Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

TUESDAY, JANUARY 6, 2009
9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems. The first four problems are worth 10 points each and the last two are worth 15 points each. Solve all six problems using a separate booklet for each problem. On the front cover of each booklet, write the following information:

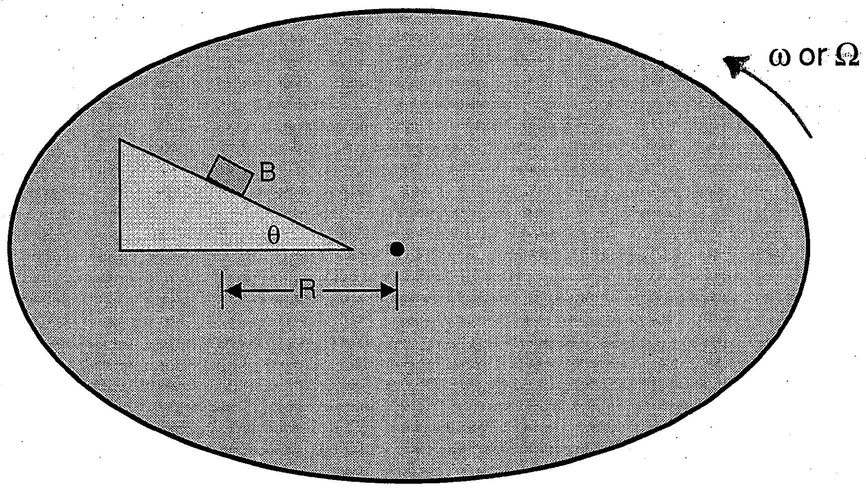
1) your special ID number that you obtained from Delores Cowen;
2) the problem number and the title of the exam (i.e., Problem 1, Part I);

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

Problem 1. (10 points)

An incline is fixed to a rotating turntable. A block B rests on the incline as shown in the figure. You are given the rotation radius of the block, R, the slope of the incline, θ, and the coefficient of static friction, μ.

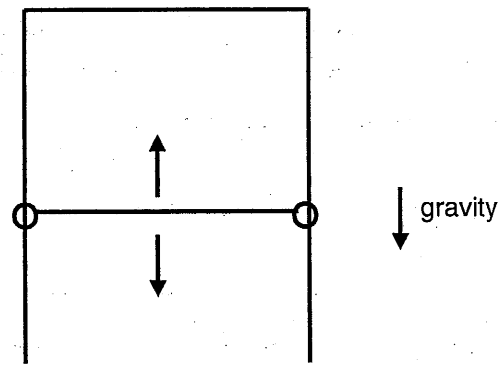


(a) (5 points) Determine the minimum angular speed of the turntable, ω, for which the block will not slide down the incline.

(b) (5 points) Determine the maximum angular speed of the turntable, Ω, for which the block will not slide up the incline.

Problem 2. (10 points)

Two parallel wires with vertical orientation are fixed in position near the surface of the Earth. A horizontal wire is fixed in position connecting the two vertical wires at the top. Another horizontal wire below the top wire can slide up or down while remaining horizontal and keeping in contact with the vertical wires, as shown in the figure. The four wires form a rectangular conducting loop at any given time.



The movable bottom wire has length L and mass m. A uniform magnetic field of magnitude B is present and points in a direction perpendicular to the plane of the loop. The bottom section of the wire is released from rest at a height of $z = 0$. Calculate the height $z(t)$ and speed $v(t)$ of the loose wire as a function of time t.

Problem 3. (10 points)

A pion at rest decays into a muon and a neutrino.

(a) (5 points) Find the total energy E_μ of the outgoing muon in terms of the two masses m_π and m_μ. Assume $m_\nu = 0$.

(b) (5 points) Find the velocity v_μ in terms of m_π and m_μ.

Problem 4. (10 points)

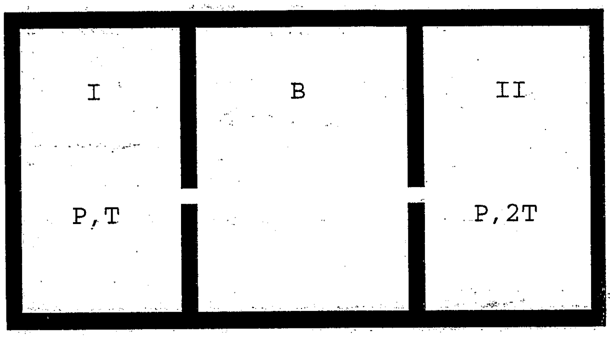
A quantum-mechanical, non-relativistic particle of total energy $3V_0$ is incident from the $-x$ axis on a potential given by

$$V = \begin{cases} 2V_0 & x < 0 \\ 0 & x > 0. \end{cases}$$

Find the probability (numerical value) that the particle will be transmitted to the positive side of the x axis.

Problem 5. (15 points)

Two containers I and II, filled by an ideal gas, are each connected by a small opening of area A to a common "ballast" container B as shown in the figure. The temperature and pressure in containers I and II are kept at (P, T) and $(P, 2T)$ respectively. The container B is thermally isolated. There is a unique equilibrium pressure and temperature (P_B, T_B) in container B when the pressures and temperatures in I and II remain unchanged.



(a) (6 points) For any of the containers, determine the average flow rate through an opening, i.e., the number of gas molecules hitting a wall per unit area per unit time, in terms of the pressure and temperature in that container. Disregard any numerical coefficients.

(b) (9 points) Using the result of (a) and taking into account that the energy in container B is conserved, and that the gas can flow in either direction through the openings, find the equilibrium pressure and temperature P_B and T_B in terms of the variables given in the problem.

Problem 6. (15 points)

A particle of mass m is in the state

$$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$$

where A and a are positive real constants.

(a) (3 points) Find A.

(b) (3 points) For what potential energy function does Ψ satisfy the Schroedinger equation?

(c) (6 points) Calculate the expectation values of x, x^2, p, and p^2.

(d) (3 points) Find the standard deviations σ_x and σ_p. Is their product consistent with the uncertainty principle?

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PART II

THURSDAY, JANUARY 8, 2009
9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems. The first four problems are worth 10 points each and the last two are worth 15 points each. Solve all six problems using a separate booklet for each problem. On the front cover of each booklet, write the following information:

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2) the problem number and the title of the exam (i.e., Problem #1, Part #II);

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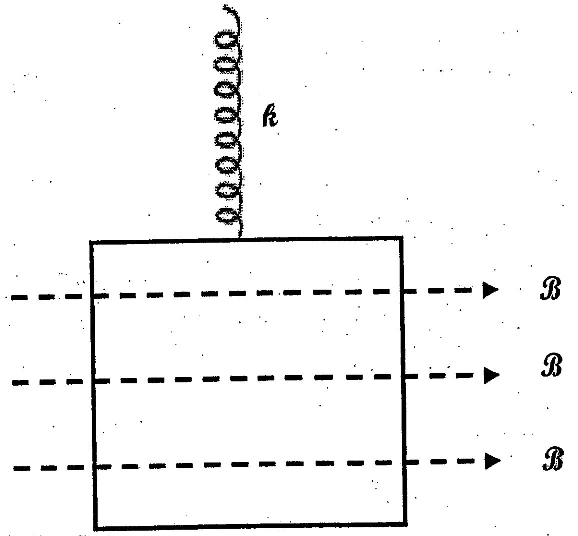
__Problem 1__. (10 points)

An ideal gas is contained in a cubical box with sides of length L located near the surface of the Earth. The gas has N particles, each of mass m, and the gas is in thermal equilibrium at absolute temperature T. Calculate the pressure as a function of the height z above the bottom of the box.

Problem 2. (10 points)

A square conducting loop has area A and is made of wire with total resistance R and linear mass density (mass/length) μ. As shown in the figure, the loop is suspended by a torsion spring of constant k in a uniform magnetic field $\vec{B} = (B, 0, 0)$. Assume the y-coordinate is vertical. The equilibrium orientation of the loop is in the (x, y) plane. Assume that the spring is non conductive, and neglect any self-inductance in the loop.



(a) (2 points) Find the moment of inertia I for the square loop about the axis of the torsion spring.

(b) (4 points) Write the equation of motion for the loop.

(c) (4 points) At time $t = 0$ the loop is given a small initial displacement θ_0, around the torsion spring axis. Then it is released. Using these initial conditions, solve the differential equation of part (b) assuming that θ is small and that the restoring torque due to the spring is larger than that due to the magnetic field.

Problem 3. (10 points)

An electron is in the spin state

$$\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}.$$

(a) (2 points) Determine the normalization constant A.

(b) (3 points) Find the expectation values of S_x, S_y and S_z using

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(c) (3 points) Find the "uncertainties" (standard deviations) σ_{S_x}, σ_{S_y} and σ_{S_z}.

(d) (2 points) Confirm that your results are consistent with the uncertainty relation

$$\sigma_{S_x}^2 \sigma_{S_y}^2 \geq \left(\frac{1}{2i} < i\hbar S_z >\right)^2.$$

Problem 4. (10 points)

Consider a non-interacting two-dimensional gas of N identical non-relativistic spin 1/2 particles of mass m at $T = 0$ confined to a square box of area A.

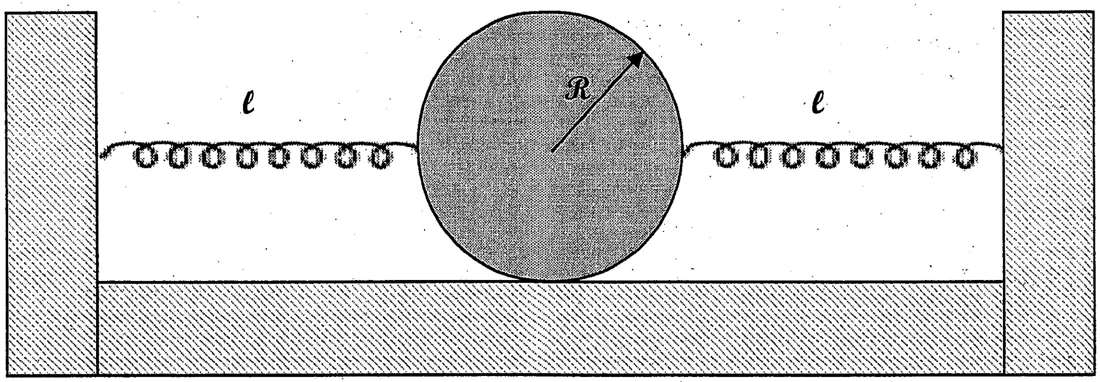
(a) (4 points) Find the momentum eigenvalues p_x and p_y for a single particle in the box in terms of quantum numbers n_x and n_y.

(b) (3 points) Find the density of states function dN/dE which gives the number of eigenstates dN per energy interval dE as a function of E.

(c) (3 points) Find the Fermi energy.

Problem 5. (15 points)

Two equal springs are attached to a uniform disk of mass M and radius R as shown in the figure. The springs have natural length l_0 and spring strength k. They are stretched to $l, l > l_0$. The disk is constrained to move in a plane, but there is no friction between the disk and the horizontal surface.



(a) (2 points) Give a set of coordinates that describe the possible motion and illustrate with a figure.

(b) (2 points) Derive the moment of inertia of the disk.

(c) (8 points) Write the equations of motion for the disk. Apply small oscillation approximations.

(d) (3 points) Find the angular frequencies of the normal oscillation modes for the disk, assuming small oscillations.

Problem 6. (15 points)

A long coaxial cable gas length L and is made of two conducting concentric cylinders separated by air. The outer cylinder has radius b and the inner cylinder has radius a.

(a) (5 points) Calculate the capacitance per unit length.

(b) (5 points) Calculate the inductance per unit length.

(c) (3 points) Calculate the pressure on the outer cylinder when a static charge Q is distributed uniformly on the outer cylinder and a static charge $-Q$ is distributed uniformly on the inner cylinder. (Hint: Apply the work-energy theorem.)

(d) (2 points) Calculate the pressure on the outer cylinder when a constant electrical current I flows on the inner cylinder along the direction of the central axis and a constant current $-I$ flows on the inner cylinder, also along the direction of the central axis.