 3 \left( \frac{2}{x-2} \right) + 2 = 0 \)

23. (6 pts.) Find:
   a) \( \log_{27} \left( \frac{1}{3} \right) \)
   b) \( \log_2 (16) \)
   c) \( \log_{10} (0.01) \)

24. (6 pts.) Given the approximate values \( \log_3 (11) = 2.18 \) and \( \log_3 (2) = 0.63 \) find:
   a) \( \log_3 (121) \)
   b) \( \log_3 (22) \)
   c) \( \log_3 \left( \frac{9}{11} \right) \)

25. (7 pts.) Solve: \( \log_5 (x+3) = 2 - \log_5 (2x+1) \)

26. (7 pts.) Identify and sketch the curve given by: \( x^2 - y^2 = 9 \)

27. (7 pts.) Arrange the following numbers in order from smallest to largest:
   \( \sin(\pi) \quad \cos(0) \quad \log_2 \left( \frac{1}{3} \right) \quad \sqrt{3} \)

28. (6 pts.)
   a) Convert \( \pi^2 \) radians to degrees.
   b) Convert \( -3^\circ \) to radians
29. (6 pts.) In the right triangle shown here, find:

   a) \( \tan(\angle B) \)
   
   b) \( \cos(\angle A) \)

30. (7 pts.) Marvin travels 4 miles downstream and 4 miles back upstream in a total time of 1 hour. If the river has a current of 3 mph, what is Marvin’s speed in still water?