Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

WEDNESDAY, January 4, 2017
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

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1. Consider circular orbits in the field of a central force $f(r) = -kr^n$
   a) For circular orbits, what is the relation between orbital velocity v and radius r? [3 Points]
   b) For what values of n are these circular orbits stable to small perturbations? [5 Points]
   c) Specially, are the circular orbits stable or unstable for n=-3? [2 Points]

2. Two identical rods of mass m and length l ($I = 1/12\ ml^2$ about its center of gravity) are connected to the ceiling and together vertically by small flexible pieces of string. The system then forms a physical double pendulum.
a) Determine the Lagrangian of the system in terms of the angles θ and ϕ, as shown in the figure below. [5 Points]
b) Find the equations of motion assuming θ and ϕ are small. [3 Points]
(Hint: $\cos\alpha \approx 1 - \alpha^2/2$)
c) Find the frequencies of the normal modes of this system for small oscillations around the equilibrium position. [2 Points]

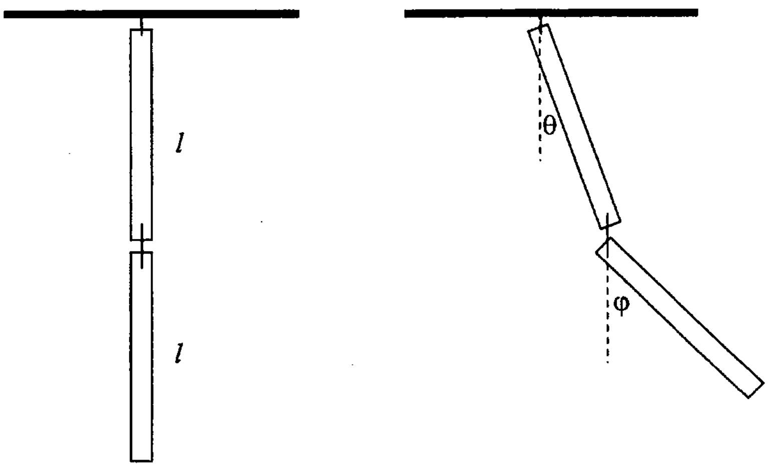

Figure 1: Figure for problem 2.

3. A cannonball is fired with velocity v at an angle θ to the horizontal up a steady incline that makes an angle ϕ to the horizontal.
a) Ignoring air resistance, calculate the horizontal distance the cannonball travels before hitting the ground. [4 points]
b) If $\theta = 30$ deg and $\phi = 10$ deg and $v = 100\ ms^{-1}$: What is the height of the tallest wall (measured vertically from the incline) you could build on the hill and still have the cannonball clear the top of it, again ignoring air resistance? Assume the wall has zero thickness. [6 Points]

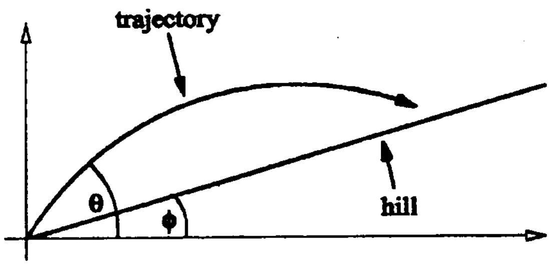

Figure 2: Figure for problem 3.

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PART II

WEDNESDAY, January 4, 2017
13:30 — 15:30

ROOM 245 PHYSICS RESEARCH BUILDING

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1. **10 points** A charge q is located at a distance d above an infinite grounded conductor plane. A second charge Q is located directly above q at elevation $3d$. See Figure.

   (3pt.) Compute the force on q.

   (3 pts.) Compute the surface charge density σ on the conducting plane.

   (2 pts.) Find the value of Q for which there is zero charge density at a radius $\rho = 2d$ on the conductor surface.

   (2 pts.) Evaluate the range of Q as a function of q for which both positive and negative surface charge densities are present on the plane. For simplicity assume that $q > 0$.

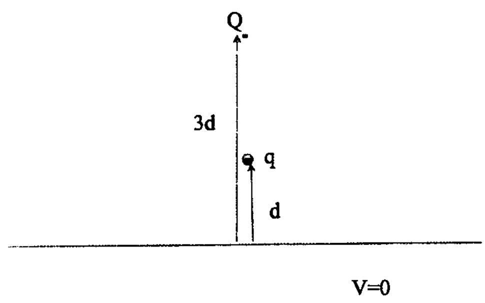

Figure 3: Two charges and infinite grounded plane. Not to scale.

2. **10 points** A metal disk of radius a rotates with angular velocity ω after having accelerated to a steady state. There is a uniform magnetic field B parallel to the disk axis. A circuit of resistance R is wired so that it is connected to both the center of the disk and the edge of the rotating disk through a sliding contact. See Figure.

   (4pt.) Find the force (both magnitude and direction) per unit charge on the disk.

   (4 pts.) Find the e.m.f. of the circuit

   (2pts.) Find the current, if any, circulating through the resistance.

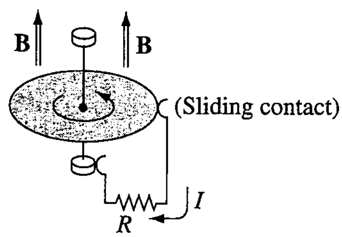

Figure 4: Rotating metal disk in a magnetic field, connected to a circuit.

3. **10 points** A circuit consists of a battery with e.m.f. ϵ, an inductor L, a resistance R_1 and a light bulb R_2. Initially the switch is open and no current flows. See figure.

(5pt.) The switch is closed at $t = 0$. Evaluate the current $I_1(t)$ flowing through R_1 as a function of time.

(3 pts.) After the switch has been closed for a long time, the switch is re-opened. Ignoring the spike created by the switch (there is no spike if $R_1 = R_2$), evaluate the current $I_2(t)$ flowing through the light bulb.

(2 pts.) Assume now that $L = 1$ Henry, $\epsilon = 12V$, $R_1 = 10\Omega, R_2 = 1\Omega$, and the switch opens over a time of order 10^{-4} seconds. This time consider that there will be a spike created by the switch. Describe how the current I_2 (or equivalently, the light of the light bulb) would behave when the switch is opened.

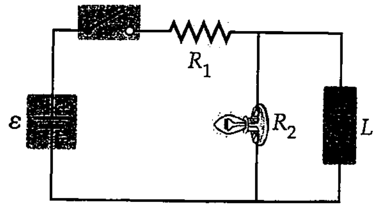

Figure 5: RL circuit with two resistors and one inductor.

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PART III

FRIDAY, January 6, 2017
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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Problem Number 1

Consider a one-dimensional potential well of depth V_o and width $2a$ centered at the origin. A particle of mass m and energy $E(E>0)$ is scattered by this potential well.

(a) [5 points] Solve the Schrodinger equation for $E>0$ and obtain the transmission coefficient T.

(b) [3 points] For finite E find the condition under which T approaches 1. Find the values of E for which this condition is satisfied.

(c) [2 points] For $T=1$, what is the ratio of the width of the well and the de Broglie wavelength of the particle while it is traversing the well? Interpret this ratio.

Problem Number 2

Consider the Hamiltonian of two identical noninteracting linear harmonic oscillators given by

$$H_o = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{m\omega^2}{2}\left(x_1^2 + x_2^2\right).$$

Here m and ω are, respectively, the mass and frequency of each oscillator.

(a) [2 points] What are the energies of the two lowest-lying energy states (that is, the ground state and the first excited state)? Are any of these two states degenerate? If yes, what is the degeneracy?

(b) [4 points] Now, suppose the two harmonic oscillators have a weak interaction between them of the form,

$$H' = \lambda m \omega^2 x_1 x_2$$

where λ is a dimensionless real number much smaller than unity. Using time-independent perturbation theory, find the energy of the ground state of this perturbed two-oscillator system to the lowest non-zero order.

(c) [4 points] Obtain an exact solution of the Hamiltonian, $H + H'$. Compare the exact eigenvalues of the perturbed oscillator system with the results obtained in part (b).

[HINT: For part (c), write p_1 and p_2 in terms of $p_\pm = (p_1 \pm p_2)/\sqrt{2}$.
Make similar replacements for x_1 and x_2 in terms of $x_\pm = (x_1 \pm x_2)/\sqrt{2}$.]
[USEFUL INFORMATION: For a one-dimensional harmonic oscillator of mass m and frequency ω ,

$$\langle p|x|q\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{q}\,\delta_{p,q-1} + \sqrt{q+1}\,\delta_{p,q+1}\right).¿$$

Problem Number 3

The wave function of a one-dimensional quantum system is

$$\psi(x)=\frac{N}{x^2+a^2}$$

with $-\infty \le x \le +\infty$.

(a) [1 point] What value of N normalizes the wave function $\psi(x)$?

(b) [4 points] Using normalized $\psi(x)$, determine the expectation values $\langle x\rangle, \langle x^2\rangle$ as well as Δx .

(c) [4 points] Using normalized $\psi(x)$, also determine the expectation values $\langle p\rangle, \langle p^2\rangle$ as well as Δp .

(d) [1 point] Explicitly verify that the uncertainty principle, $\Delta x \Delta p \ge \hbar/2$, is satisfied.

[USEFUL INFORMATION: Consider the integral

$$I_n=\int_0^\infty \frac{dx}{\left(x^2+a^2\right)^n}.$$

For $n=1$, $I_1=\frac{\pi}{2a}$.

For $n\ge 2$,

$$I_n=\frac{2n-3}{2(n-1)}\frac{1}{a^2}I_{n-1}.¿$$

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PART IV

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1. One mole of ideal monatomic gas undergoes a cycle as shown in the figure below.
   (a) [3 pts] Calculate the net work for a single cycle.
   (b) [3 pts] What is the difference in the internal energy between state C and A.
   (c) [4 pts] Calculate the heat generated when the system goes from $A \to B \to C$.
   Note: You might find the following integral helpful

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}(x\sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a}) + C \tag{1}$$

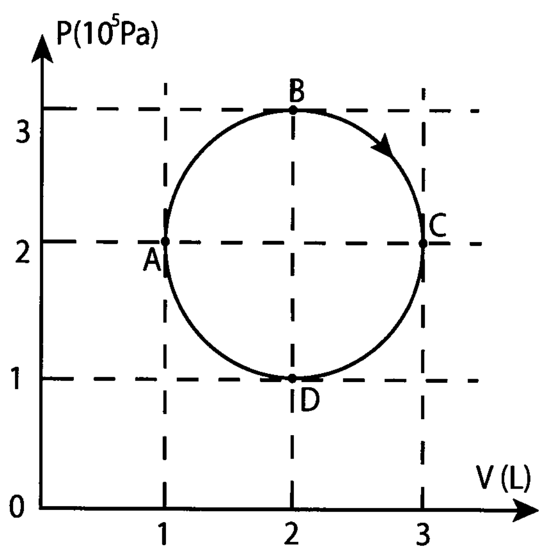

Figure 6: Thermodynamic cycle.

2. Consider a non-relativistic electron gas in a one-dimensional (1D) metal of length L with one free electron per atom. The atomic spacing d is 2.5 angstroms at $T = 0$. The mass of an electron is $m_0 = 9.1 \times 10^{-31}$ kg. Calculate the Fermi energy. [note: you must provide both analytical and numerical answers].

3. In the laboratory frame, a particle of mass M and momentum P directed along the $z-$axis decays in flight into a particle of mass m_1 and a particle of mass m_2. The two particles make angles of θ_1, θ_2 with the $z-$axis.

(a) [2pts.] compute the magnitude of the momenta of the two particles p_1, p_2 as a function of the given variables.

(b) [2pts] From this point onward make sure you treat the problem relativistically. Compute the corresponding angle θ^* made by the particle 1 in the M rest frame, as a function of the given variables.

(c) [3pts.] Compute the energies E_1^*, E_2^* and the momenta p_1^*, p_2^* in the M rest frame, as a function of the given variables.

(d) [3pts.] Evaluate the momentum P for which momentum p_1 in the laboratory frame is always greater than the momentum p_2 (assume that $m_1 > m_2$).