Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

MONDAY, JAN. 7, 2008
9:00 A.M. — 1:00 P.M.

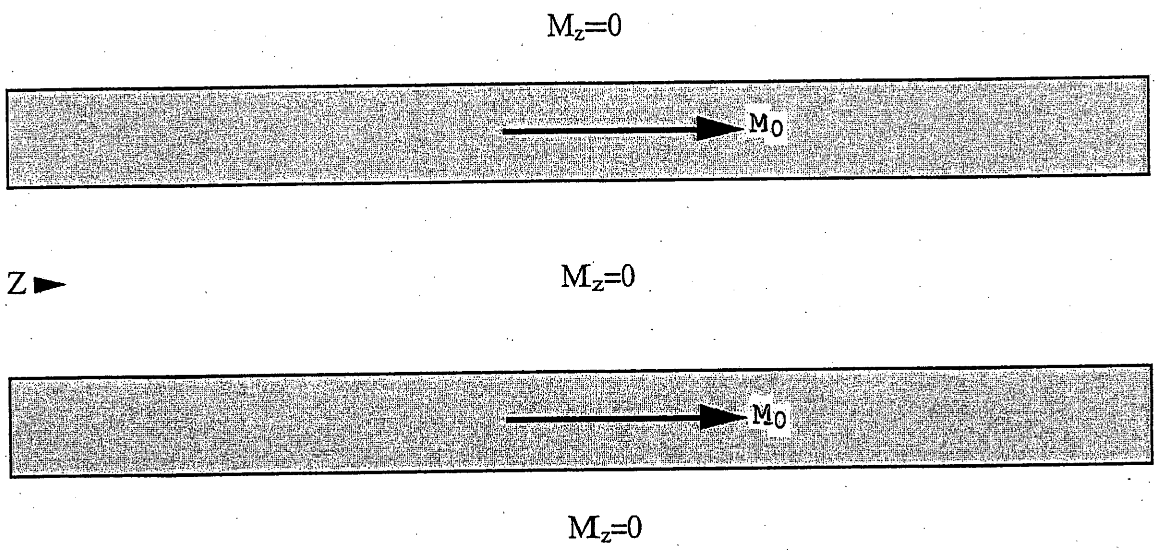
ROOM 224 PHYSICS BUILDING

INSTRUCTIONS: This paper contains six problems. The first four problems are worth 10 points per problem and the last two are worth 15 points. You are to solve all six using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

1) Your special ID number that you obtained from Delores Cowen
2) The problem number and the title of the exam (i.e. problem #1, part #1)
3) Please press hard and make your answers legible.

Please do NOT write your name on the cover or anywhere else in the booklet!

1. (10 points) The material of a very long, hollow, rod is uniformly magnetized with magnetization $M_z=M_0$ as shown in the sketch. (Although the rod is shown as having a finite length in the sketch, it is actually infinitely long).

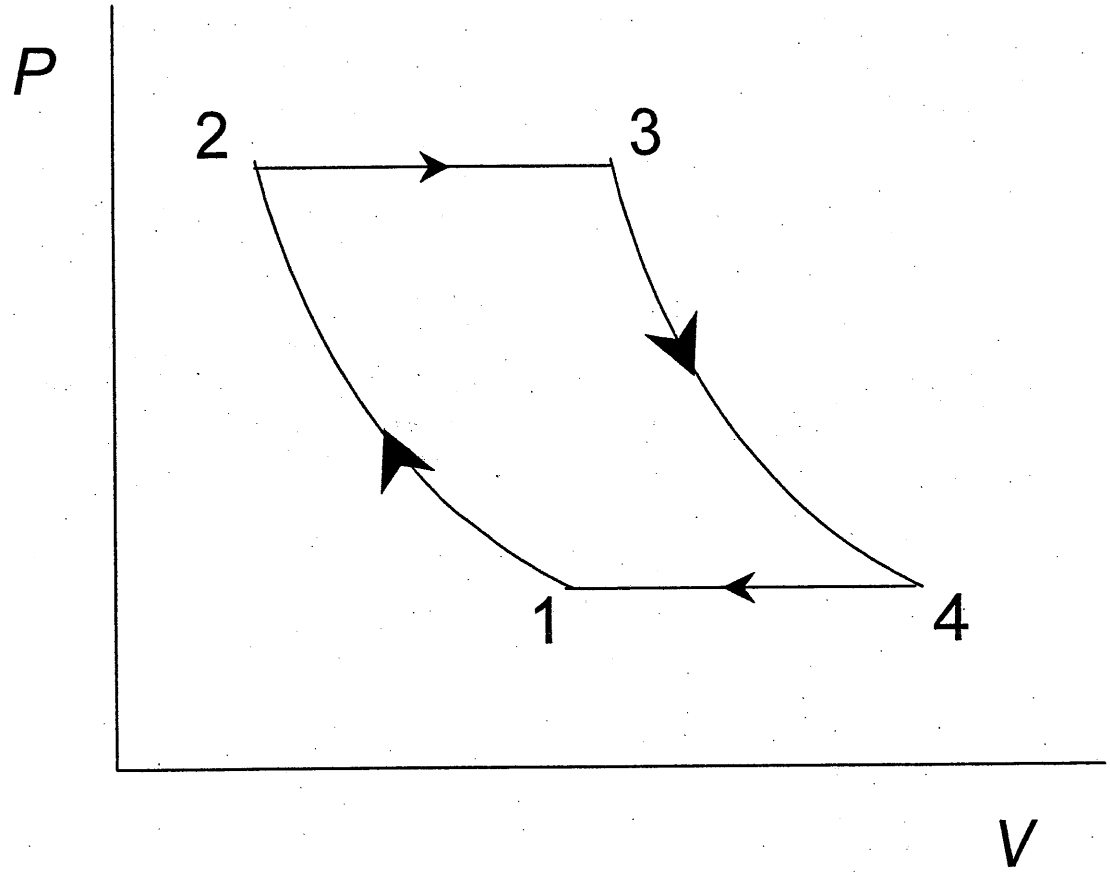


(a) (3 points) What is the value of the magnetic field **B** outside the rod?

(b) (3 points) What is the value of the magnetic fields **H**, **B** in the central hollow region where $M_z=0$?

(c) (4 points) What are the values of **B**, **H** in the material of the rod where the magnetization is M_0?

2. (10 points) Find the efficiency of the Joule cycle, consisting of two adiabats and two isobars (see figure). Express your answer in terms of the minimum and maximum pressures, P_1 and P_2, and $\gamma = C_P/C_V$. Assume that the gas is an ideal gas and that the heat capacities C_P (constant pressure) and C_V (constant volume) are constant.



3. (10 points) The Hamiltonian of a one-dimensional simple harmonic oscillator is given by

$$\hat{H} = \hbar\omega(\hat{a}_+\hat{a}_- + 1/2),$$

where

$$\hat{a}_\pm = (m\omega\hat{x} \mp i\hat{p})/\sqrt{2\hbar m\omega}, \qquad \hat{a}_+\psi_n = \sqrt{n+1}\,\psi_{n+1}, \qquad \hat{a}_-\psi_n = \sqrt{n}\,\psi_{n-1}$$

with ψ_n the eigenfunction.

(a) (2 points) What is the energy eigenvalue in the nth stationary state? Derive the normalized eigenfunction of the ground state.

(b) (1 point) Express the Hamiltonian in terms of $\hat{x}$ and $\hat{p}$.

(c) (5 points) Now an electric field E is switched on, adding an additional term $\hat{H}_1 = eE\hat{x}$ to the Hamiltonian. What is the exact value of the new ground state energy (without using perturbation theory)? Also find the new ground state eigenfunction.

(d) (2 points) A particle is initially in the ground state before the field is switched on. Assuming that the field switching process is very fast, what is the probability for finding the particle in the new ground state instantaneously after the switch?

4. (10 points) At time t=0 the spatial wave function for the electron in a hydrogen atom is described by

$$\psi(\mathbf{r},0)=\frac{1}{\sqrt{10}}\left(2\psi_{100}+\psi_{210}+\sqrt{2}\psi_{211}+\sqrt{3}\psi_{21-1}\right),$$

where ψ_{nlm} is the eigenfunction of the hydrogen atom, with n the principle quantum number, l the quantum number associated with the orbital angular momentum operator $\hat{L}^2$, and m the quantum number associated with $\hat{L}_z$.

(a) (3 points)What is the expectation value of the energy in the above system?

(b) (7 points)A measurement at later time finds the hydrogen atom in the eigenstate of $\hat{L}_x$ with eigenvalue $\hbar$. This measurement also shows that n=2 and l=1. Find the normalized wave function immediately after such a measurement (express it in terms of ψ_{nlm}).

Possibly useful information:

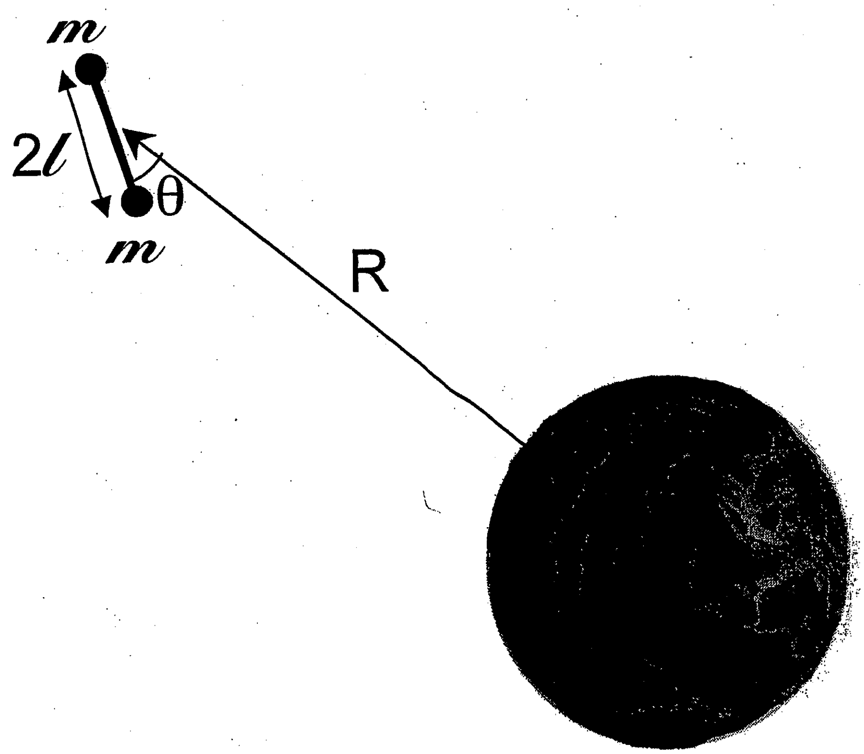
$$\hat{L}_\pm Y_l^m=\hbar\sqrt{l(l+1)-m(m\pm1)}\,Y_l^{m\pm1}$$

with Y_l^m the spherical harmonics and $\hat{L}_\pm=\hat{L}_x\pm i\hat{L}_y$ the raising and lowering operators for orbital angular momentum.

5. (15 points) Automatic stabilization of the orientation of orbiting satellites utilizes the torque from the Earth's gravitational pull on a non-spherical satellite in a circular orbit of radius R. Consider a dumbbell-shaped satellite consisting of two point masses of mass m connected by a massless rod of length 2ℓ much less than R where the rod lies in the plane of the orbit (see Figure). The orientation of the satellite relative to the direction toward the Earth is measured by angle θ. Motion only occurs in the plane of the orbit.

   (a) (10 points) Determine the value of θ for the stable orientation of the satellite.

   (b) (5 points) Show that the angular frequency of small-angle oscillations of the satellite about its stable orientation is $\sqrt{3}$ times the orbital angular velocity of the satellite.



6. (15 points) Two long (ignore end effects) cylindrical conductors of length l and negligible thickness and radii a and b $(a<b)$ are arranged coaxially as shown. The conductors are connected to a battery that maintains a constant potential difference V between the cylinders.

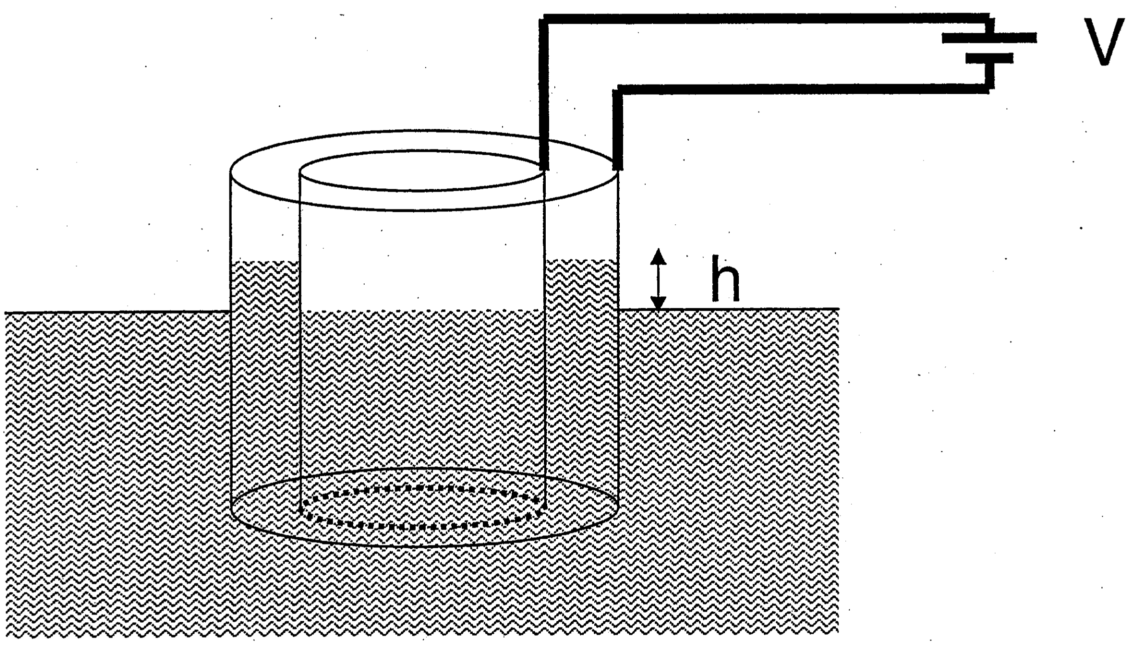
(a) (2 points) Find the electric field in the gap between the cylinders in terms of the radial distance r and the potential V before the cylinders are partly submerged in fluid, while there is still air everywhere between the cylinders.

(b) (4 points) The assembly is now partly submerged into a dielectric fluid of density ρ and dielectric constant ε and the fluid rises in the gap between the cylinders. Consider that the dielectric fluid rises a distance z (presently an unknown distance) and calculate W_E the energy content of the electric field as a function of z and dW_E the change in this energy for a change in the height of the fluid, dz.

(c) (4 points) As the fluid rises a distance dz the charge on the capacitor changes an amount dQ. Calculate dQ in terms of V, ε, a and b. Calculate the work done by the battery $dW_B = VdQ$ as the fluid rises a height dz.

(d) (5 points) Calculate the height h that the fluid rises.

Hint: Consider energy conservation in the form $dW_B = dW_E + dW_M$ where dW_B is energy drawn from the battery, dW_E is energy stored in the electric field and dW_M is the mechanical energy increase in the fluid as it rises in the gravitational field.



Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

WEDNESDAY, JAN. 9, 2008
9:00 A.M. — 1:00 P.M.

ROOM 224 PHYSICS BUILDING

INSTRUCTIONS: This paper contains six problems. The first four problems are worth 10 points per problem and the last two are worth 15 points. You are to solve all six using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

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1. (10 points) In a particular liquid, the drag force experienced by plastic spheres moving with speed v with respect to the liquid is

$$f = bv + cv^2$$

where b and c are positive constants for a given sphere, ρ_s is the density of the sphere, and ρ_l is the density of the fluid. Assume that $\rho_s < \rho_l$.

(a) (7 points) Determine the velocity as a function of time for spheres of radius R starting at rest.

(b) (2 points) Find the velocity for the limiting cases of short and long times ($t \to 0$ and $t \to \infty$) and briefly explain why these results are expected.

(c) (1 points) Assume that the constants b and c are related to the sphere's radius R by $b = \beta R$ and $c = \gamma R^2$ where β and γ are constants independent of radius. Will larger or smaller spheres rise faster? Explain why.

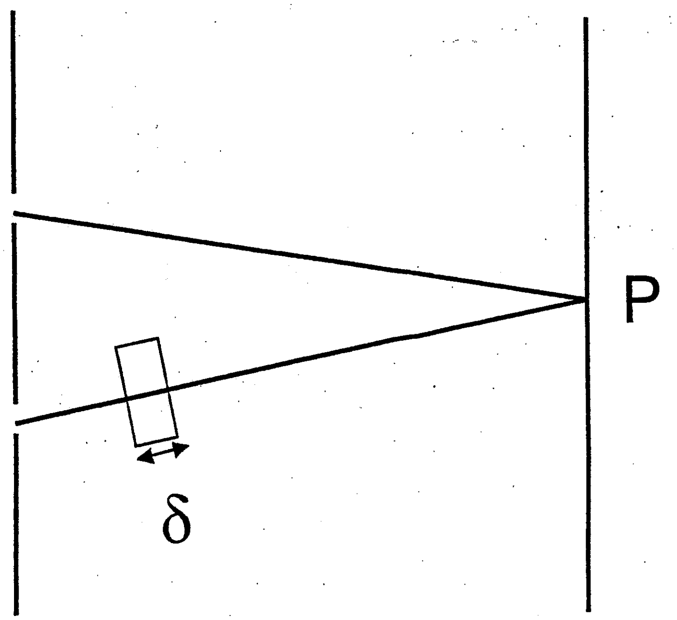
2. (10 points) The diagram below shows a standard double-slit experiment in which coherent monochromatic light of wavelength λ from a distant source is incident upon two narrow slits. The resultant interference pattern is viewed on a distant screen. A thin piece of glass of thickness δ and index of refraction n is placed between one of the slits and the screen, perpendicular to the light path for the point P. Assume that the glass does not absorb or reflect any light. The point P is midway between the two slits.

If the intensity at the point P for $\delta = 0$ is I_0:

(a) (3 points) What is the intensity at the point P as a function of the thickness δ?

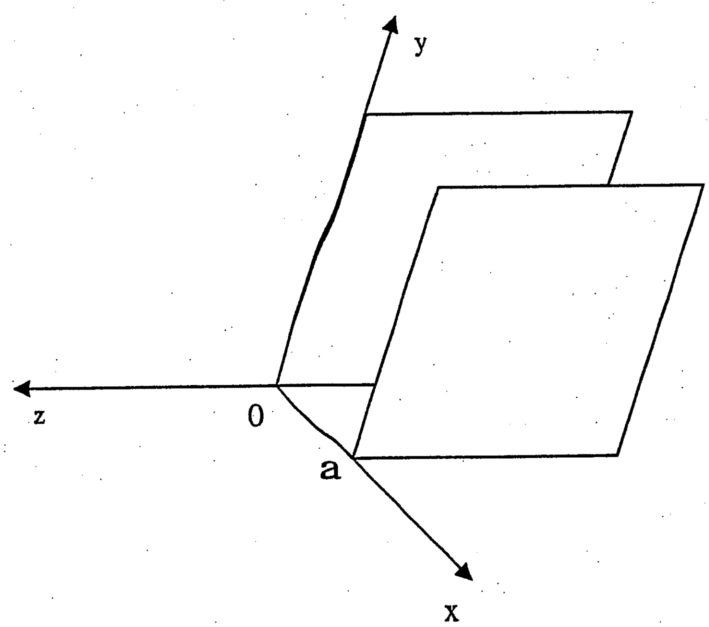
(b) (3 points) For what values of δ is the intensity at point P a minimum?

(c) (4 points) Suppose the width of the top slit is increased so that the amplitude of the light falling on the screen from this slit is doubled. What is the intensity now at point P as a function of δ?



3. (10 points). A semi-infinite channel is formed from three conducting plates all of which extend indefinitely in the positive and negative z-direction and satisfy:

$x=0, \ y>0$
$x=a, \ y>0$
$y=0, \ a>x>0$



The two plates parallel to the y-z plane are fixed at zero electrostatic potential $\Phi=0$, and the plate which lies in the x-z plane is fixed at an electrostatic potential of $\Phi=V$, where V is a constant.

Take note of the translational symmetry in the z-direction and find the electrostatic potential for $a>x>0$ and $y>0$.

4. (10 points) A particle of mass m is confined in a one-dimensional infinite square well, with potential $V(x)=0$ for $0<x<L$ and $V(x)=\infty$ elsewhere.

(a) (2 points) What are the energy eigenvalues and normalized eigenfunctions of this system?

(b) (5 points) Given the following initial wave function at $t=0$:

$$\Psi(x,t=0)=\left(\frac{8}{5L}\right)^{1/2}\left[1+\cos\left(\frac{\pi x}{L}\right)\right]\sin\left(\frac{\pi x}{L}\right),$$

what is the wave function at a later time $t=t_0>0$?

(c) (2 points) What is the expectation value of the energy at $t=0$, given the above initial condition?

(d) (1 point) What is the expectation value of the energy at time $t=t_0>0$?

5. (15 points) The elasticity of a rubber band can be described in terms of a one-dimensional model of N "molecules" linked together end-to-end with N fixed. All of the molecules in the chain align along a single axis, which we may call the x-axis, and each successive molecule in the chain is equally likely to extend the chain in either the $+x$ or $-x$ direction, as shown below. Taking the length of a molecule to be d, the contribution of each molecule to the displacement between the two ends of the chain is therefore equally likely to be $+d\hat{\mathbf{x}}$ or $-d\hat{\mathbf{x}}$.

(a) (5 points) Show that the number of arrangements that give an overall length of $L = 2md$ is given by

$$g(N,m) = \frac{2(N!)}{(N/2+m)!(N/2-m)!},$$

where m is a positive integer. Clearly explain the reasoning to get this result.

(b) (3 points) For $m \ll N$ the number of arrangements is approximately

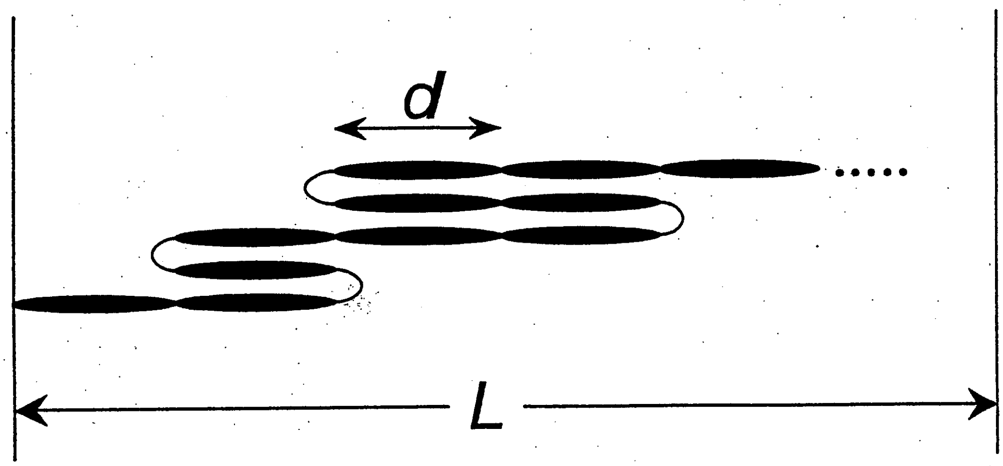
$$g(N,m) \approx g(N,0)e^{-2m^2/N}.$$

Find the entropy of the system as a function of L for $N \gg 1$ and $L \ll Nd$.

(c) (3 points) Find the force required to maintain the length L for $L \ll Nd$.

(d) (3 points) Express the length as a function of the applied force f and temperature τ for arbitrary length L, but still assuming $N \gg 1$.

(e) (1 point) Under what condition does your result yield Hooke's law?



6. (15 points) Consider a system of two electrons bound to a proton by the Coulomb interaction. Neglect the Coulomb repulsion between the two electrons. Also neglect the motion of the proton, and only consider the electronic states.

(a) (3 points) What are the allowed values of the total spin quantum number s and the spin z-component quantum number m_s for this two-electron system? Express the corresponding spin states in terms of $\chi_+^{(i)}, \chi_-^{(i)}$, which are eigenspinors of the z-component of spin for electron i =1,2.

(b) (3 points) What is the ground state energy? Also write down the general form of the total wave function in the ground state (in terms of the spatial wave functions ψ_{nlm} and the spinor).

(c) (9 points) Now consider a weak potential between the two electrons, with the form

$$V(\mathbf{r}_1 - \mathbf{r}_2) = V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2.$$

Here V_0 is a constant, and $\hat{\mathbf{S}}_1, \hat{\mathbf{S}}_2$ are the spin operators for electrons 1 and 2 respectively. Treat the above potential as a small perturbation $\hat{H}'$ to the original Hamiltonian, and use first-order perturbation theory to calculate the change in the ground state energy.

Possibly useful information: the eigenfunction of the hydrogen atom ground state is

$$\psi_{100}(r, \theta, \varphi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0},$$

with a_0 the Bohr radius.