Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

Friday, May 4, 2018
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part III).

Please make sure your answers are dark and legible.

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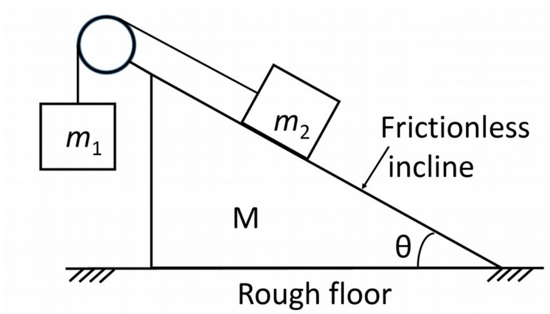
1. 10 points

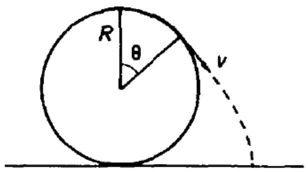
An inclined plane of mass M rests on a rough floor with coefficient of static friction μ. A mass m_1 is suspended by a string which passes over a frictionless and massless peg at the upper end of the incline and attaches to a mass m_2 which slides on the frictionless incline. The incline makes an angle θ with the horizontal.



(a) (6 pts) Solve for the accelerations of m_1, m_2 and the tension in the string when μ is very large.
(b) (4 pts) Find the smallest coefficient of friction for with the inclined plane will remain at rest.

2. 10 points

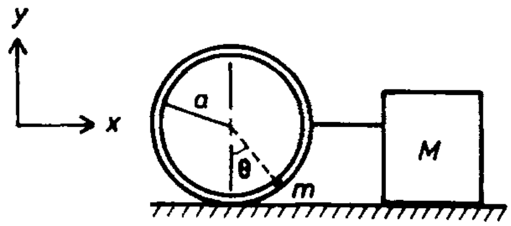
A smooth sphere is attached to a horizontal plane and does not move. A point particle of mass *m* slides frictionlessly down the sphere, starting from rest at the top ($\theta = 0$). Let *R* be the radius of the sphere.



(a) (3 pts) Write the equation describing the radial force that the sphere exerts on the particle.
(b) (4 pts) What are the speed and angle θ of the particle when it leaves the sphere?
(c) (3 pts) Determine the velocity of the particle (both the magnitude and direction) at the instant right before it strikes the horizontal plane.

3. 10 points

A block of mass M is rigidly connected to a massless circular track of radius a on a frictionless horizontal table as shown in the figure below. A particle of mass m is confined to move without friction on the circular track which is vertical.



(a) (3 pts) Write the Lagrangian of the system, using θ as one coordinate.
(b) (3 pts) Find the equations of motion.
(c) (4 pts) In the limit of small angles, find the angular frequency of oscillation.

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PART II

Friday, May 4, 2018
13:30 — 15:30

ROOM 245 PHYSICS RESEARCH BUILDING

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4. **10 points** The $x-y$ infinite plane has a surface charge density $\sigma(x)=\sigma_0\cos{(kx)}$. This problem can not be solved with separation of variables.

   a) (2 pts.) Find which components of the electric field are zero, on which Cartesian coordinates the electric field depends, and its periodicity if any.

   b) (2pt.) Evaluate the electric field $z-$ component at the boundaries.

   c) (3pt.) Evaluate the electric field in the $x=0$ plane. Integrals that you may need are in Chapter 18 (yellow book) or 15 (brown book).

   d) (3 pts.) Evaluate the electric field everywhere. This integral may be useful

$$\int_{-\infty}^{\infty} dx \frac{x\sin{(kx)}}{x^2+z^2}=\pi e^{-kz}.$$

5. **12 points** A very long solid cylinder of length D and radius R is immersed in a magnetic field $\mathbf{B} = B\hat{z}, (B > 0)$ which is parallel to the cylinder axis.

The cylinder is made of an insulator with total charge $+Q$ uniformly distributed through the material, and a shell of negligible mass and thickness on the outside, which has a charge $-Q$ uniformly distributed on the surface.

a) (2pt.) Compute the electric field everywhere.

b) (2 pts.) Compute the Poynting vector $\mathbf{S}$ for the configuration.

c) (2 pts.) the electromagnetic momentum density $\mathbf{p}$ is defined as $\mathbf{S}/c^n$, where c is the speed of light. Use dimensional analysis to find the power n.

d) (2 pts.) Find the total momentum $\mathbf{P}$ for the configuration.

e) (2 pts.) Find the angular momentum density and total angular momentum, $\mathbf{l}$ and $\mathbf{L}$ of the configuration.

f) (2 pts.) Over a very long period of time charge leaks through the cylinder and the shell and cylinder become neutrally charged. Describe the motion acquired by the cylinder including any directionality. Assume that the cylinder has mass M. (even if you were unable to answer the previous two questions you can derive the result based on the Lorentz force on the migrating charges).

6. **8 points**

In the figure there are four resistors, R_1, R_2, R_3, R_4, and four terminals A, B, C, D.

a) (2pts.) Evaluate the resistance when a battery is connected to terminals A and B.

b) (2 pts.) Evaluate the resistance when a battery is connected to terminals A and C.

c) (2 pts.) Evaluate the resistance when a battery is connected to terminals A and D.

d) (2 pts.) Assuming that the first three resistances are 1Ω while $R_4 = 10\Omega$, find which of the three configurations described above delivers the largest power to the circuit and compute the delivered power for an applied voltage of 12V.

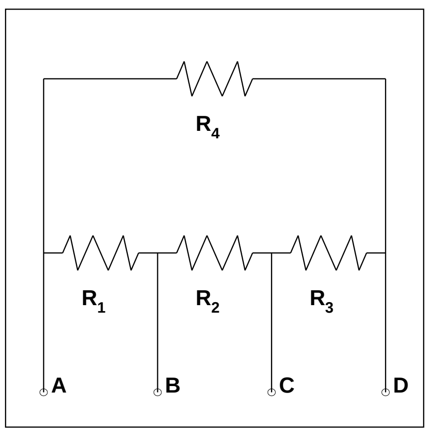

Figure 1:

Ph.D. QUALIFYING EXAMINATION
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PART III

MONDAY, May 7, 2018
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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7. **10 points**

a) (2 pts.) Write the time-independent Schroedinger equation for the hydrogen atom in spherical coordinates. Assume that the proton is stationary. Ignore spin angular momenta. Use the S.I. (mks) system of units. See p. 126 of Shaum's mathematical reference book regarding spherical coordinates.

b) (5 pts.) Consider the stationary state

$$\psi = Are^{-r/2a}\cos\theta$$

where A is a constant. Find the energy eigenvalue and the constant a.

c) (3 pts.) Find the expectation value of L^2 where

$$L^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right]$$

8. **10 points**

a) (4 pts.) Using Cartesian (rectangular) coordinates, find the operator L^2, where L is the total orbital angular momentum.

b) (2 pts.) Evaluate the commutators $[x, p_x]$, $[L_x, L_y]$, $[L^2, L_x]$, and $[L^2, \vec{L}]$.

c) (4 pts.) Suppose that a particle is in a state with orbital angular momentum quantum numbers $(l = 1, m_l)$, and spin quantum numbers $(s = 3/2, m_s)$. State the possible eigenvalues of L^2, L_z, S^2, and S_z. State the largest and smallest possible eigenvalues of J^2 where $\vec{J} = \vec{L} + \vec{S}$.

9. **10 points**

A particle of mass m is confined to move in one dimension inside an infinite square well with $V = 0$ for $-a < x < a$.

a) (6 pts.) Find the normalized time-independent wavefunctions for the ground state and the first and second excited states. Show that each wavefunction obeys the required boundary conditions and sketch each wavefunction.

b) (2 pts.) Find the energies of each of the three states.

c) (2 pts.) Suppose that a small perturbation $\alpha\delta(x)$ is applied to the potential energy, where α is a positive constant. Find the change in energy for each of the three states.

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PART IV

Monday, May 7, 2018
13:30 — 15:30

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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Some constants:
$m_e = 9.1 \cdot 10^{-31}$ kg,
$N_A = 6.02 \cdot 10^{23}$ mol^{-1}
$\hbar = 1.05 \cdot 10^{-34}$ J $\cdot$ s
$k_B = 1.38 \cdot 10^{-23}$ J/K

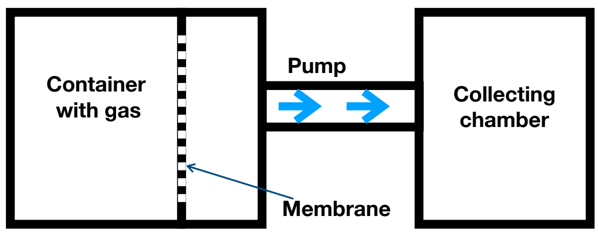
10. Consider a photon of energy $\varepsilon_\gamma = m_e c^2$, where m_e is the electron mass, scattering off an electron at rest. Both the scattered photon and the recoil electron are detected.

   (a) (1 pts.) Write down the relations between the energy and the magnitude of the momentum for the photon and the electron.

   (b) (6 pts.) Find the relation between the direction of the recoiled electron (relative to the direction of the incident photon) and its energy.

   (c) (3 pts.) What is the maximum possible energy the electron can have after scattering? What is the relative change in the photon wave length in that case?

11. Consider an ideal gas of molecules of mass m at temperature T and pressure p in a container of volume V. One of the container walls is a membrane containing holes of total area A. Gas molecules can pass through these holes by effusion and then be pumped off to the collecting chamber, such that the gas pressure from the other side of membrane is negligible (see Figure below). The temperature and pressure inside the container are maintained constant by continuously adding gas to compensate for particles effused through the membrane.



(a) (4 pts.) Starting from canonical distribution, derive the particle distribution in speed, dw/dv, normalized to unity (the probability density for a particle to have a speed v). Calculate the mean speed $\langle v \rangle$.

(b) (3 pts.) Show that the rate of effusion (number of particles "escaped" via the membrane per unit time) equals $An \langle v \rangle /4$, where n is the particle density in the container.

(c) (3 pts.) Suppose now that the gas consists of two particle species, with masses m_1 and m_2. Assume that the concentrations c_1 and c_2 in the container are maintained constant. Find the concentrations of gases, c_1' and c_2', in the collecting chamber after the effusion. How does the ratio c_1'/c_2' depend on temperature?

12. To a good approximation, the conducting electrons (of mass m_e) in solid copper (one conducting electron for each atom) can be treated as an ideal Fermi gas.

    (a) (4 pts.) Calculate the Fermi energy of the electron gas .

    (b) (3 pts.) Formulate the condition for a Fermi gas to be *degenerate*. Make a quantitative comparison of the Fermi energy and show that at room temperature the gas of electrons is degenerate. (Recall that for copper atomic number $A = 64$, density is $8.96\ \mathrm{g/cm^3}$).

    (c) (3 pts.) Treating the gas as a *completely* degenerate ideal Fermi gas of particles with magnetic moment μ_e, calculate the *paramagnetic* (Pauli) part of its magnetic susceptibility.