Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

Friday, January 6, 2023
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number (*i.e.* Problem 7).

Please make sure your answers are dark and legible.

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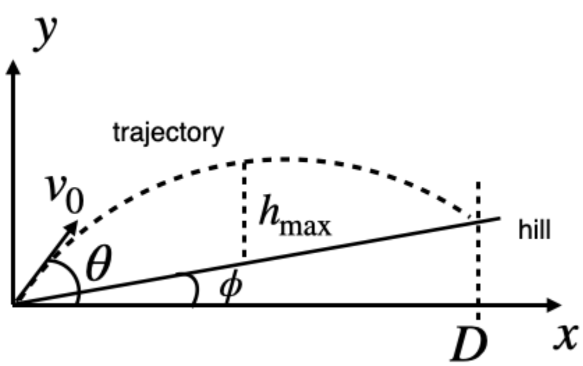
Problem 1 (10 points)

A cannonball is fired with velocity v at an angle θ to the horizontal up a steady incline that makes an angle ϕ to the horizontal.
(a) Ignoring air resistance, calculate the horizontal distance D the cannonball travels before hitting the ground. [4 points]
(b) If $\theta = 35$ deg and $\phi = 15$ deg and $v = 110\,\mathrm{m/s}$: What is the height of the tallest wall h_{max} (measured vertically from the incline) you could build on the hill and still have the cannonball clear the top of it, again ignoring air resistance? ($g = 9.81\,\mathrm{m/s^2}$) [6 Points]



Problem 2 (10 points)

Consider a body that is confined to move in a vertical plane, the $x - z$ plane, with a gravitational force in the $-\hat{z}$ direction. The body has mass m and moves in the plane subject to the (constant) gravitational force mg and an additional "central" force of the form $f = -Ar^{-1/2}$, where $r^2 = x^2 + z^2$. This additional force is thus directed towards the origin. Choose the appropriate generalized coordinates and let the gravitational potential be zero along a horizontal line through the origin ($z = 0$).

(a) Find the Lagrangian equations of motion for the system. [6 points]

(b) Show whether or not angular momentum about the origin is conserved. [4 points].

Problem 3 (10 points)

Suppose a comet has a highly elliptical orbit about the Sun with an orbital period of 86 yr. The eccentricity is $\epsilon = 0.8$. Express your answers in Astronomical Units, which is the average distance between Sun and Earth (1 AU $= 1.5 \times 10^{11}$ m).
(a) Find the length of the semimajor axis. [4 points]
(b) Find the distance of the closest approach r_p to the Sun (also called perihelion) and the farthest distance r_a (also called aphelion) the comet lies from the Sun. [6 points]

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PART II

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13:30 — 15:30

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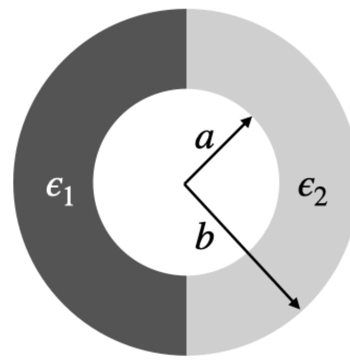
Problem 4 (10 points)

One half of the region between the plates of a spherical capacitor of inner and outer radii a and b is filled with a linear isotropic dielectric of permittivity ϵ_1 and the other half has permittivity ϵ_2, as shown in the figure. If the inner plate has total charge Q and the outer plate has a total charge $-Q$, find:
(a) the electric displacements D_1 and D_2 in the region of ϵ_1 and ϵ_2 [4 points]
(b) the electric fields in ϵ_1 and ϵ_2 [3 points]
(c) the total capacitance of this system [3 points]



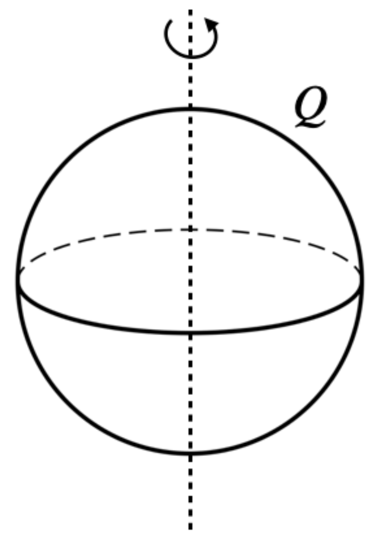
Problem 5 (10 points)

A charge Q is uniformly distributed over the surface of a sphere of radius R. The material inside and outside the sphere has the properties of the vacuum.

(a) Calculate the electrostatic energy in all space. [3 points]

(b) Now, the sphere rotates around an axis through a diameter with constant angular speed ω. Calculate the magnetic field at the center of the sphere. [4 points]

(c) What is the magnetic dipole moment of the rotating sphere? [3 points]



Problem 6 (10 points)

Consider a possible solution to Maxwell's equations given by

$$\vec{A}(\vec{r},t) = \vec{A}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \quad \phi(\vec{r},t) = \phi_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{1}$$

where $\vec{A}$ is the vector potential and ϕ is the scalar potential. Assuming $\vec{A}_0$, ϕ_0, $\vec{k}$, and ω are constants in space-time.
(a) [2 points] Show that the electric and magnetic fields from the provided scalar and vector potentials are

$$\begin{aligned}\vec{E}(\vec{r},t) &= -i\vec{k}\phi(\vec{r},t) + i\frac{\omega}{c}\vec{A}(\vec{r},t).\\ \vec{B}(\vec{r},t) &= i\vec{k}\times\vec{A}(\vec{r},t).\end{aligned}$$

Individual Maxwell's equations may or may not impose constraints on the constants, $\vec{A}_0$, ϕ_0, $\vec{k}$, and ω in the vector and scalar potentials. Examine each of Maxwell's equations given below and determine the constraints they impose on these constants if any.

(b) $\vec{\nabla}\cdot\vec{B} = 0$ [2 points]
(c) $\vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = 0$ [2 points]
(d) $\vec{\nabla}\cdot\vec{E} = 0$ [2 points]
(e) $\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = 0$ [2 points]

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PART III

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Problem 7 (10 points)

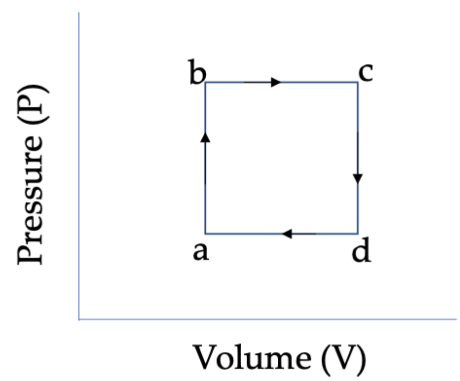
The Figure shows a reversible cycle through which 1 mole of a monoatomic ideal gas is taken. The point 'a' corresponds to (V_0, P_0) and the point 'c' corresponds to $(2V_0, 2P_0)$. Assume that $P_0 = 10^5\,\text{Pa}$ and $V_0 = 0.05\,\text{m}^3$. The molar specific heats, $C_v = (3/2)R$ and $C_p = (5/2)R$

(a) Calculate the work done during the cycle [2 points].

(b) Identify the steps where the energy is added as heat and calculate the amount of heat added [2 points].

(c) What is the efficiency of the engine [3 points]?

(d) What is the efficiency of a Carnot engine operating between the highest and lowest temperatures that occur in the cycle [3 points]?



Problem 8 (10 points)

Consider a system of three spins arranged in an equilateral triangle, each spin interacting with the other two. Each spin can only point up or down with the values of $S = \pm 1$, respectively. The energy of the spins in a magnetic field B is described by the Hamiltonian

$$H = -J(S_1 S_2 + S_2 S_3 + S_1 S_3) - F(S_1 + S_2 + S_3),$$

where $F = \mu B$.
(a) Find the partition function for the system [4 points].
(b) Determine the average spin [3 points].
(c) Calculate the average energy [3 points].

Problem 9 (10 points)

A single slit of width, $a = 5.5$ micron is illuminated with a light signal containing only two wavelengths, $\lambda_1 = 400\,\text{nm}$ and $\lambda_2 = 500\,\text{nm}$. The lights incident perpendicular on the screen containing the slit.
(a) What is the angular separation between the second order minima of these two wavelengths? [4 pionts]
(b) What is the smallest angle at which two of the resulting minima are superimposed? [3 points]
(c) What is the highest order for which minima for both wavelengths are present in the diffraction pattern? [3 points]
[Useful relation: $a \sin\theta = n\lambda$, where $n = 1, 2, 3, \ldots$]

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PART IV

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Problem 10 (10 points)

A particle of energy E and mass m moves from $-\infty$ towards a potential of the form

$$V(x) = C(\delta(x-a) + \delta(x+a)).$$

(a) Write the Schroedinger equation for the one-dimensional problem. [1 point]
(b) In view of calculating the transmission coefficient, write the wave solution for the three regions $x < -a, -a < x < a, x > a$. [2 points]
(c) Write the boundary conditions for the wavefunction at the two δ. [4 points]
(d) Calculate the transmission coefficient T for the particle to cross the potential. [3 points]

Problem 11 (10 points)

A 3-dimensional isotropic harmonic oscillator has energy eigenvalues $E_n = \hbar\omega(n + 3/2)$, where $n = 0, 1, 2, \cdots$.

(a) Find the degree of degeneracy D_2 of the quantum state $n = 2$. [4 points]

(b) Find the degree of degeneracy D_n of the quantum state n for general n. [6 points]

Problem 12 (10 points)

The ground state of a 1-dimensional harmonic oscillator has the Gaussian form $\psi(x) = Ce^{-k^2x^2}$. The corresponding Hamiltonian is $H_0 = (1/2)(p^2/m + m\omega^2x^2)$.
(a) Using Schroedinger equation, find k and find the energy of the ground state E_0. [4 points]
(b) Find the normalization constant C. [1 point] (Hint: $\int_{-\infty}^{\infty} dx e^{-x^2} = \sqrt{\pi}$.)
(c) A small defect is introduced in the Hamiltonian, $H_1 = \lambda|x|$, with λ a constant, so that $H = H_0 + H_1$. Using perturbation theory evaluate the change ΔE to the ground state energy E_0. [5 points]