CLASSICAL EXAM I

08/26/2002

Exam 1, Problem 2

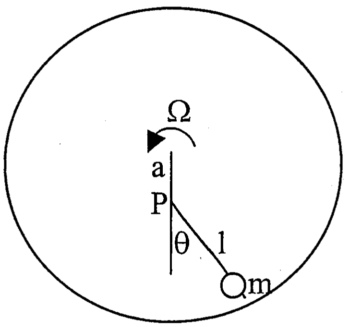
A horizontal turntable rotates at constant angular speed Ω. A distance a away from the center is the pivot point P of a pendulum consisting of a mass m at the end of a rigid massless rod of length l. Let θ be the angle made by the pendulum with a line drawn from the center of the turntable through P extended, as shown. Neglect friction.

(3 pts.) (a) Find the equation of motion for θ. Do not assume small θ.

(3 pts.) (b) Find an expression that is a constant of the motion.

(2 pts.) (c) Explain how your results for (a) and (b) would change for the special case where $a = 0$.

(2 pts.) (d) Show that (a) and (b) are of exactly the same form you would have for a vertical pendulum in a uniform gravitational field g. What does g correspond to in your case ?



Exam 1, Problem 4

In the early universe electromagnetic radiation and matter were strongly coupled. As the universe expanded and cooled to about 3000 K, neutral atoms began to form and photons and matter decoupled. From then on, the background radiation continued to cool, and is today measured as the 'cosmic microwave background (CMB)' at a temperature of 2.728 K and has been shown to have the characteristic energy distribution of a blackbody radiation at that temperature.

(3 pts.) (a) The free energy of a volume V filled with blackbody radiation at temperature T is given by:

$$F = -\alpha VT^4 \qquad (1)$$

Where α is a known constant.
If the universe expanded adiabatically (isentropically), what is the volume of the universe (V_{now}) compared to the volume when radiation and matter decoupled, V_0 (i.e. find V_{now}/V_0) ?
Hint: First find the entropy of radiation using Eq. (1)

(2 pts.) (b) What is the internal energy of a volume V filled with blackbody radiation at temperature T in terms of V, T, and α ?

(2 pts.) (c) Find an expression for radiation pressure.

(3 pts.) (d) Find an expression for the work done by the background radiation to adiabatically expand the universe.
Hint: Convert the integral over dV to an integral over dT using results of part (a)

E & M; EXAM II
8/28/2002

__Exam 2, Problem 2__

The semi-classical model of a spinning electron can be thought of as follows. A uniform charged solid sphere of radius R and mass M carries a total charge Q and is spinning with an angular velocity ω about the *z-axis*.

(5 pts.) (a) Show that the magnetic dipole moment of the sphere is:

$$\vec{m} \equiv \tfrac{1}{5} Q\omega R^2 \hat{z}$$

(3 pts.) (b) Calculate the gyromagnetic ratio γ of the sphere, which is defined as the ratio of its magnetic dipole moment to its angular momentum.

(2 pts.) (c) A precise calculation of the electron's magnetic dipole moment using relativistic quantum electrodynamic yields $1\ \mu_B$ (Bohr magneton). Show that the above semiclassical model gives a value off by a factor 2 (i.e. the g-factor).

Exam 2, Problem 4

A mode-locked laser can be thought of conceptually as a short pulse of light bouncing back and forth between two parallel mirrors separated by a distance L large compared to the length of the light pulse. One of the mirrors is semi-transparent. Each time the light pulse reflects from it a fraction of the light exits from the laser as the output. (The light energy lost in the output is replenished by an active 'gain-medium' situated between the mirrors). The equal-intervaled pulses of this laser can therefore be used as the ticks of a clock.
Without invoking any special relativity formulas, analyze the time interval between successive pulses in the following three different states of motion of the laser (clock).

(1 pt.) (a) The clock is at rest. Calculate the time interval between successive ticks in the rest frame of the clock t_{rest}.

(3 pts.) (b) The clock is moving with velocity v in a direction perpendicular to the length of the clock. Find the time interval t_{perp}.

(3 pts.) (c) The clock is moving with velocity v in a direction parallel to the length of the clock. Find the time interval t_{para}.

(3 pts.) (d) Now use special relativity and show that t_{para} and t_{perp} are the same and relate them to t_{rest}.

QUANTUM EXAM III
8/30/2002

Exam 3, Problem 2

The two-dimensional simple harmonic oscillator has the Hamiltonian

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}m\omega^2(x^2 + y^2)$$

(4 pts.) (a) Write down the energy of the two-dimensional oscillator using what you know about the one-dimensional oscillator. What is the degeneracy of each energy level ?

(2 pts.) (b) Show that the ground state is an eigenstate of any operator that commutes with H.

(2 pts.) (c) Show that [L,H] = 0, where the angular momentum operator $L = xp_y - yp_x$

(2 pts.) (d) Construct eigenfunctions of L from the first excited states.

Hint: convert the explicit form of the harmonic oscillator wavefunction into plane polar coordinates and use:

$$L = \frac{\hbar}{i}\frac{\partial}{\partial\theta}$$

where $x = r\cos\theta$ and $y = r\sin\theta$.

The eigenfunctions of the one-dimensional harmonic oscillator are:

$$\Psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right) \exp\left(-\frac{m\omega x^2}{\hbar}\right)$$

where $H_0(u) = 1$, $H_1(u) = 2u$, $H_2(u) = 4u^2 - 2, \ldots\ldots$

Exam 3, Problem 4

A spin ½ particle is in the state:

$$\chi_z = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

in the z-representation for which

$$S_z = \frac{\hbar}{2}\sigma_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

(2 pts.) (a) What are the possible results and their probabilities for a measurement of S_z ?

(2 pts.) (b) Show that there is a direction $\hat{z}'$ such that in the z'-representation the state is

$$\chi_{z'} = \begin{pmatrix}1\\0\end{pmatrix}$$

Now suppose that the particle is in a static uniform magnetic field

$$\vec{B} = B_0\hat{z}$$

so that the Hamiltonian is:

$$H = \mu_B B_0 \sigma_z$$

where $\mu_B = \dfrac{\hbar e}{2mc}$. (You may leave your answers in terms of μ_B without making that substitution.)

(2 pts.) (c) If the particle is in the state χ_z at time $t = 0$, what state is it in at time t and what is the expectation value of $\vec{S}$ as a function of time ?

(2 pts.) (d) What are the energies and the corresponding eigenfunctions of the eigenstates of this system ?

(2 pts.) (e) If a weak sinusoidal magnetic field $\vec{B}_{rf}$, whose magnitude is much less than B_0, is to be applied, which is (are) the most effective direction(s) to choose for $\vec{B}_{rf}$ to produce transitions?