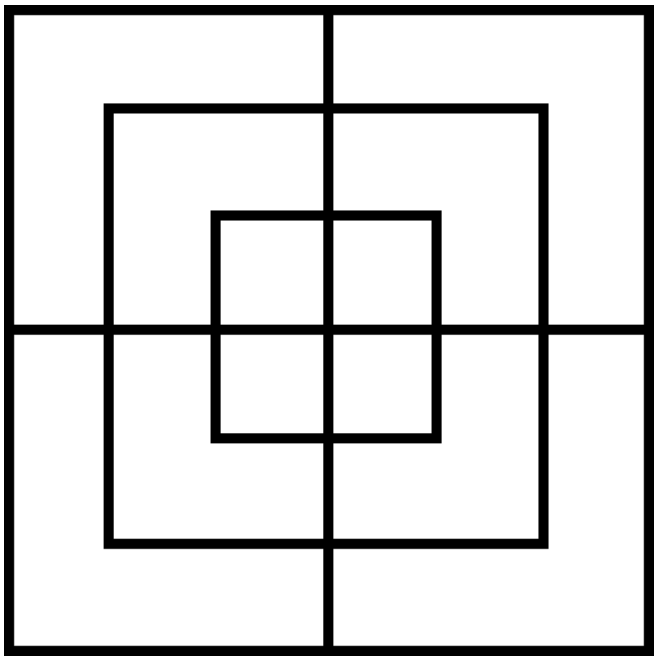
Week #1 Fall 2022

Problem of the Week Solution

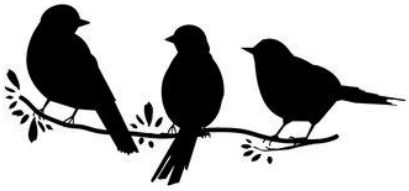


How many squares are in the image above?

Solution: If the total picture is 6x6, then the number of squares by size is 1x1: 4, 2x2: 5, 3x3: 4, 4x4: 1, 5x5: 0, 6x6: 1 so in total there are 15 squares.

Week #2 Fall 2022

Problem of the Week Solution



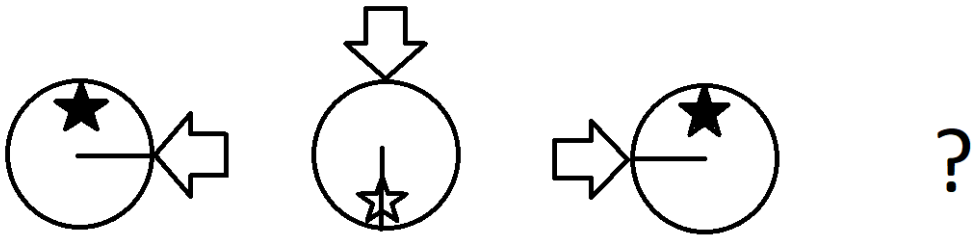
How many birds will there be on the 5th day?

A bird enthusiast has been watching the birds in her garden each day and has noticed something interesting. On the first day, there were just three birds—one cardinal, one nut hatch, and one finch. On the second day there were 12 birds (more of each type). On the third day there were 50 birds, but even more interesting was the number of birds of each species. The enthusiast noticed that when she paid attention to the individual species, the number of birds was growing by a constant factor. For example, there was 1 finch on the first day, 3 finches on the second day, and 9 finches on the third day (each day had three times as many as the previous day). The growth factors from species to species was different, but each species of bird was growing by a constant factor. On the fourth day the same thing happened and in total there were 216 birds. How many birds will there be on the fifth day?

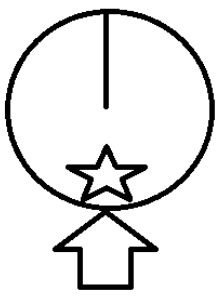
Solution: Using the information above, the missing growth factors must be 4 and 5. So on the fifth day there must be 81+256+625=962 birds.

Week #3 Fall 2022

Problem of the Week Solution



What is the next image?



Solution:

Week #4 Fall 2022

Problem of the Week Solution

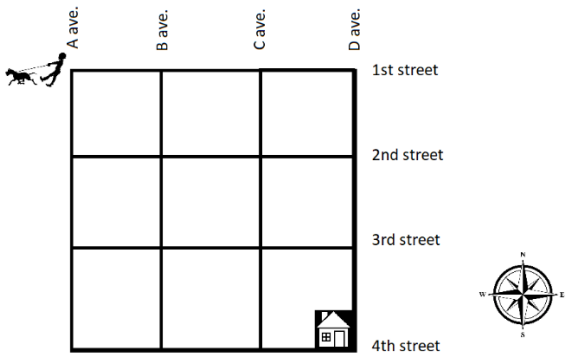
There are four students who decided to hold a race each month for the past three months. The freshman, who came in last during the July race, came in behind the sophomore during the August race. The sophomore won the September race but came behind the Junior during the July race. The Junior won the August race. How did the Senior do in all three races if no student placed in the same spot for any two races?

	July	August	September
First	Senior	Junior	Sophmore
Second	Junior	Sophmore	Freshman
Third	Sophmore	Freshman	Senior
Fourth	Freshman	Senior	Junior

The above table shows how everyone did. This is the only way it could have worked out. Red indicates placings we were told directly, blue indicates deductive placings, Green is how the Senior did.

Week #5 Fall 2022

Problem of the Week Solution



Steven lives on the corner of 4th street and D avenue, but he is currently walking his dog and at the corner of 1st street and A avenue. At the moment he is trying to get home before his friend Tina gets to his house. As he walks towards home he will use probability to determine which way to go at each intersection. 25% of the time he will head East, and 75% of the time he will head South. He will do this until he reaches D avenue or 4th street (once he reaches either he will head straight home). Tina has actually arrived at Steven's home at the same time that Steven starts heading back. She wants to surprise him and plans to walk towards the corner of 1st street and A avenue. She will choose North 20% of the time and West 80% of the time until she gets to either 1st street or A avenue. What is the probability that they cross paths if they walk at the same speed?

Solution: In order for Steven and Tina to cross paths, they would need to meet after each walking three blocks so they could only possibly meet at the corner of D and 1st or the corner of C and 2nd, or the corner of B and 3rd, or the corner of A and 4th. The probabilities that they meet at each corner respectively is 1/8000, 108/8000, 1296/8000, and 1728/8000 so in total, the odds that they meet is up 3133/8000 or 39.1625%

Week #6 Fall 2022

Problem of the Week Solution

Your boss asked you to pick up his son from a Halloween party. Unfortunately, you don't know which kid he is because all the kids are dressed up. You have narrowed it down to either the mummy, the pirate, or the ghost. Two of them are liars and one of them is honest. The ghost tells you that he is not your boss's son. The pirate tells you that the mummy is your boss's son, and the mummy tells you that the pirate is not telling the truth. Which of these little monsters is actually your boss's son?

Solution: The only scenario that works is that the Ghost is your boss's son. In this case the Ghost and Pirate are both telling lies and the mummy is telling the truth.

Week #7 Fall 2022

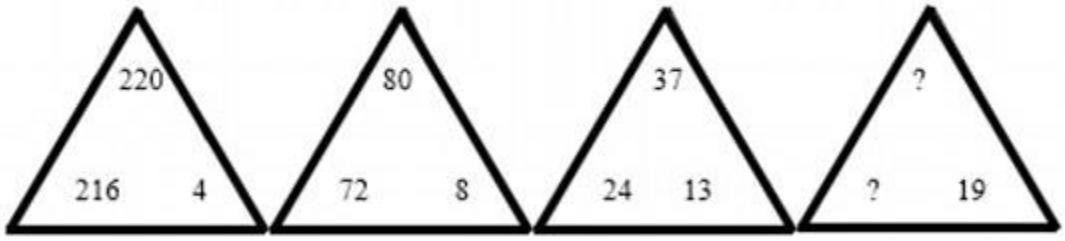
Problem of the Week Solution

Grandma has just made a batch of cookies. She is about to eat them all, when her husband arrives home. Fortunately the number of cookies in the batch will allow her to evenly split them with her husband. They are about to share the cookies when their daughter walks in. Thankfully, there is enough cookies to share among all three of them evenly. Grandma's batch of cookies is tested again when her grandson shows up, and again when her granddaughter shows up, and again when her grandchildren's 7 friends show up. Each time, the number of cookies she has made can be evenly split among the cookie eaters. What is the smallest number of cookies she could have made in a batch (assume she did make cookies)?

Solution. The number must be a multiple of 1, 2, 3, 4, 5, and 12 so it must be 60.

Week #8 Fall 2022

Problem of the Week Solution

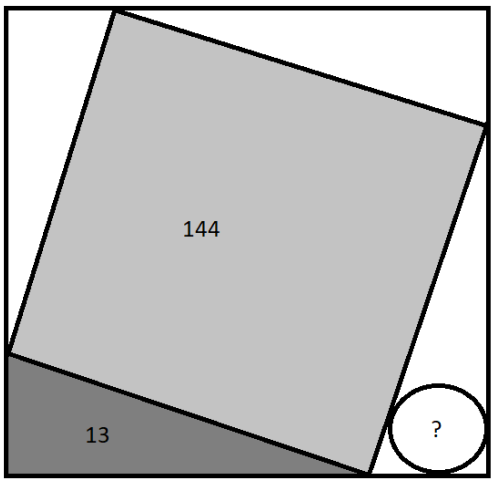


What numbers should replace the question marks in the fourth triangle?

Solution: If you add the two bottom numbers you always get the number at the top. To figure out what goes in the lower left corner, notice that 216/3=72 and 72/3=24 so since 24/3=8, it seems that 8 should go in the lower left corner and since 19+8=27, 27 should go in the upper corner.

Week #9 Fall 2022

Problem of the Week Solution



The area of the triangle and the inscribed square are given above. What is the area of the inscribed circle?

Solution: If r is the radius of the circle, x is the length of one of the legs of the triangle and y is the length of the other leg then the area of the triangle is $r^2 + r(x - r) + r(y - r)$ which yields r=1 so the area is π

Week #10 Fall 2022

Problem of the Week Solution

What's my number?

Use the digits 1 through 9 once only to form a nine-digit number such that the first (leftmost) eight digits form a number divisible by 8, the first seven digits form a number divisible by 7, and so on. How many such nine-digit numbers are there that satisfy these requirements?

Solution: Using rules such as "numbers divisible by 5 must end in 0 or 5" will give us only one possibility. It is 381654729