Ph.D QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

FRIDAY, MAY 7, 2010 9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth of 10 points. Solve all six problems using a separate booklet for each problem. On the front cover of each booklet, write the following information:

1) your special ID number that you obtained from Delores Cowen
2) the problem number and the title of the exam (e.g., Problem 1, Part I)

Please make sure your answers are dark and legible.

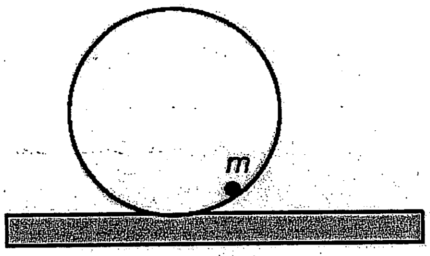
Do NOT write your name on the cover or anywhere else in the booklet!

Problem 1

A point mass m is attached to the rim of an otherwise uniform hoop of mass M and radius R, rocking on a horizontal plane along a straight line.

a) (5 pts) Construct the Lagrangian. Derive the equations of motion.

b) (5 pts) Find the frequency of small oscillations around the equilibrium point.



Problem 2

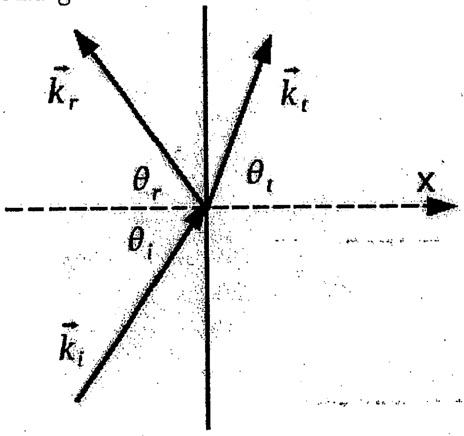
Consider a photon of energy E scattering off an electron of mass m_e at rest. Starting from energy-momentum conservation, derive the relationship between the shift in wavelength and the scattering angle.

Problem 3

Consider a 2d potential given by

$$V(x, y) = \begin{cases} 0, & x < 0, \\ V_0, & x > 0, \end{cases}$$

A stream of particles of mass m and energy E impinges on the $x = 0$ plane and gives rise to the reflected and transmitted waves as shown in the figure.



Consider a solution of the form:

$$\psi(\mathbf{r}) = \begin{cases} Ae^{i\mathbf{k}_i\cdot\mathbf{r}} + Be^{i\mathbf{k}_r\cdot\mathbf{r}}, & x < 0, \\ Ce^{i\mathbf{k}_t\cdot\mathbf{r}}, & x > 0, \end{cases}$$

a) Find the angle θ_r in terms of θ_i and constants E, V_0, m, and compare to the law of reflection.

b) Find the angle θ_t in terms of θ_i and constants E, V_0, m, and compare to the law of refraction.

c) Is there an angle $\theta_i = \theta_c$ above which all particles are reflected? If yes, what is this angle?

d) Find the fraction of particles being transmitted as a function of the incident angle.

Problem 4

Consider a particle of mass m in the infinitely deep potential well of width L, $0 \leq x \leq L$. At $t = 0$ the particle is localized in the left half of the well, with

$$\Psi(x,0)) = \begin{cases} \sqrt{2/L}, & 0 < x \leq L/2, \\ 0 & x > L/2, \end{cases}$$

a) Find $\Psi(x,t)$.

b) What is the probability that an energy measurement at time t will yield the ground state energy?

Problem 5

a) (5 pts) State the condition for two phases to coexist at a given pressure and temperature. From this condition derive the Clausius-Clapeyron equation for the slope of the liquid-gas coexistence curve

$$\left.\frac{dP}{dT}\right|_{coexistence} = \frac{L}{T\Delta v},$$

where L is the latent heat and v is the specific volume.

b) (5 pts) Using the equation derived in the first question, estimate the boiling temperature at a mountain top where the pressure is 2/3 of that at sea level.

The following data are provided:

- The boiling temperature at sea level is 373 K.
- The latent heat of water vapor transformation is $2.25{\cdot}10^6$ J/kg and can be approximated to be independent of temperature
- The gas constant $R = 8.314\ \frac{\mathrm{J}}{\mathrm{mol\ K}}$
- There are 18 g in one mole of water.

Problem 6

Consider a conducting sphere of radius a embedded in a medium whose conductivity is σ. The medium is a sphere that extends to such a large radius that we may consider it to be extended to infinity. The sphere is at potential V_s above the potential at infinity.

a) Is it possible to set the potential at infinity to be zero? Why?

b) What differential equation does $V(r)$ satisfy for $r > a$?

c) What is the current density in the medium?

d) What is the total current?

e) What is the total resistance (from a to ∞)?

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PART II

MONDAY, MAY 10, 2010 9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth of 10 points. Solve all six problems using a separate booklet for each problem. On the front cover of each booklet, write the following information:

1) your special ID number that you obtained from Delores Cowen
2) the problem number and the title of the exam (e.g., Problem 1, Part II)

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

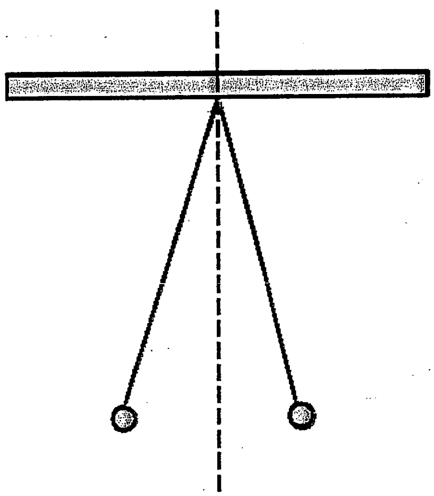
Problem 1

Two bodies with equal masses m are suspended in Earth's gravity by massless strings of length l as shown in the figure. The bodies have equal electric charges q. Consider only motion in the plane of the paper. Assume all angles $\ll 1$.

a) (4 pts) Derive the equations of motion

b) (3 pts) Find the equilibrium position.

c) (3 pts) Find the frequency(ies) of small oscillations.



Problem 2

Consider a spin 1/2 particle with Hamiltonian

$$\hat{H} = \gamma \hat{S}_z,$$

where $\gamma > 0$ is a constant.

a) Suppose the physical observable S_y is measured. What are the possible outcomes of the measurement?

b) Let the result of the S_y measurement at $t = 0$ be $\hbar/2$. Calculate the wave function for $t > 0$.

c) If S_y is measured again at a later time, does a t exists for which the probability to find $S_y = \hbar/2$ is zero? If yes, what is the time, if not why is there no such time?

Problem 3

Consider a particle with mass m in the potential

$$U(x) = \begin{cases} \infty, & x < 0, \\ -\alpha\, \delta(x - a), & x > 0, \end{cases}$$

where $\alpha > 0$ and $a > 0$. Find the condition for a bound state in such a potential.

Problem 4

Consider a cavity of volume V heated to a high temperature $T \gg m_e c^2$, m_e being the electron mass. In such a system electron-positron pair production can be viewed as a chemical reaction:

$$e^+ + e^- \leftrightarrow \gamma,$$

where γ represents a photon. Assume everything is in thermodynamical equilibrium. The densities of electron and positrons are equal.

a) What is the photon chemical potential? Calculate the chemical potential of the positrons and electrons.

b) Calculate the density of states of electrons and positrons. Assume that they are extremely relativistic and hence the mass can be ignored.

c) Calculate the densities of electrons and positrons as a function of T.

Problem 5

Two equal bodies have a thermal capacity that can be expressed as

$$C(T) = a\sqrt{T}$$

over the temperature range of interest. Initially they are at unequal temperatures $T_1 < T_2$.

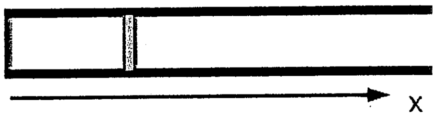
a) Consider two bodies brought into thermal contact.

- (3pts) Calculate the equilibrium temperature T_f.
- (3pts) Calculate the entropy change ΔS.

b) (4pts) Now consider a Carnot engine that does work by exchanging heat between the two bodies until the temperature of the two bodies become equal. Calculate the final common temperature of the two bodies and the total amount of work done by the engine.

Problem 6

The figure below presents a top view of the apparatus situated in a horizontal plane. There is a uniform magnetic field out of the page and the metal bar (of length L and mass m) is free to slide on the frictionless metal rails shown as thick black lines. Initially the bar is sliding at speed v_0 to the right. The rails can be taken to have no electrical resistance, and the resistance of the sliding bar is R.



a) Will the bar speed up, slow down, or maintain a constant velocity?

b) What is the current in the bar when it is moving at speed v?

c) What is the force on the bar?

d) What location does the bar approach as $t \to \infty$?

e) How much energy is dissipated due to the current in the bar?