Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

FRIDAY, January 4, 2013
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

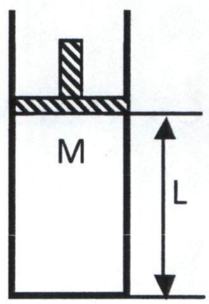
Do NOT write your name on the cover or anywhere else in the booklet!

1. **10 points** Consider a long cylindrical vessel, with the bottom end closed and the top end open. A piston of mass M can move freely up and down without friction (see figure below). The vessel is filled with an ideal gas. In equilibrium the distance between the piston and the bottom of the vessel is L. The cross section of the vessel is A, the external (atmospheric) pressure is P_A.

   a) The piston is initially shifted downwards by a small distance $x \ll L$. Find the pressure in the cylinder after the piston is shifted, assuming that the process is isothermal. (1 pts)

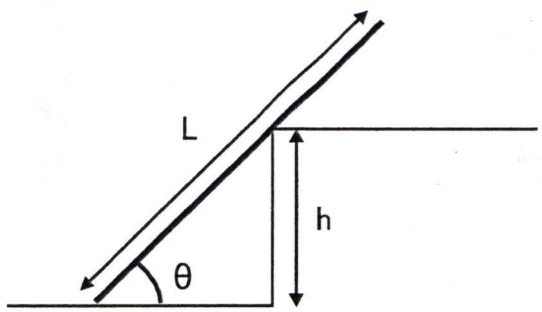
   b) After the piston is released, it starts to oscillate about the equilibrium point. Find the period of small oscillations (make the same assumptions as in part a)). (7 pts)

   c) What is the period of oscillations if the external pressure is zero? Comment on the result. (2 pts)



2. **10 points** A uniform ladder of weight W and length L is leaning at an angle θ against a step of height $h < L$ as shown in the figure below. There is static friction between the ladder and the ground, but no friction between the ladder and the vertical step. Find the minimal coefficient of friction between the ladder and the ground that would be necessary to keep the ladder from sliding.

Hint: The normal force at the corner is perpendicular to the ladder.



3. **10 points** An interstellar dust grain has a diameter of 1μm, and a density of $\rho = 3g/cm^3$. Consider a spherical, completely light-absorbing grain at the Earth's orbit, (1 A.U., or 1.5×10^8km), subject to the radiation pressure from the Sun. The Sun's total radiation intensity is $I = 1400\text{W}/m^2$ at 1 A.U.

   a) Calculate the force on the grain due to radiation pressure. (5 pts)

   b) Calculate the force due to the Sun's gravity. (4 pts)

   c) In which direction will a grain move, if it is initially at rest at 1 A.U.? (1 pts)

4. **10 points** The Spin operators for a spin-1/2 particle can be described by the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

a) Write the normalized eigenvectors of σ_z, $|+\rangle$ and $|-\rangle$ which are defined such that $\sigma_z |+\rangle = |+\rangle$ and $\sigma_z |-\rangle = -|-\rangle$, as column vectors in the same basis as the Pauli matrices given above. (You can assume without loss of generality that these eigenvectors are real.) (3 pts)

b) Consider an eigenvector $|\psi\rangle = a|+\rangle + b|-\rangle$. Assuming that a is a real number, $0 \leq a \leq 1$, show that $b = e^{i\phi}\sqrt{1-a^2}$, where ϕ is an arbitrary angle. (3 pts)

c) Find the expectation values of σ_x, σ_y and σ_z in the state $|\psi\rangle$ in terms of a and ϕ. (4 pts)

5. **10 points** Two energy levels E_1 and E_2, with $\Delta = E_2 - E_1$, are populated by N distinguishable non-interactive particles at temperature T.

a) Find the average energy of the system. (3 pts)

b) Consider the limits $T \to 0$ and $T \to \infty$ and find the average energy per particle in these two limits to the lowest order in Δ. (2 pts)

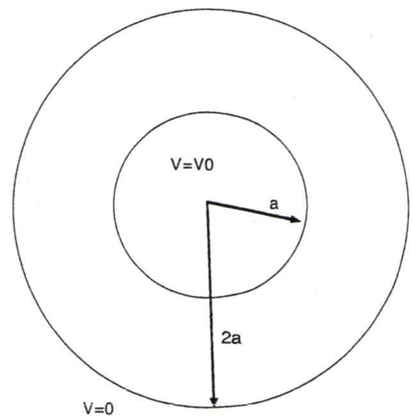
c) Calculate the specific heat C for this system. (3 pts)

d) Consider the limits $T \to 0$ and $T \to \infty$ and evaluate the specific heat per particle in these limits to the lowest order in Δ. (2 pts)

6. **10 points** A coaxial cable of length L consists of one cylinder of radius a and another of radius $2a$. The space between the two cylinders is filled with a medium of conductivity σ. The inner cylinder is held at V_0 and the outer cylinder is grounded (see figure below).

   a) Using Poisson's equation, compute the electric field and electric potential inside the medium. (4 pts)

   b) Compute the resistance between the two cylinders. (6 pts)



Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

MONDAY, January 7, 2013
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
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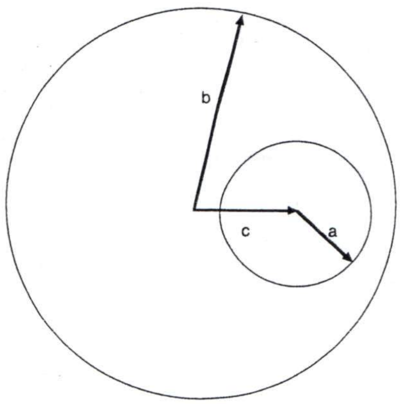
1. **10 points** An infinite conducting cylinder of radius b carries a current I. The current density J is uniform across the cylinder.

   a) Calculate the magnetic field $\mathbf{B}$ inside the cylinder. (3 pts)

   b) Express the magnetic field vector $\mathbf{B}$ inside the cylinder as a function of the current density vector $\mathbf{J}$ and transverse radius vector $\mathbf{r} = r(\cos\theta, \sin\theta, 0)$. (3 pts)

   c) Next, consider the cylinder with a cylindrical hole of radius $a < b$, centered at $c < b - a$ (see figure below). Assume that the center of the hole is along the x-axis. Compute the magnetic field inside the hole. (4 pts)

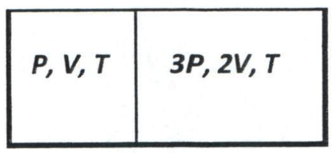


2. **10 points** Consider a thermally isolated system consisting of two volumes of an ideal gas separated by a thermally conducting movable partition, which is initially fixed. Initially, the temperatures, pressures, and volumes of the two parts are P, V, T and $3P$, $2V$, and T respectively (see figure below). The partition is now allowed to move without gases mixing, until the equilibrium is reached.

a) What is the change of the internal energy of the system after the equilibrium is reached? (1 pts)

b) What is the equilibrium temperature and pressure? (3 pts)

c) What is the change in the total entropy of the system after the equilibrium is reached? (6 pts)



3. **10 points** Consider a one-dimensional step potential of the form:

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x \geq 0 \end{cases}$$

where $V_0 > 0$. A quantum particle with mass m and energy $E > V_0$ is incident on this step *from the left* as shown in the figure below.

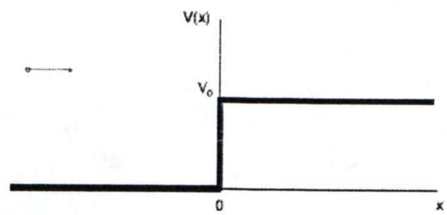
a) Solve the time-independent Schrödinger equation for this particle in the regions $x < 0$ and $x > 0$. (3 pts)

b) Apply the boundary conditions at the point $x = 0$ to match these solutions. (3 pts)

c) Derive expressions for the probabilities that the particle is reflected (R) and transmitted (T) by the step potential. (4 pts)

Hint: The probability density current is given by:

$$j(x) = \frac{\hbar}{2im}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right)$$



4. **10 points** A pendulum consists of a mass m attached to one end of a massless, extensionless string, whose other end is attached to the uppermost point of a fixed vertical disk of radius R, as shown in the figure below. Assume that the total length of the string is l and that $\pi R < l$.

a) Express the x and y position of the mass m in terms of the angle θ, the radius of the disk R and the total length of the string l, as shown in the figure. (1 pts)

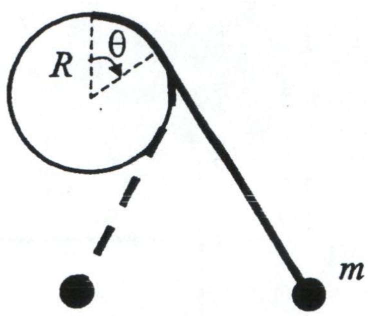
Hint: The string extends tangentially to the disk.

b) Write the Lagrangian in terms of the angle θ. (2 pts)

c) Use the Lagrangian formalism to find the equation of motion for small angles θ. (3 pts)

d) What is the equilibrium angle θ_0? (1 pts)

e) Find the frequency of oscillations about the equilibrium angle θ_0. (3 pts)



5. **10 points** The time-independent ground state and first excited state wave functions for a particle of mass m which moves in a one-dimensional potential $V(x)$ has the forms

$$\psi_0(x) = Ae^{-a^2x^2} \text{ and } \psi_1(x) = Bxe^{-a^2x^2}, \text{ respectively, where } a = \sqrt{\frac{m\omega}{2\hbar}},$$

and A and B are the normalization constants.

Using the time-independent Schrödinger equation, find the difference in energy eigenvalues between the ground and first excited states in terms of ω and $\hbar$.

6. **10 points** The gravitational potential near a black hole of mass M and Schwarzchild radius a can be described by a modified classical potential: $U(r) = -\frac{GM}{(r-a)}$.
(No further knowledge concerning black holes is required to solve this problem!)

a) Find an expression for the force acting on a particle of mass m in this gravitational potential. (1 pts)

b) Expand your answer using the result from a) to find the lowest order correction to the classical gravitational force at a distance $r \gg a$. (3 pts)

c) Express your answer in the form $r = r(\theta)$, where θ is the angle between a fixed axis and the radius vector to the particle as shown in the figure below. Consider just the classical and first order correction terms. You only need to find the solutions periodic in θ.(6 pts)

Hint: The radial equation in polar coordinates is $f(r) = m(\ddot{r} - r\dot{\theta}^2)$. Use the standard substitution $u = 1/r$.